Chapter 4: Lateral-torsional buckling

4.1. Introduction

Beams subjected to bending with no lateral bracing to resist lateral deformation are at risk of lateral-torsional buckling (Figure 4.1), even if the bending stress has not yet reached its yield limit.

Lateral-torsional buckling is a lateral instability phenomenon (a distortion) characterized by the buckling of the compressed portions of a section flexed about its strong axis (y-y). This lateral instability (relative to the z-z axis) means that sections where the minor-axis inertia (z-z) is significantly lower than the major-axis inertia (y-y)—such as IPE profiles—are more sensitive to buckling than sections where these inertias are closer (such as HE profiles or tubular sections).

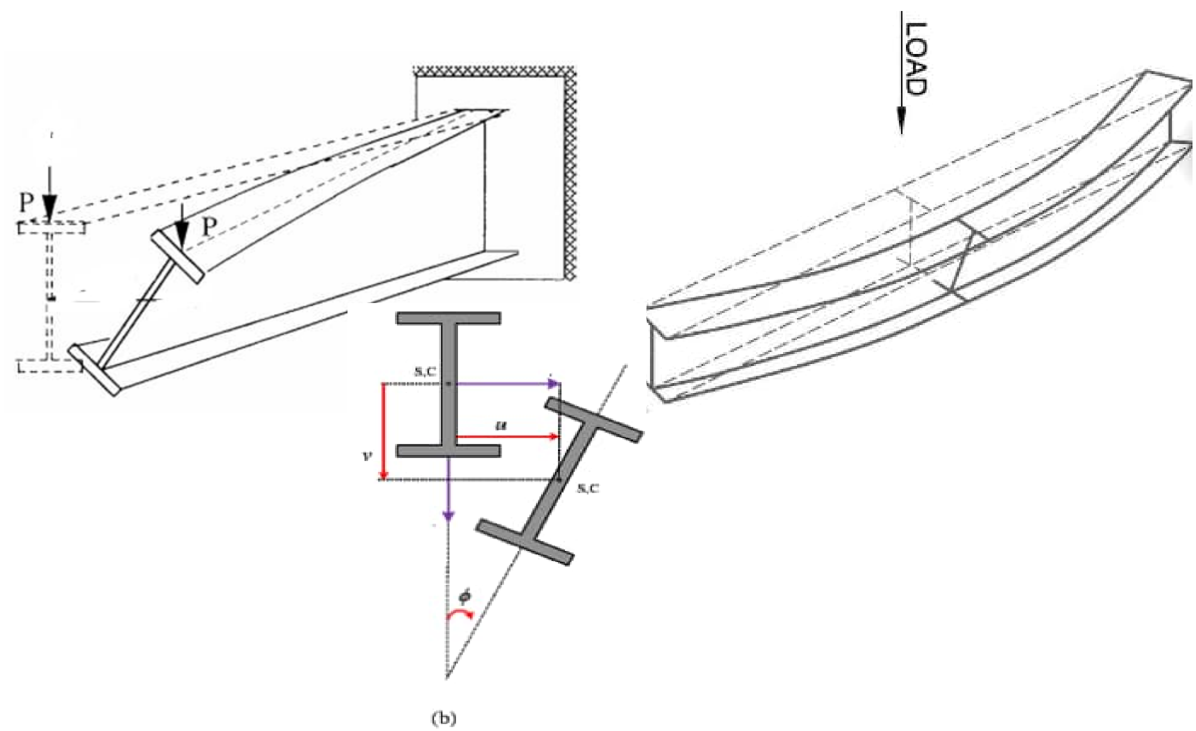


Figure 4.1. Lateral-torsional buckling of the I-beam

Regarding this phenomenon, the Algerian regulation CCM97 (or Eurocode 3) maintains the same verification method as for flexural buckling (utilizing the non-dimensional slenderness ratio $\bar{\lambda}$). Beams with a lateral-torsional relative slenderness exceeding 0.4 ($\bar{\lambda}_{LT}$> 0.4) are susceptible to buckling. Therefore, it must be verified that their yield strength is not exceeded under the effect of the distortion generated by this lateral instability.

4.2. Verification of lateral-torsional buckling stability

The non-dimensional slenderness for lateral-torsional buckling $\bar{\lambda}_{LT}$ is the ratio between the geometric slenderness of the beam for lateral-torsional buckling λ_{LT} and the elastic slenderness:

$$\bar{\lambda}_{LT} = \sqrt{\frac{\beta_w . W_{pl,y} . f_y}{M_{cr}}} = \frac{\lambda_{LT}}{\lambda_l} . \sqrt{\beta_w} \quad (4.1)$$

M_{cr}: is the elastic critical moment for lateral-torsional buckling

$$\lambda_l = \pi \left[\frac{E}{f_y}\right]^{0.5} = 93.9 \times \varepsilon$$

With $$\varepsilon = \left[\frac{235}{f_y}\right]^{0.5}$$

$$\lambda_{LT} = \frac{\frac{L}{i_z}}{\sqrt{C_1 . \left[1 + \frac{1}{20} . \left(\frac{\frac{L}{i_z}}{\frac{h}{t_f}}\right)^2\right]}}$$

Coefficient C1 depends on the loading and support conditions. Its values are obtained from the tables provided in the following section.

The risk of lateral-torsional buckling must be considered if:

$$\bar{\lambda}_{LT} > 0.4 \quad (4.2)$$

In this case, it must be verified that:

$$M_{Sd} \leq M_{b,Rd} = \chi_{LT}.\beta_W.W_{pl,y}.\frac{f_y}{\gamma_{M1}} \tag{4.3}$$

Where: χ_{LT} is the reduction factor for lateral-torsional buckling.

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1 \tag{4.4}$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2\right] \tag{4.5}$$

Where: α_{LT} is the imperfection factor

Table 4.1- Imperfection factor α_{LT} and buckling curve

Cross-section	Buckling curve	α_{LT}
Rolled I-sections	a	0.21
Welded I-sections	c	0.49

Values for β_W:

- $\beta_W = 1$: For Class 1 and 2 cross-sections
- $\beta_W = \frac{W_{el,y}}{W_{pl,y}}$: For Class 3 cross-sections
- $\beta_W = \frac{W_{eff,y}}{W_{pl,y}}$: For Class 4 cross-sections

4.2.1- Calculation of the elastic critical moment for lateral-torsional buckling

For a constant cross-section, the elastic critical moment for lateral-torsional buckling is calculated for the gross section (including Class 4 sections with I_t = 0). It is primarily governed by the lateral moment of inertia of the section I_z combined with those of the warping torsion I_w and St. Venant torsion I_t. It is as follows:

$$M_{cr} = C_1 \frac{\pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g - C_3 \cdot z_j)^2} - (C_2 \cdot z_g - C_3 \cdot z_j) \right\} \quad (4.6)$$

E: Young's modulus (Modulus of elasticity).

G: Shear modulus.

I_z: Second moment of area about the weak axis.

I_t: Torsional modulus (St. Venant torsion constant).

I_w: Warping modulus.

L: Length of the beam between lateral restraints.

k and k_w: Effective length factors related to end rotations and warping.

z_g: Distance between the point of load application and the shear center.

C_1, C_2, C_3: Coefficients depending on the loading conditions and end restraint conditions.

Case of a beam subjected to transverse loading only.

The values of the factor C_1 (as well as C_2 and C_3) can be derived from table 4.2.

Case of a beam subjected to end moments.

The values of the coefficient C_1 (as well as C_2 and C_3) can be derived from table 4.3.

The factors k (and k_w) depend on the end boundary conditions, and their values are provided in table 4.4.

Table 4.2- The factor C_1 (as well as C_2 and C_3) in the Case of a beam subjected to transverse loading only

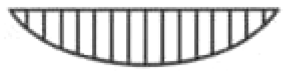
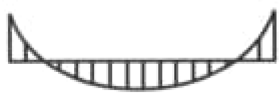
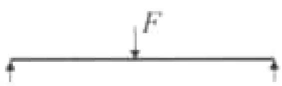
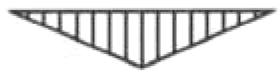
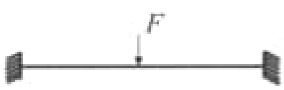
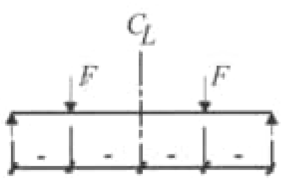
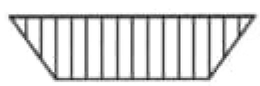
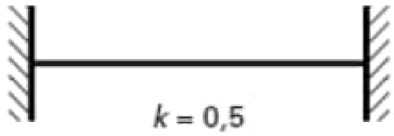
Chargement et conditions d'appuis	Diagramme de moment de flexion	Valeur de k	Coefficients		
			C_1	C_2	C_3
		1,0 0,5	1,132 0,972	0,459 0,304	0,525 0,980
		1,0 0,5	2,57 0,712	1,562 0,652	0,753 1,070
		1,0 0,5	1,365 1,070	0,553 0,432	1,730 3,050
		1,0 0,5	1,565 0,938	1,267 0,715	2,640 4,800
		1,0 0,5	1,046 1,010	0,430 0,410	1,120 1,890

Table 4.3- C1, (C2, and C3) factors for the case of end moment loading.

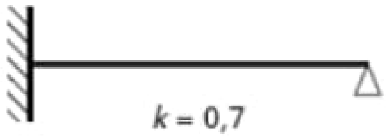
Chargement et conditions d'appuis	Diagramme de moment de flexion	Valeur de k	Coefficients C_1	C_2	C_3
M ψM For cases where k= 1.0 C_1 can be given by: $C_1 = 1.88 - 1.40\psi^2$	$\psi = +1$	1,0 0,7 0,5	1,000 1,000 1,000	-	1,000 1,113 1,144
	$\psi = +3/4$	1,0 0,7 0,5	1,141 1,270 1,305	-	0,998 1,565 2,283
	$\psi = +1/2$	1,0 0,7 0,5	1,323 1,473 1,514	-	0,992 1,556 2,271
	$\psi = +1/4$	1,0 0,7 0,5	1,563 1,739 1,788	-	0,977 1,531 2,235
	$\psi = +0$	1,0 0,7 0,5	1,879 2,092 2,150	-	0,939 1,473 2,150
	$\psi = -1/4$	1,0 0,7 0,5	2,281 2,538 2,609	-	0,855 1,340 1,957
	$\psi = -1/2$	1,0 0,7 0,5	2,704 3,009 3,093	-	0,676 1,059 1,546
	$\psi = -3/4$	1,0 0,7 0,5	2,927 3,258 3,348	-	0,366 0,575 0,837
	$\psi = -1$	1,0 0,7 0,5	2,752 3,063 3,149	-	0,000 0,000 0,000

The k factor relates to the end rotation in the plane of loading. It is analogous to the ratio of the effective buckling length to the actual length of a compression member $\frac{l_f}{L}$.

- k is taken as 0.5 when rotation is restrained by lateral support at both ends.



- k = 0.7 for lateral restraint at one end.



- k = 1 In the absence of lateral supports preventing the rotation of the beam ends.

In the case of a cantilever beam, it is evident that the ratio k = 2.

The k_w: relates to the end warping.

k_w = 0.5: for warping restraint (prevention of deformation) at both ends of the beam.

k_w = 1: for simple supports (no measures taken regarding warping at both beam supports).

k_w = 0.7: for warping restraint at one end only.

L : is the span between two beam supports or the distance between two consecutive lateral restraints of the beam flange, if they exist

4.2.2- Influence of the load application point position for a vertical load on I – Section

$z_g = z_a - z_s$

z_g **:** distance from the point of load application to the center of gravity of the section

z_a: coordinate of the point of load application.

z_S: coordinate of the shear center.

a. Load Position (z_g)

Three cases are possible for an "I" section:

- $z_g > 0$: The load is applied above the center of gravity (G). This occurs when a load is placed on the top flange.
- $z_g = 0$: The load is applied exactly at the center of gravity (G).
- $z_g < 0$: The load is applied below the center of gravity (G). This occurs when a load is suspended from the bottom flange.

b. Effect on Stability (Lateral-Torsional Buckling)

The lower part of the image shows how this position affects the rotation of the section during lateral-torsional buckling (lateral and torsional instability):

- **Load applied at the top ($z_g = +h/2$):**

  When a small rotation occurs, the point of application of the force moves further away from the vertical axis passing through the shear center (S). This creates a moment that increases the rotation. The load is referred to as "destabilizing".

- **Load applied at the center ($z_g = 0$):**

  The effect is neutral relative to the vertical position; this is the baseline reference.

- **Load applied at the bottom ($z_g = -h/2$):**

  During rotation, the point of application shifts in a way that creates a restoring moment, which tends to return the section to its equilibrium position. The load is referred to as "stabilizing".

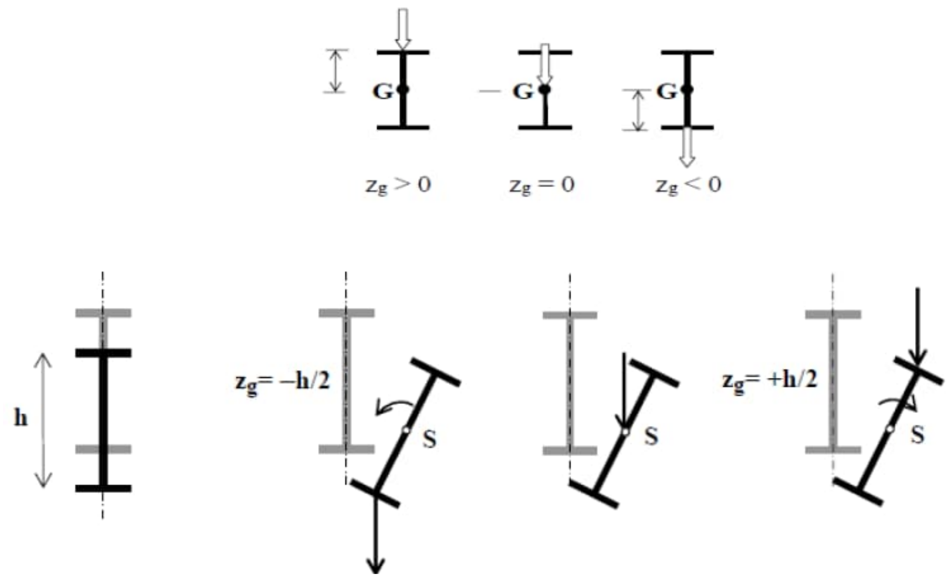


Figure 4.2. Sign of z_g as a function of the point and direction of load application

$$z_j = z_s - \frac{\int_A z \cdot (y^2 + z^2)\, dA}{2 \cdot I_y}$$

z_j : is a symmetry factor. For doubly symmetric sections, $z_j = 0$.

For mono-symmetric I-sections with unequal flanges (Figure 4.3 (a)), the approximations from Table 4.4 can be used to calculate z_j.

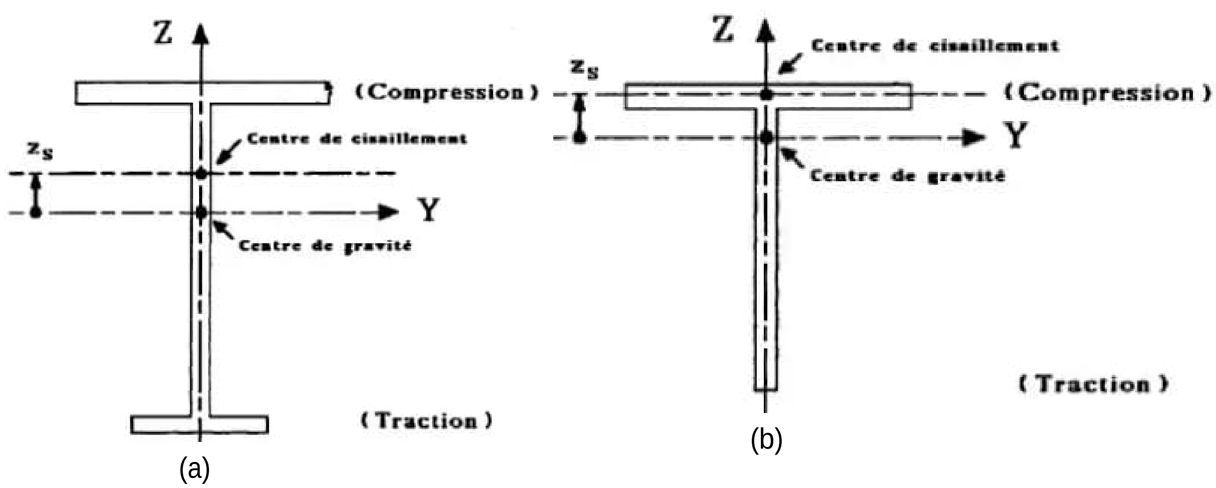


Figure 4.3. Sign convention for the determination of z_j and z_s

Table 4.4- Calculation of z_j for mono-symmetric I-sections

Profiles with	$\beta_f = \dfrac{I_{fc}}{I_{fc}+I_{ft}} < 0.5$	$\beta_f = \dfrac{I_{fc}}{I_{fc}+I_{ft}} \geq 0.5$
Unequal Flanges	$z_J = 1.(2.\beta_f - 1).\dfrac{h_s}{2}$	$z_J = 0.8.(2.\beta_f - 1).\dfrac{h_s}{2}$
Compressed flange with dropped edges	$z_J = 1.(2.\beta_f - 1).(1+\dfrac{h_L}{h}).\dfrac{h_s}{2}$	$z_J = 0.8.(2.\beta_f - 1).(1+\dfrac{h_L}{h}).\dfrac{h_s}{2}$

With:

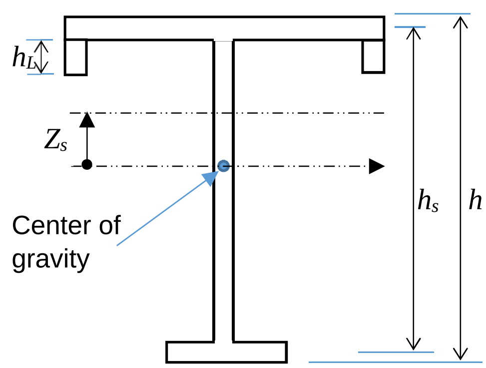
h: the height of the cross-section of the profile;

h_s: distance between the shear centers of the flanges;

h_L: the height of the dropped edges;

I_{fc}: moment of inertia of bending of the compressed flange along the axis of weak inertia of the section;

I_{ft}: moment of inertia of bending of the tensioned flange along the axis of weak inertia of the section;



G : transverse modulus of elasticity, $G = \frac{E}{2(1+\nu)} = 8077\ MPa$

I_t : torsional inertia modulus..

for plates where $t_i < \frac{b_i}{5}$, we have $I_t = \dfrac{1}{3}\sum_{i=1}^{n} b_i t_i^3$

Thus, for a built-up I or H section whose plate thicknesses satisfy the previous condition:

$$I_t = \frac{1}{3}\left(d \cdot t_w^3 + 2 \cdot b \cdot t_f^3\right)$$

For class 4 sections, the torsional moment of inertia is to be considered zero ($I_t = 0$).

I_w : warping modulus

• For doubly symmetric I-sections: $I_w = I_z \cdot \left(\dfrac{h - t_f}{2}\right)^2$

• For mono-symmetric I-sections with unequal flanges, the following expression may be used: $I_w = \beta_f \cdot (1 - \beta_f) \cdot I_z \cdot h_s^2$

I_z : second moment of area about the weak axis.

$h_s,\ I_{fc},\ I_{ft},\ \beta_f$: see the table 4.4

4.2.3 - Special Cases for the Elastic Critical Moment of Lateral-Torsional Buckling:

For a constant and doubly symmetric beam, the equation for the elastic critical moment of lateral-torsional buckling can be reduced in the following cases:

- For a constant and doubly symmetric cross-section, since $z_j = 0$, or for a cantilever beam $C_3 = 0$:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + (C_2 z_g)^2} - C_2 z_g \right\}$$

- If, in addition to the previous cases, there is a loading by end moments ($C_2 = 0$), or a load applied at the shear center ($z_g = 0$):

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z}} \right\}$$

- Under the previous cases, if there is no warping restraint at the ends ($k_w = k = 1$):

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \left\{ \sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}} \right\}$$

4.2.4- Practical calculation of lateral-torsional buckling

- Depending on the restraint and loading conditions, take the values of coefficients C_1, C_2, and C_3 from Table 4.2 or Table 4.3.
- Depending on the load position within the cross-section, calculate the z_g coordinate (Figure 4.2).
- Determine the z_j coordinate (taking into account the cross-sectional geometry) (Table 4.4).
- Determine the values of L, k, and k_w. (L is the buckling length, designated as L_{LT} in Figure 4.5).
- Calculation of the elastic critical moment M_{cr}.

- Calculate the non-dimensional slenderness for lateral-torsional buckling $\bar{\lambda}_{LT}$.

Designate the imperfection curve to be used:

- Rolled sections $\Longleftrightarrow$ Buckling curve a.
- Welded sections $\Longleftrightarrow$ Buckling curve c.

- χ_{LT} is calculated. It can also be obtained by interpolation from Table 2.6 as a function of the non-dimensional slenderness $\bar{\lambda}_{LT}$.
- Calculation of the design buckling resistance moment $M_{b,Rd}$ for verification against M_{Sd} (design bending moment).

4.2.5- Verification of stability against lateral-torsional buckling in biaxial bending

In biaxial bending, as in the case of a purlin or a side rail, the non-dimensional slenderness for lateral-torsional buckling $\bar{\lambda}_{LT}$ is calculated in the same way as previously described. If the risk of lateral-torsional buckling must be considered ($\bar{\lambda}_{LT} > 0.4$), the verification shall be performed using $M_{b,Rd}$:

a) <u>For Class 1 and 2 sections</u>: (If $\gamma_{M0} = \gamma_{M1} = 1.1$, we obtain $M_{pl,z,Rd}$)

$$\left[\frac{M_{y,Sd}}{M_{b,Rd}}\right] + \left[\frac{M_{z,Sd}}{\frac{W_{pl,z}\cdot f_y}{\gamma_{M1}}}\right] \leq 1 \Longleftrightarrow \left[\frac{M_{y,Sd}}{\chi_{LT}\cdot\frac{W_{pl,y}\cdot f_y}{\gamma_{M1}}}\right] + \left[\frac{M_{z,Sd}}{\frac{W_{pl,z}\cdot f_y}{\gamma_{M1}}}\right] \leq 1$$

b) <u>For Class 3 sections</u>: (If $\gamma_{M0} = \gamma_{M1} = 1.1$, we obtain $M_{el,z,Rd}$)

$$\left[\frac{M_{y,Sd}}{M_{b,Rd}}\right] + \left[\frac{M_{z,Sd}}{\frac{W_{el,z}\cdot f_y}{\gamma_{M1}}}\right] \leq 1 \Longleftrightarrow \left[\frac{M_{y,Sd}}{\chi_{LT}\cdot\frac{W_{el,y}\cdot f_y}{\gamma_{M1}}}\right] + \left[\frac{M_{z,Sd}}{\frac{W_{el,z}\cdot f_y}{\gamma_{M1}}}\right] \leq 1$$

c) <u>For Class 4 sections</u>: (If $\gamma_{M0} = \gamma_{M1} = 1.1$, we obtain $M_{eff,z,Rd}$)

$$\left[\frac{M_{y,Sd} + e_{Ny}\cdot N_{Sd}}{M_{b,Rd}}\right] + \left[\frac{M_{z,Sd} + e_{Nz}\cdot N_{Sd}}{\frac{W_{eff,z}\cdot f_y}{\gamma_{M1}}}\right] \leq 1$$

$$\Longleftrightarrow \left[\frac{M_{y,Sd} + e_{Ny}\cdot N_{Sd}}{\chi_{LT}\cdot\frac{W_{eff,y}\cdot f_y}{\gamma_{M1}}}\right] + \left[\frac{M_{z,Sd} + e_{Nz}\cdot N_{Sd}}{\frac{W_{eff,z}\cdot f_y}{\gamma_{M1}}}\right] \leq 1$$

4.3. Structural arrangements to prevent lateral-torsional buckling

The lateral restraint of the compression flange of a beam (Figure 4.4) makes the latter less susceptible to lateral-torsional buckling (LTB). This is due to the fact that it provides a rotational restraint (blocks rotation) and reduces the buckling length (between two successive restraints) (Figure 4.5)

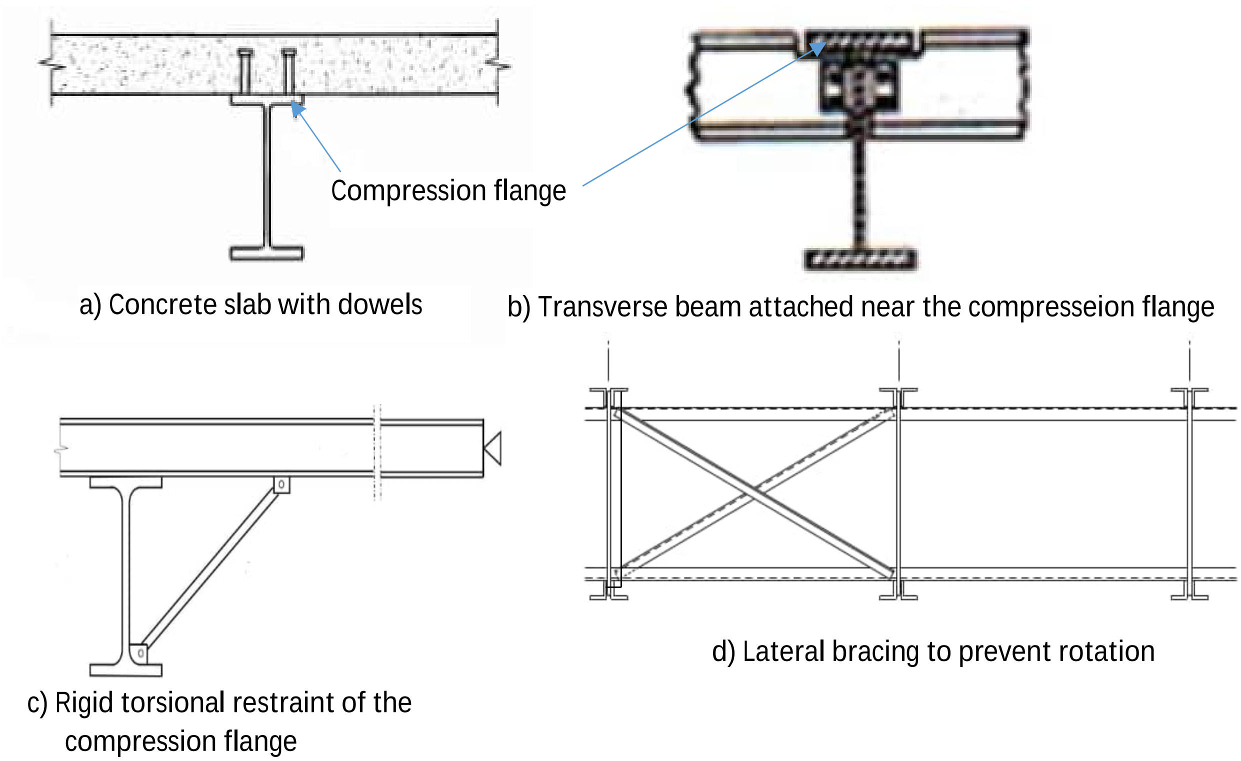


Figure 4.4. Example of lateral restraint of a beam subjected to bending

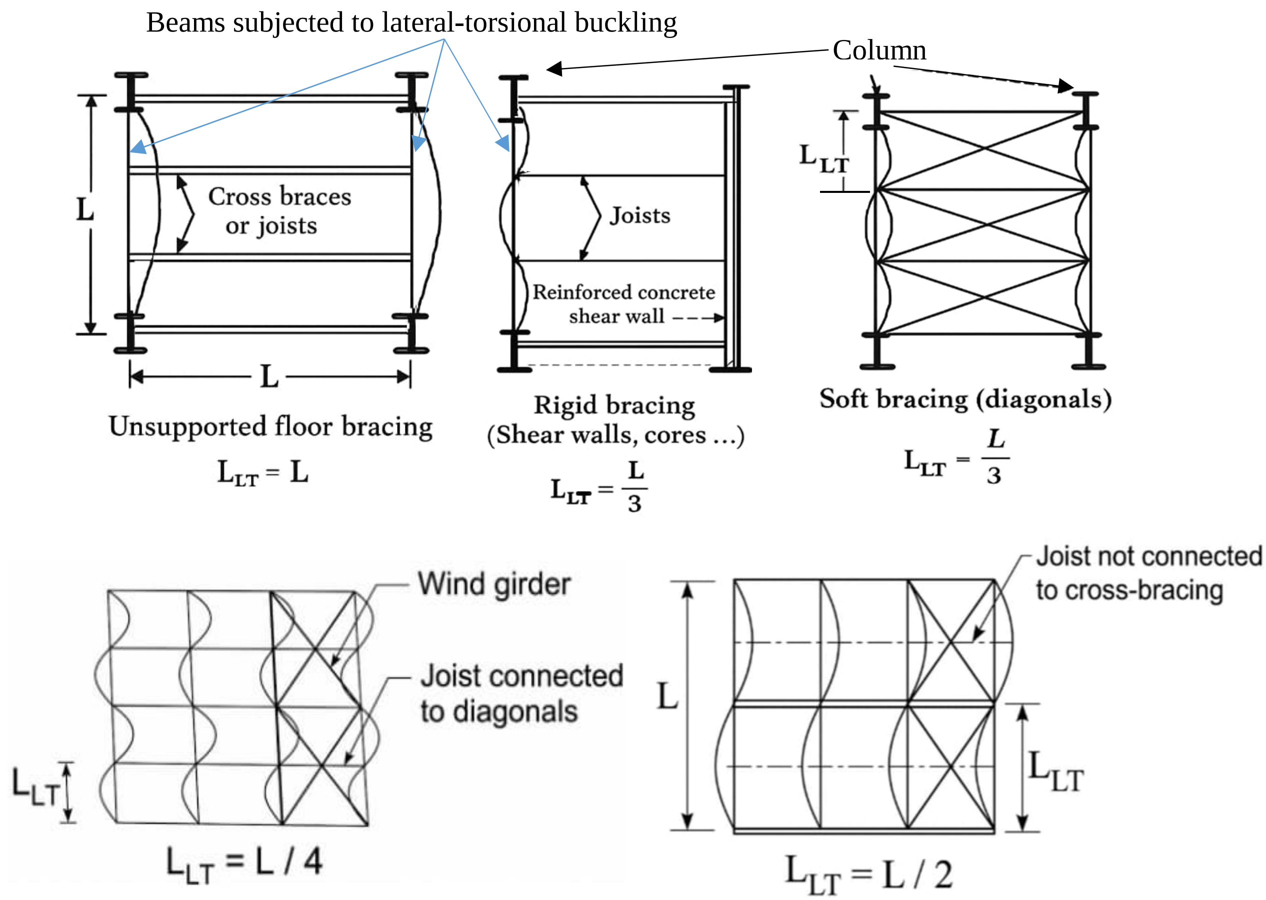


Figure 4.5. Lateral–torsional buckling length L_{LT} of a beam with joists

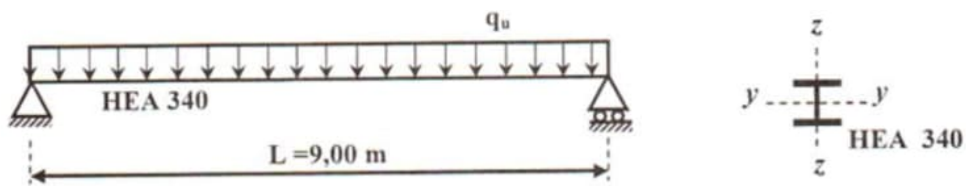
Furthermore, circular or square hollow sections, whether hot-rolled or welded, are also not susceptible to lateral–torsional buckling, due to their high torsional constant I_t and moment of area I_z, which result in a high critical moment M_{cr}.

4.4 Exercises

Exercise No. 1 (Lateral–torsional buckling check of a simply supported beam)

Consider a HEA 340 beam made of S235 steel, simply supported, with a span L = 9.00 m. It supports an ultimate uniformly distributed load applied to its top flange q_u = 20 kN/m (including the self-weight of the section).

- Verify the lateral–torsional buckling resistance of this beam.



Exercise No. 2 (Lateral-Torsional Buckling Stability Verification of a Cantilever)

Consider a cantilever beam with an HEA 340 section and a span of L = 2.00 m. It is loaded at its free end by an ultimate concentrated load applied at the center of gravity of the section, P_u = 150 kN.

The steel grade is S235. Neglecting the self-weight of the section.

- Verify the lateral-torsional buckling resistance of this cantilever.