Ex N 01.

$$\text{State 1 } \{\text{\_\_\_\_ isothermal expansion \_\_\_\_ State 2 }\} \quad \{\text{\_\_\_\_ isothermal compression \_\_\_\_}\}$$

$$V_1 = 2 \text{ liters} \quad T_1 = 298 \text{ K} \quad P_1 = 5 \text{ atm}$$

$$V_2 = 10 \text{ liters} \quad T_2 = T_1 = 298 \text{ K} \quad P_2 = ?$$

The transformation is **isothermal** ($T = \text{constant}$)

Given: $P_1V_1 = P_2V_2 = nRT \Rightarrow P_2 = P_1V_1/V_2 = 5 \times 2/10 = 1 \text{ atm}$

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a. Work done (reversible isothermal expansion)

Reversible transformation (transformation that passes through **equilibrium**) $\Rightarrow P_{\text{ext}} = P_{\text{gaz}}$ at each instant

$$W_{\text{rev}}(1 \to 2) = -\int_1^2 P_{\text{ext}} dV = -\int_1^2 P_{\text{gaz}} dV = -\int_1^2 \frac{nRT}{V} dV = -nRT \ln\left(\frac{V_2}{V_1}\right) = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$W_{\text{rev}}(1 \to 2) = -(5 \times 1.013 \times 10^5)(2 \times 10^{-3}) \ln\left(\frac{10}{2}\right) = -1630.4 \text{ J}$$

b. Work done (irreversible expansion)

Irreversible transformation (very rapid transformation) $\Rightarrow P_{\text{ext}} = P_{\text{final}} = P_2$, so we can assume that the isochoric transformation is followed by an isobaric transformation.

$$W_{\text{irr}}(1 \to 2) = -\int_1^2 P_{\text{ext}} dV = -\int_1^2 P_{\text{final}} dV = -P_2 \int_1^2 dV = -P_2(V_2 - V_1)$$

$$W_{\text{irr}}(1 \to 2) = -(1 \times 1.013 \times 10^5)((10 - 2) \times 10^{-3}) = -810.4 \text{ J}$$

We can note that the **work done in irreversible expansion** is smaller than in reversible expansion:

$$W_{\text{irr}}(1 \to 2) < W_{\text{rev}}(1 \to 2)$$

c. Work done (reversible isothermal compression)

Reversible transformation (very slow transformation) $\Rightarrow P_{\text{ext}} = P_{\text{gas}}$ at each instant

$$W_{\text{rev}}(2 \to 1) = -\int_2^1 P_{\text{ext}}\, dV = -\int_2^1 P_{\text{gaz}}\, dV = -\int_2^1 \frac{nRT}{V}\, dV = -nRT \ln \frac{V_1}{V_2}$$

$$= P_1V_1 \ln \frac{V_1}{V_2} = -(5 \times 1.013 \times 10^5)(2 \times 10^{-3}) \ln \frac{2}{1} = 1630.4\, J$$

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d. Work done (irreversible isothermal compression)

Irreversible transformation (very fast transformation) $\Rightarrow P_{\text{ext}} = P_{\text{final}} = P_2$, so we can assume that it is an isochoric transformation followed by an isobaric transformation:

$$W_{\text{irr}}(2 \to 1) = -\int_2^1 P_{\text{ext}}\, dV = -\int_2^1 P_{\text{final}}\, dV = -P_{\text{final}} \int_2^1 dV = -P_1(V_1 - V_2)$$

$$= -(5 \times 1.013 \times 10^5)((2 - 10) \times 10^{-3}) = 40523\, J$$

EXNO2:

Kinetic energy is the only form of mechanical energy that the wind possesses, and it can be completely converted. Therefore, the potential power of the wind is its kinetic energy, which is equal to $V^2/2$ per unit of mass.

Using the **1st law of thermodynamics** (open system):

a. Mechanical energy per unit mass

$$Q + W = \Delta H + \Delta E_{\text{chim}} + \Delta E_{\text{pot}} + \Delta E_{\text{cin}} + \dots$$

$$w = \Delta e_{\text{cin}} = \frac{V_{\text{out}}^2}{2} - \frac{V_{\text{in}}^2}{2} = \frac{V_{\text{in}}^2}{2} = \frac{(12\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)$$

$$w = 0.07\ \text{kJ/kg}$$

b. Actual wind energy production potential of the wind turbine

$$\dot{m}_{\text{air}} = \rho \times V \times A = \rho \times V \times \frac{\pi D^2}{4} = (1.25\ \text{kg/m}^3) \times (12\ \text{m/s}) \times \frac{\pi(50\ \text{m})^2}{4} = 29,437.5\ \text{kg/s}$$

$$\dot{W}_{\text{max}} = \dot{m}_{\text{air}} \times w = (29,437.5\ \text{kg/s}) \times (0.072\ \text{kJ/kg}) = 2,119.5\ \text{kW}$$

c. Actual electricity production

$$W_{\text{elec}} = \eta_{\text{wind turbine}} \times \dot{W}_{\text{max}} = (0.30)(2,119.5\ \text{kW}) = 635.85\ \text{kW}$$

EX. N°3

a. Overall efficiency of the pump-motor unit

$$Q + W = \Delta H + \Delta E_{\text{chim}} + \Delta E_{\text{pot}} + \Delta E_{\text{cin}} + \dots$$

$$w = \Delta e_{\text{pot}} = (gz_2 - gz_1) = gz_2 = (9.81\frac{m}{s^2})(20m)\left(\frac{1\frac{kJ}{kg}}{1000\frac{m^2}{s^2}}\right)$$

$$w = 0.1962\,kJ/kg$$

$$\dot{m}_{\text{water}} = \rho \times \dot{V}_{\text{water}} = \left(1000\frac{kg}{m^3}\right) \times \left(0.07\frac{m^3}{s}\right) = 7\,kg/s$$

$$\dot{W}_{\text{ideal}} = \dot{m}_{\text{water}} \times w = (7\,kg/s) \times (0.1962\,kJ/kg) \approx 13.7\,kW$$

$$\eta_{\text{pump-motor}} = \frac{\dot{W}_{\text{ideal}}}{\dot{W}_{\text{elec}}} = \frac{13.734\,kW}{20.4\,kW} = 0.6732 \approx 67.32\%$$

b. The pressure difference between the pump inlet and outlet

$$\dot{W}_{\text{ideal}} = \dot{m}_{\text{water}}(e_{\text{out}} - e_{\text{in}}) = \dot{m}_{\text{water}}\frac{P}{\rho} \implies \Delta P = \frac{\dot{W}_{\text{ideal}}}{\dot{V}_{\text{eau}}}$$

$$\Delta P = \frac{13.734\,kW}{0.07\frac{m^3}{s}}\left(\frac{1\,kPa \cdot m^3}{1\,kJ}\right) \approx 196.2\,kPa$$

Ex N 04

Reminder!

$$\Delta S_{\text{system}} = \Delta S_{\text{exchanged}} + \Delta S_{\text{created}}$$

For any transformation: $dS_{\text{sys}} = \frac{\delta Q}{T}$

1. For a reversible transformation: $\Delta S_{\text{created}} = 0 \Rightarrow \Delta S_{\text{system}} = \Delta S_{\text{exchanged}}$
2. For an irreversible transformation: $\Delta S_{\text{created}} > 0 \Rightarrow \Delta S_{\text{system}} > \Delta S_{\text{exchanged}}$

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1. During irreversible isothermal expansion

a. Variation of entropy of an ideal gas

$$dS_{\text{sys}} = \frac{\delta Q}{T} \quad \text{and} \quad dU = \delta Q + \delta W$$

Isothermal transformation $\Rightarrow dU = nC_v dT = 0 \Rightarrow \delta Q = -\delta W = PdV \Rightarrow dS_{\text{sys}} = \frac{PdV}{T} = \frac{nRTdV}{TV} = nR\frac{dV}{V}$

Entropy of the system: $\Delta S_{\text{system}}(1 \to 2) = nR\int_{V_1}^{V_2} \frac{dV}{V} = nR\ln\frac{V_2}{V_1} = 2 \times \ln\frac{50}{30} = 2.04 \text{ Cal. K}^{-1}$

Exchanged entropy: $\Delta S_{\text{exchanged}}(1 \to 2) = \frac{Q_{\text{irr}}}{T} = \frac{W_{\text{irr}}}{T} = P_{\text{final}}\frac{\Delta V}{T} = nR\frac{\Delta V}{V} = 2 \times \frac{20}{50} = 1.6 \text{ Cal. K}^{-1}$

b. Created entropy

$$\Delta S_{\text{created}} = \Delta S_{\text{system}} - \Delta S_{\text{exchanged}} = 2.04 - 1.6 = 0.44 \text{ Cal. K}^{-1}$$

2. During non-isothermal expansion

$$dS_{\text{sys}} = \frac{\delta Q}{T} = \frac{dU - \delta W}{T} = n\left(\frac{C_v dT}{T} + \frac{RTdV}{TV}\right) \Rightarrow \Delta S = \int_{T_1}^{T_2} n\left(C_v\frac{dT}{T} + R\frac{dV}{V}\right)$$

$$= 2\left(\ln\frac{290}{300} + 2\ln\frac{50}{30}\right) = 1.7 \text{ Cal. K}^{-1}$$

Ex. N°5

1. Reversible adiabatic expansion:

a. Final temperature of the gas

$$dW + dQ = dU = nC_v dT, \quad dQ = 0 \text{ (adiabatic)} \implies -PdV = C_v dT = -\frac{RT}{V}dV = -(C_p - C_v)\frac{dV}{V}dT$$

$$\frac{dT}{T} = -\frac{(C_p - C_v)}{C_v}\frac{dV}{V} \quad \implies \quad \frac{dT}{T} = -(\gamma - 1)\frac{dV}{V} \quad \implies \quad \frac{dT}{T} = (1 - \gamma)\frac{dV}{V}$$
