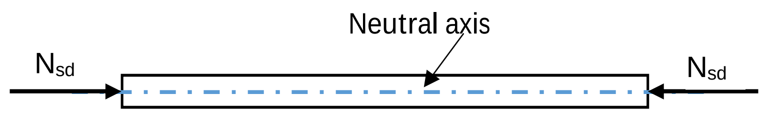
Chapter 2 : Design of members subjected to simple compression

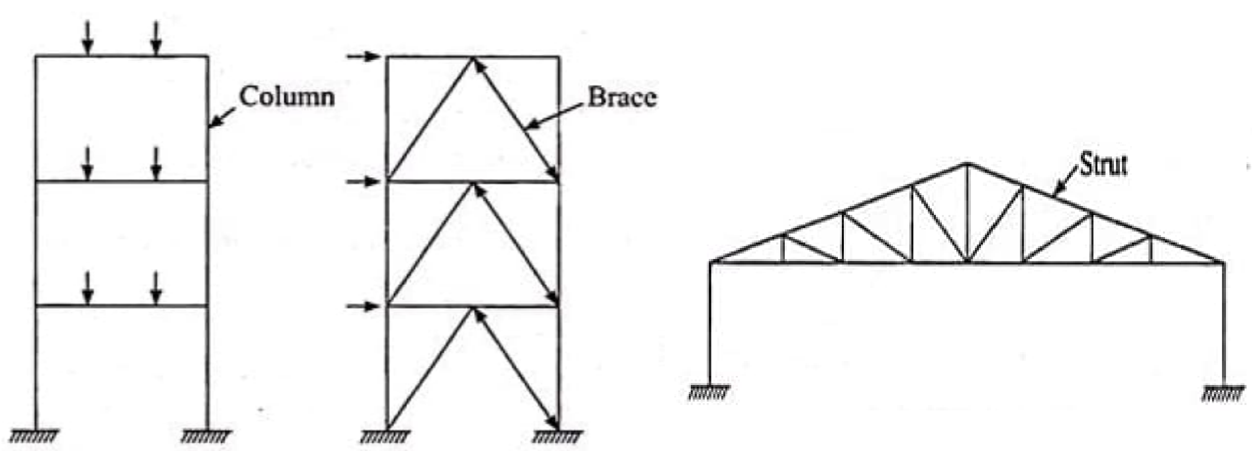
2.1. Area of application of simply compressed members

Compression members are structural elements that are subjected only to axial compressive forces; that is, the loads are applied along a longitudinal axis through the centroid of the member cross section, and the stress can be taken as $f = P/A$, where f is considered to be uniform over the entire cross section.



In a steel frame structure, several members can be subjected to compression.

Following are some examples of compression members:



Strut: a structural member subjected to an axial compressive force is called strut. As per definition strut may be horizontal, inclined or even vertical. Vertical strut is called a column. It can be also a top chord member of a roof truss girder.

Column: It is a vertical member of a steel building frame that is used to support a floor girder or floor subjected to heavy axial compressive loads.

Bracing members : are structural elements designed to resist lateral forces (such as wind or earthquakes) and to prevent instability or excessive deformation of a structure by providing stiffness and maintaining its geometric shape.

2.2. Classification of cross-sections

Cross-sections are classified into four classes according to EC3 (Eurocode 3).
This classification is based on several criteria, including:

- plate slenderness ratios,
- design strength,
- plastic rotation capacity,
- susceptibility to local buckling, etc.

Determining the class of a cross-section provides information about its behavior and resistance, and therefore makes it possible to select the appropriate design method.

Table 2.1- Design method depending on the cross-section class.

Class	Design method
1	Plastic (allowing the formation of a plastic hinge)
2	Plastic (no hinge formation)
3	Elastic analysis on the full section
4	Elastic analysis on the effective section

2.3. Verification of compressed members without risk of buckling

The compressive force N_{Sd} must remain less than or equal to the resisting force N_{Rd} of the section.

- For cross-sections of Class 1, 2 and 3: $N_{Sd} \leq N_{pl.Rd} = \frac{A * f_y}{\gamma_{M0}}$; $\gamma_{M0} = 1$

$N_{pl.Rd}$: The plastic compressive resistance of the cross-section.

A : The gross cross-sectional area

- <u>For cross-sections of Class 4:</u> $N_{Sd} \leq N_{eff.Rd} = \frac{A_{eff} * f_y}{\gamma_{M1}}$; $\gamma_{M1} = 1{,}1$

$N_{eff.Rd}$: The plastic compressive resistance of the cross-section.

A_{eff} : The elastic compressive stress of the effective section

2.4. Section classes of some rolled I and H profiles

For practical use, the following table provides the section classes of some common rolled I and H profiles subjected to pure compression.

Table 2.2- Section classification of rolled I and H profiles in pure compression

Steel grade	S235		S275		S355	
Profile type	**Profile number**	**Section class**	**Profile number**	**Section class**	**Profile number**	**Section class**
IPE	80 to 240	1	80 to 220	1	80 to 160	1
	270 to 360	2	240 to 300	2	180 to 240	2
	400 to 500	3	330 to 400	3	270	3
	550 to 600	4	450 to 600	4	300 to 600	4
HEA	100 to 240	1	100 to 160	1	100 to 120	1
	260 to 300	2	180 to 240	2	140 to 160	2
	320 to 500	1	260 to 300	3	180 to 340	3
	550 to 600	2	340 to 450	1	360	2
	650 to 700	3	500 to 550	2	400 to 450	2
	800 to 1000	4	600	3	500	3
	—	—	650 to 1000	4	550 to 1000	4
HEB	100 to 600	1	100 to 550	1	100 to 450	1
	650 to 700	2	600 to 700	2	500 to 650	2
	800 to 900	3	800	3	600 to 650	3
	1000	4	900 to 1000	4	700 to 1000	4
HEM	100 to 800	1	100 to 700	1	100 to 650	1
	900	2	800	2	700	2
	1000	3	900	3	800	3
	—	—	1000	4	900 to 1000	4

2.4. Simple buckling

Simple buckling is a lateral bending deformation caused by the geometry of the member: its length is large compared to one of the cross-section dimensions. The buckled shape lies in the buckling plane, which is orthogonal to the axis with the smallest moment of inertia.

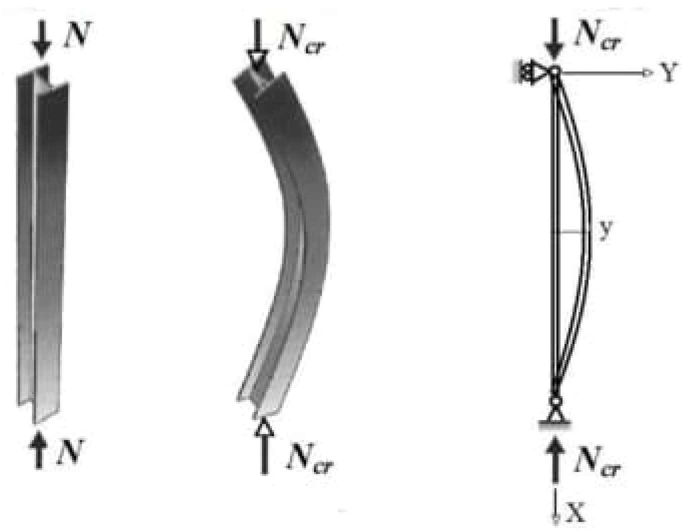


Figure 2.3. Simple buckling

Simple buckling affects members subjected to axial compression. The first to study this phenomenon was the Swiss engineer and mathematician Leonhard Euler in 1759.

Elastic buckling occurs at low compressive stresses, and the material behavior remains elastic. Simple buckling is due to a normal compressive load.

Euler’s theory is based on the following assumptions:

- A perfectly straight column
- Axial load with no eccentricity
- Column pinned at both ends

2.4.1 The critical buckling load

Euler defined the critical compressive force N_{cr}, at which the buckling phenomenon occurs, by the following expression:

$$N_{cr} = \frac{\pi^2 E I_{min}}{l_f^2} \tag{2.1}$$

Where:

E: is the modulus of elasticity of the material,

I_{min}: is the moment of inertia of the cross-sectional area with respect to the minor principal axis, and

l_f : is the length of the member between points of support.

At the Euler critical force N_{cr} corresponds a critical stress σ_{cr}

$$\sigma_{cr} = \frac{N_{cr}}{A} \tag{2.2}$$

Where: A is the cross-sectional area of the column

By replacing N_{cr} with its expression, we obtain:

$$\sigma_{cr} = \frac{\pi^2 E}{l_f^2} . \frac{I_{min}}{A}$$

- The minimum radius of gyration is given by $i_{min} = \sqrt{\frac{I_{min}}{A}}$
- the slenderness ratio $\lambda = \frac{l_f}{i_{min}}$

Finally, we obtain the expression of Euler's critical stress σ_{cr} as a function of the maximum slenderness:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{2.3}$$

The limit of curvilinear equilibrium (for which $\sigma_{cr} = f_y$) corresponds to a critical slenderness ratio λ_l.

In the case of a pinned-pinned S235 grade steel column (f_y = 235 MPa), the Euler critical slenderness ratio is:

$$\lambda_l = \pi\sqrt{\frac{E}{f_y}} = \pi\sqrt{\frac{210000}{235}} = 93.9 \qquad (2.4)$$

For different steel grades: $\lambda_l = \pi\sqrt{\frac{E}{f_y}} = 93.9.\varepsilon$ with $\varepsilon = \sqrt{\frac{235}{f_y}}$

Table 2.3- λ_l for different steel grades

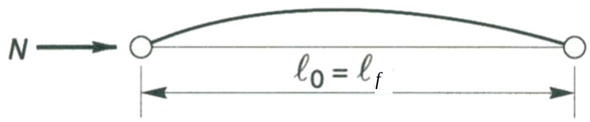
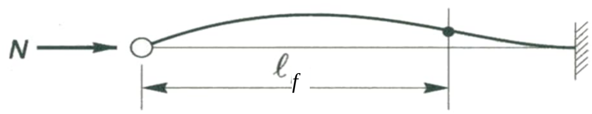
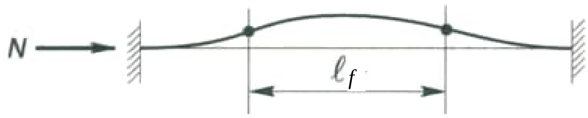
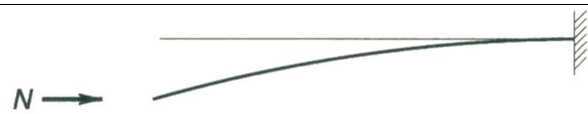
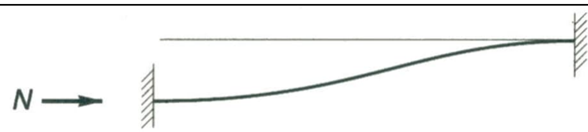
Steel grade	S235 (f_y =235 MPa)	S235 (f_y =275 MPa)	S235 (f_y =355 MPa)
λ_l	93,9	86,8	76,4

2.4.1 The effective length

The buckling length of a column pinned at both ends is equal to its actual length (l_0).For other types of end connections, the potential deformation of the column's centroidal axis is examined. The buckling length *(l_f)* is then taken as the greatest distance between two real or fictitious points of articulation or inflection. The following table illustrates the buckling length for several common cases based on the support conditions.

The critical buckling length is defined as $l_f = \frac{l_0}{\sqrt{m}}$ with the values of m given in Table below.

Table 2.4- The critical buckling length

Support Conditions	m	l_f
1. No Translation at Ends		
	1	l_0
	2	0.7 l_0
	4	0.5 l_0
2. Freedom of Displacement at Ends		
	1/4	2 l_0
	1	l_0

2.4.2 Buckling stability verification of real columns

Euler's theory, established for ideal columns, is insufficient given the imperfections in centering, straightness, verticality, and the presence of residual stresses. It is therefore necessary to take these imperfections or their effects into account. Regulations (Eurocode 3 and CCM97) have notably defined an imperfection factor α.

***2.4.2.a -** <u>Verification of simple buckling stability according to CCM97</u>*

The reduced slenderness $\bar{\lambda}$ is the ratio of the geometric slenderness λ (calculated using the gross section of the element) to the Euler critical slenderness λ_l given by equation (2.4).

$$\bar{\lambda} = \sqrt{\frac{\beta_A . A . f_y}{N_{cr}}} = \frac{\lambda}{\lambda_l} . \sqrt{\beta_A} \qquad (2.5)$$

β_A=1 : for Class 1, 2 and 3 cross-sections

$\beta_A = \frac{A_{eff}}{A}$: for Class 3

- Experimentally, it has been shown that buckling does not occur for low slenderness ratios of the order of $\lambda \leq 20$.
- The risk of buckling is only to be considered in a direction if $\bar{\lambda} > 0.2$; in this case, one must verify that:

$$N_{Sd} \leq N_{b,Rd} = \chi \cdot \beta_A \cdot A \cdot \frac{f_y}{\gamma_{M1}} \qquad (2.6)$$

With **:**

- $N_{b,Rd}$: the design buckling resistance strength.
- χ : the buckling reduction factor.
- β_A: the cross-section class factor.
- A: the column cross-section area.
- f_y: the yield strength of the column steel.

For columns with a constant cross-section, subjected to constant axial compression, the value of χ for the reduced slenderness $\bar{\lambda}$ can be determined by the formula:

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1 \qquad (2.7)$$

With $\chi \leq 1$; and the function $\phi = 0.5[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2]$

α : is the imperfection factor corresponding to the appropriate buckling curve considered. Table 2.5 defines the curve to be used (a, b, c, or d) depending on the type of section and the buckling axis, and its value is:

Table 2.5- Values of the imperfection factors α

Buckling curve	a	b	c	d
Imperfection factor α	0.21	0.34	0.49	0.76

The calculation is carried out for both the strong axis and the weak axis, in order to obtain χ_x and χ_y respectively. The value to be retained will be the smaller of the two:

$$\chi = \min(\chi_x, \chi_y)$$

The buckling curves used by the C.C.M.97 Rules (Eurocode 3) depend on the imperfection factor α. Depending on the types of steel sections and their buckling axes, these imperfection curves correspond to four families of instability behavior

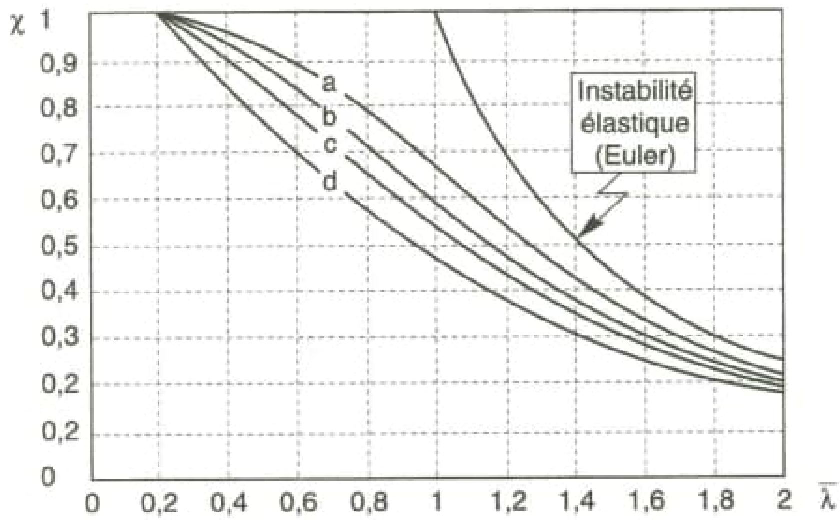
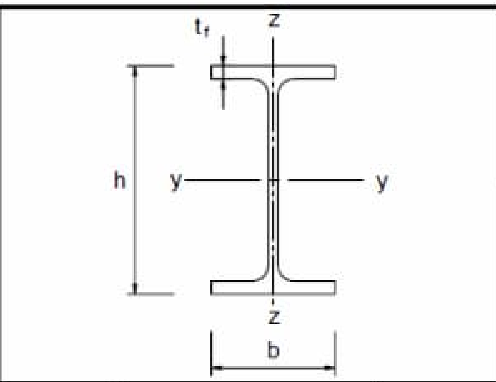
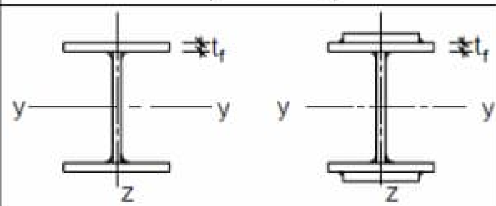
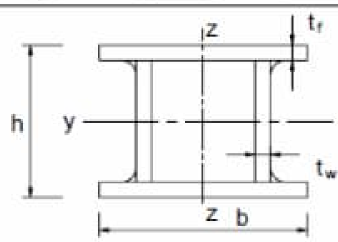
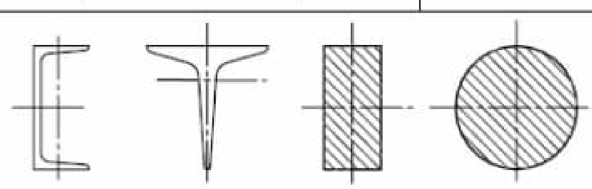
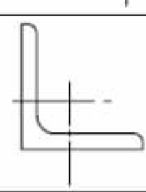


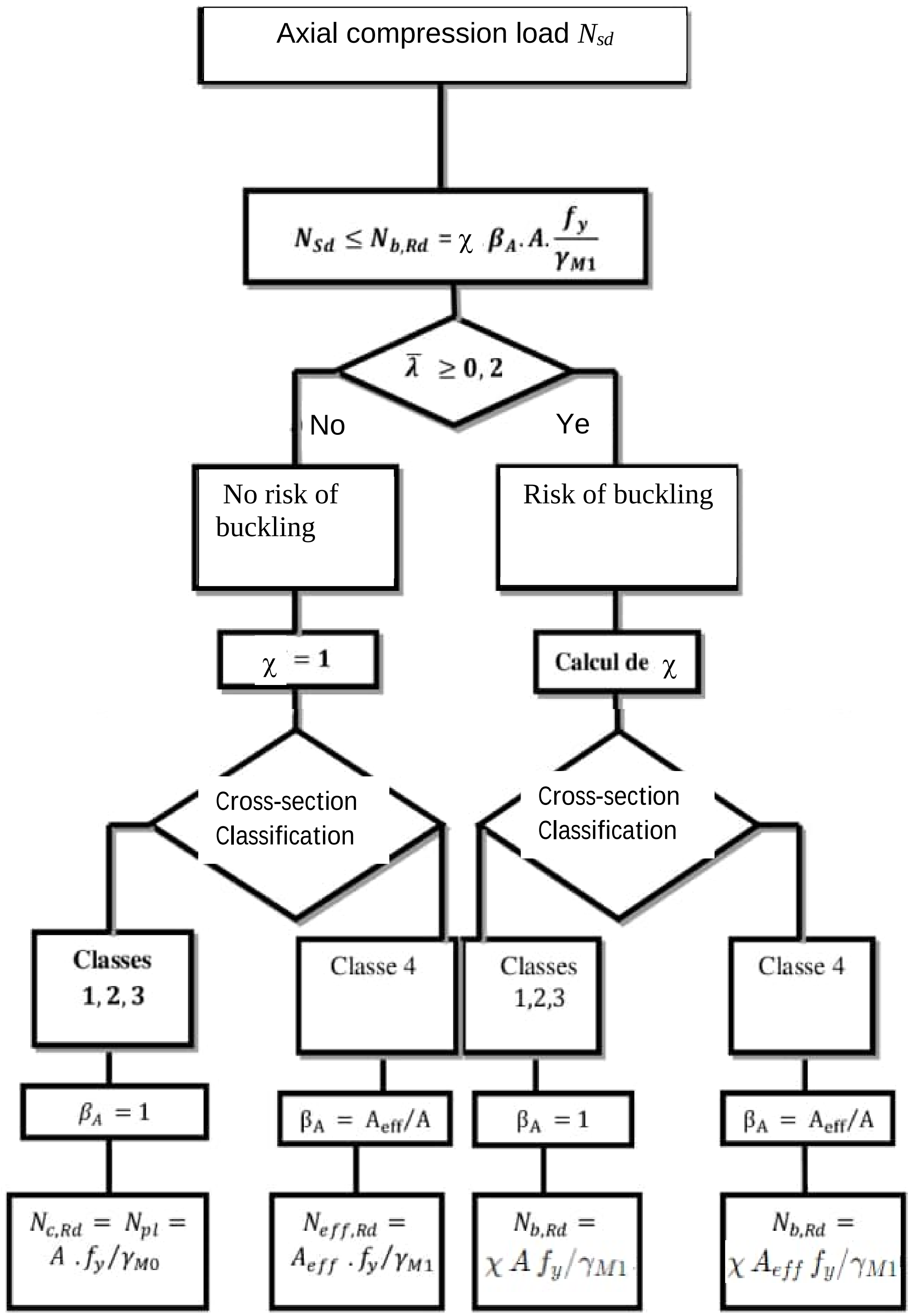
Figure: 2.4. Shapes of the buckling curves

Table 2.6-Reduction factor χ as a function of the reduced slenderness $\bar{\lambda}$

Reduction factors				
$\bar{\lambda}$	χ values for buckling curve			
	a	b	c	d
0.2	1	1	1	1
0.3	0.9775	0.9641	0.9491	0.9235
0.4	0.9528	0.9261	0.8973	0.8504
0.5	0.9243	0.8842	0.843	0.7793
0.6	0.89	0.8371	0.7854	0.71
0.7	0.8477	0.7837	0.7247	0.6431
0.8	0.7957	0.7245	0.6622	0.5797
0.9	0.7339	0.6612	0.5998	0.5208
1	0.6656	0.597	0.5399	0.4671
1.1	0.596	0.5352	0.4842	0.4189
1.2	0.53	0.4781	0.4338	0.3762
1.3	0.4703	0.4269	0.3888	0.3385
1.4	0.4179	0.3817	0.3492	0.3055
1.5	0.3724	0.3422	0.3145	0.2766
1.6	0.3332	0.3079	0.2842	0.2512
1.7	0.2994	0.2781	0.2577	0.2289
1.8	0.2702	0.2521	0.2345	0.2093
1.9	0.2449	0.2294	0.2141	0.192
2	0.2229	0.2095	0.1962	0.1766
2.1	0.2036	0.192	0.1803	0.163
2.2	0.1867	0.1765	0.1662	0.1508
2.3	0.1717	0.1628	0.1537	0.1399
2.4	0.1585	0.1506	0.1425	0.1302
2.5	0.1467	0.1397	0.1325	0.1214
2.6	0.1362	0.1299	0.1234	0.1134
2.7	0.1267	0.1211	0.1153	0.1062
2.8	0.1182	0.1132	0.1079	0.0997
2.9	0.1105	0.106	0.1012	0.0937
3	0.1036	0.0994	0.0951	0.0882

Table 2.7-Selection of buckling curve of a cross-section

Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		$h/b > 1,2$	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			$40 \text{ mm} < t_f \le 100$	y – y z – z	b c	a a
		$h/b \le 1,2$	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections		$t_f \le 40$ mm		y – y z – z	b c	b c
		$t_f > 40$ mm		y – y z – z	c d	c d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U-, T- and solid sections				any	c	c
L-sections				any	b	b


Axial compression load N_{sd}
$N_{Sd} \leq N_{b,Rd} = \chi \ \beta_A . A . \frac{f_y}{\gamma_{M1}}$
$\bar{\lambda} \geq 0,2$
No
Ye
No risk of buckling
Risk of buckling
$\chi = 1$
Calcul de χ
Cross-section Classification
Cross-section Classification
Classes 1, 2, 3
Classe 4
Classes 1,2,3
Classe 4
$\beta_A = 1$
$\beta_A = A_{eff}/A$
$\beta_A = 1$
$\beta_A = A_{eff}/A$
$N_{c,Rd} = N_{pl} = A \ . f_y/\gamma_{M0}$
$N_{eff,Rd} = A_{eff} \ . f_y/\gamma_{M1}$
$N_{b,Rd} = \chi \ A \ f_y/\gamma_{M1}$
$N_{b,Rd} = \chi \ A_{eff} \ f_y/\gamma_{M1}$

Exercise: Verification of stability against flexural buckling

Consider a column of section **HEA 340** made of **S235 steel**, with a height L = 9 m. The column is pinned at the base and at the top in the plane perpendicular to the y–y axis (⊥ y–y plane), and fixed at the base and pinned at the top in the plane perpendicular to the z–z axis (⊥ z–z plane).

The column is subjected to a factored axial load N_{Sd}= 1000 kN.

Verify the stability of the column against buckling.