Chapter 3 : Calculation of members subjected to combined buckling (buckling with bending)

3.1. Introduction

The columns of a steel building can be subjected to axial loads that may induce compression and transverse loads that may induce bending.

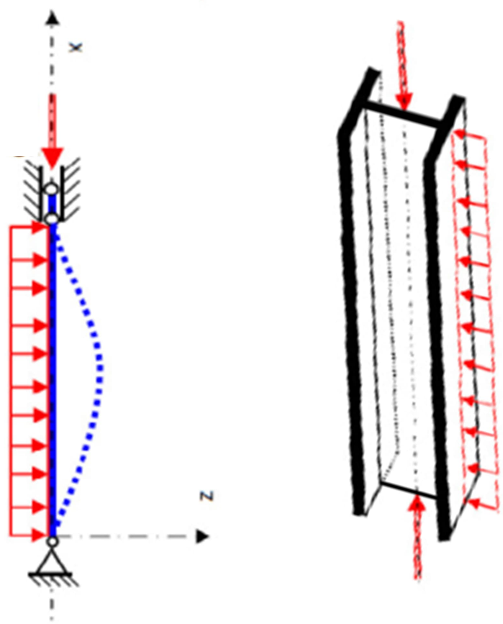


Figure 3.1. Column subjected to combined compression

3.2. Stability verification under combined loading

Columns subjected to combined compression and bending must be verified for combined buckling stability only if:

$$\bar{\lambda}_{max} > 0.2 \quad and \quad \frac{N_{sd}}{N_{b.Rd}} > 0.1$$

With: $\bar{\lambda}_{max} = \max(\bar{\lambda}_y; \bar{\lambda}_z)$

$$N_{b.Rd} = \chi_{min} \cdot \frac{A.f_y}{\gamma_{M1}} \quad and \quad \chi_{min} = \min(\chi_y; \chi_z)$$

Verification shall be performed by ensuring that:

- for Class 1 and 2 cross-sections
  - ✓ For a column subjected to N_{Sd} and $M_{y,Sd}$, it must be verified that:

$$\left[\frac{N_{Sd}}{N_{b,Rd}}\right] + \left[\frac{k_y . M_{y,Sd}}{\frac{M_{pl,y}}{\gamma_{M1}}}\right] \leq 1$$

$$==> \left[\frac{N_{Sd}}{\chi_{min} . \frac{A . f_y}{\gamma_{M1}}}\right] + \left[\frac{k_y . M_{y,Sd}}{\frac{W_{pl,y} . f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.1)$$

  - ✓ For a column subjected to N_{Sd} and $M_{z,Sd}$, it must be verified that:

$$\left[\frac{N_{Sd}}{N_{b,Rd}}\right] + \left[\frac{k_z . M_{z,Sd}}{\frac{M_{pl,z}}{\gamma_{M1}}}\right] \leq 1$$

$$==> \left[\frac{N_{Sd}}{\chi_{min} . \frac{A . f_y}{\gamma_{M1}}}\right] + \left[\frac{k_z . M_{y,Sd}}{\frac{W_{pl,z} . f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.2)$$

  - ✓ For a column subjected to N_{Sd}, $M_{y,Sd}$ and $M_{z,Sd}$, it must be verified that:

$$\left[\frac{N_{Sd}}{N_{b,Rd}}\right] + \left[\frac{k_y . M_{y,Sd}}{\frac{M_{pl,y}}{\gamma_{M1}}}\right] + \left[\frac{k_z . M_{z,Sd}}{\frac{M_{pl,z}}{\gamma_{M1}}}\right] \leq 1$$

$$==> \left[\frac{N_{Sd}}{\chi_{min} . \frac{A . f_y}{\gamma_{M1}}}\right] + \left[\frac{k_y . M_{y,Sd}}{\frac{W_{pl,y} . f_y}{\gamma_{M1}}}\right] + \left[\frac{k_z . M_{y,Sd}}{\frac{W_{pl,z} . f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.3)$$

With: $M_{pl,y}$: the plastic design resistance moment about the y-y axis, $M_{pl,y} = W_{pl,y} . f_y$

$M_{pl,z}$: the plastic design resistance moment about the z-z axis, $M_{pl,z} = W_{pl,z} . f_y$

$W_{pl,y}$ Plastic Section Modulus about the y-y axis

$W_{pl,z}$ Plastic Section Modulus about the z-z axis

The amplification factors k_y or k_z due to the applied axial force, which induces lateral flexural deformation, are given by:

$$k_y = 1 - \left(\frac{\mu_y}{\chi_y} . \frac{N_{sd}}{A. f_y}\right) \qquad with \qquad k_y \leq 1.5$$

$$\mu_y = \bar{\lambda}_y . (2. \beta_{My} - 4) + \left(\frac{W_{pl,y} - W_{el,y}}{W_{el,y}}\right) \qquad with \qquad \mu_y \leq 0.9$$

$$k_z = 1 - \left(\frac{\mu_z}{\chi_z} . \frac{N_{sd}}{A. f_y}\right) \qquad with \qquad k_z \leq 1.5$$

$$\mu_z = \bar{\lambda}_z . (2. \beta_{Mz} - 4) + \left(\frac{W_{pl,z} - W_{el,z}}{W_{el,z}}\right) \qquad with \qquad \mu_z \leq 0.9$$

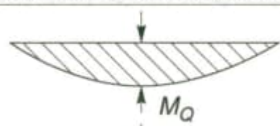
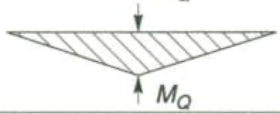
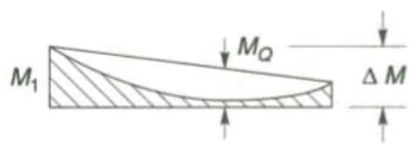
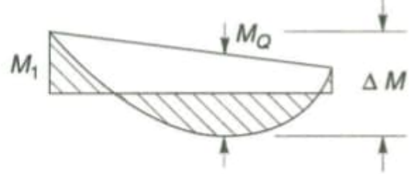
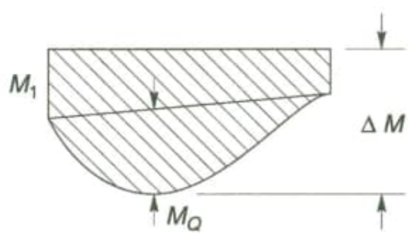
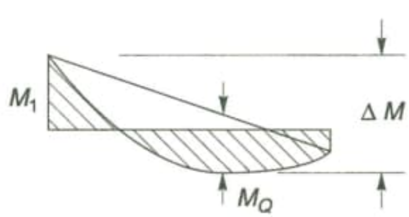
Note: the values of μ_y and μ_z can be negative

β_{My} and β_{Mz} are the equivalent uniform moment factors for flexural buckling. They must be calculated from Table 3.1 based on the shape of the bending moment diagram between the points of restraint.

Table 3.1- Classification of HEA sections under pure bending

Bending Only around y-y / z-z						
Steel Grade	S235		S275		S355	
Section Type	Profile No.	Class	Profile No.	Class	Profile No.	Class
HEA	100 to 240	1	100 to 160	1	100 to 120	1
	260 to 300	2	180 to 240	2	140 to 160	2
	320 to 500	1	260 to 300	3	180 to 340	3
	550 to 600	1	340 to 450	1	360	2
	650 to 1000	1	500 to 1000	1	400 to 1000	1

Table 3.2 - Selection of the uniform moment factor and axis of bending

Diagramme de moment	Facteurs de moment uniforme équivalent βm
Moments d'extrémités	
 $-1 \leq \psi \leq 1$	$\beta_{M,\psi} = 1{,}8 - 0{,}7\,\psi$
Moments dus à des charges latérales	
	$\beta_{M,Q} = 1{,}3$
	$\beta_{M,Q} = 1{,}4$
Moments dus à des charges latérales plus des moments d'extrémités	
   	$\beta_M = \beta_{M,\psi} + \frac{M_Q}{\Delta M}(\beta_{M,Q} - \beta_{M,\psi})$ $M_Q = \lvert \max M \rvert$ dû aux charges latérales seulement pour diagramme de moment sans changement de signe $\Delta M = \begin{cases} \lvert \max M \rvert \\ \lvert \max M \rvert + \lvert \min M \rvert \end{cases}$ pour diagramme de moment avec changement de signe

Example: a fixed-fixed beam subjected to a uniformly distributed load q.

$\Psi = 1 \Rightarrow \beta_{M,\Psi} = 1.1;\ \beta_{M,Q} = 1.3\ ;\ M_Q = \Delta M = \frac{q.l^2}{8}\ ; hence\ \beta_M = \beta_{M,Q} = 1.3$

➢ Class 3 sections: The formulas established for Class 1 and 2 sections remain valid, provided that $W_{pl,y} = W_{pl}.f_y$ is replaced by $W_{el,y} = W_{el}.f_y$. That is to say

✓ For a column subjected to N_{Sd} and $M_{y,Sd}$, the following must be verified:

$$\left[\frac{N_{Sd}}{N_{b,Rd}}\right] + \left[\frac{k_y.M_{y,Sd}}{\frac{M_{el,y}}{\gamma_{M1}}}\right] \leq 1$$

$$==> \left[\frac{N_{Sd}}{\chi_{min}.\frac{A.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_y.M_{y,Sd}}{\frac{W_{el,y}.f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.4)$$

✓ For a column subjected to N_{Sd} and $M_{z,Sd}$, the following must be verified:

$$\left[\frac{N_{Sd}}{N_{b,Rd}}\right] + \left[\frac{k_z.M_{z,Sd}}{\frac{M_{el,z}}{\gamma_{M1}}}\right] \leq 1$$

$$==> \left[\frac{N_{Sd}}{\chi_{min}.\frac{A.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_z.M_{z,Sd}}{\frac{W_{el,z}.f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.5)$$

✓ For a column subjected to N_{Sd}, $M_{y,Sd}$ and $M_{z,Sd}$, it must be verified that:

$$\left[\frac{N_{Sd}}{N_{b,Rd}}\right] + \left[\frac{k_y.M_{y,Sd}}{\frac{M_{el,y}}{\gamma_{M1}}}\right] + \left[\frac{k_z.M_{z,Sd}}{\frac{M_{el,z}}{\gamma_{M1}}}\right] \leq 1$$

$$==> \left[\frac{N_{Sd}}{\chi_{min}.\frac{A.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_y.M_{y,Sd}}{\frac{W_{el,y}.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_z.M_{y,Sd}}{\frac{W_{el,z}.f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.6)$$

Note: For asymmetrical sections, the minimum values of $W_{el,y}$ and $W_{el,z}$ are used.

- Class 4 sections

The formulas become, by introducing effective sections and modules, i.e.:

- For a column subjected to N_{Sd} and $M_{y,Sd}$, it must be verified that:

$$==> \left[\frac{N_{Sd}}{\chi_{min}.\frac{A_{eff}.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_y.(M_{y,Sd} + e_{Ny}.N_{sd}}{\frac{W_{eff,y}.f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.7)$$

- For a column subjected to N_{Sd} and $M_{z,Sd}$, it must be verified that:

$$==> \left[\frac{N_{Sd}}{\chi_{min}.\frac{A_{eff}.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_z.(M_{z,Sd} + e_{Nz}.N_{sd}}{\frac{W_{eff,z}.f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.8)$$

- For a column subjected to N_{Sd}, $M_{y,Sd}$ and $M_{z,Sd}$, it must be verified that:

$$==> \left[\frac{N_{Sd}}{\chi_{min}.\frac{A_{eff}.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_y.(M_{y,Sd} + e_{Ny}.N_{sd}}{\frac{W_{eff,y}.f_y}{\gamma_{M1}}}\right] + \left[\frac{k_z.(M_{z,Sd} + e_{Nz}.N_{sd}}{\frac{W_{eff,z}.f_y}{\gamma_{M1}}}\right] \leq 1 \qquad (3.9)$$

With: $N_{b.Rd} = \chi_{min}.\frac{A_{eff}.f_y}{\gamma_{M1}}$ and $\chi_{min} = \min(\chi_y;\chi_z)$

Exercise No. 1: Consider a column with an HEA340 profile made of S235 grade steel, simply supported (in both planes) with a span of $L = 9.00$ m. It is subjected to an axial compression force, N_{Sd} = 400 kN, and a transverse bending load applied in the plane perpendicular to y-y, q_u = 20 kN/m. Verify the resistance of this beam-column to flexural buckling.