University of M'sila (2025/2026)
Faculty of technology — 2nd year License (S4) : Mechanical Engineering
Module : **Mathematics 4** — Duration : 2 hours

Final Exam

Course Questions. (4 points = 1+1+1+1)

1) Let $f(z) = u(x, y) + iv(x, y)$ be a complex function. Give the definition of :
   - ◇ Cauchy-Riemann equations C.R.E for the function f.
   - ◇ Pole of order m for the function f.
   - ◇ Laurent series for the function f at point z_0.

2) Calculate : $\lim\limits_{z \to i} \dfrac{z^4 - 1}{z^2 + 1}$.

Exercice 1. (6 points = 2+1.5+2.5)

1) Solve in $\mathbb{C}$ the equation : $e^z = -\sqrt{3} - i$.

2) Compute the value of : $i^{\sqrt{2}}$.

3) Find the set of values of z for : $\text{Img}(z^2) < 0$. Graph this set in the complex plane.

(Hint : Take $z = x + iy$).

Exercice 2. (6 points = 2+2+2)

1) Show that the function u defined by $u(x, y) = x^2 - y^2 + xy$ is harmonic.

2) Find a function v such that the function $f(z) = u(x, y) + iv(x, y)$ is analytic.

3) What is the type of singularities of $g(z) = \dfrac{\sin(z) - 2z}{z}$.

Exercice 3. (4 points = 2+2)

1) Calculate $\displaystyle\int_\gamma y \, dz$, where γ is the circle of center $(0, 0)$ and radius 5.

2) Find the Laurent series of $f(z) = \dfrac{1}{(z+1)(z+3)}$, for $1 < |z| < 3$.

(Hint : $\dfrac{1}{1+z} = 1 - z + z^2 - z^3 + ... + (-1)^{n-1} z^n + ...$ valid for $|z| < 1$).

Good luck