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Faculty of Technology
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Module: Numerical Methods

Tutorial Series No. 4

Iterative Methods for Solving Linear Systems

Exercise 1: Reordering for Strict Diagonal Dominance

$$\begin{cases} 2x_1 + x_2 + 4x_3 = 7 \\ 5x_1 + 2x_2 - x_3 = 6 \\ x_1 + 6x_2 - 3x_3 = 4 \end{cases}$$

1. Reorder the equations (rows) so that the matrix becomes strictly diagonally dominant.
2. Using the Jacobi method, then the Gauss-Seidel method, compute the first 3 iterations starting from $X^{(0)} = (0,0,0)^T$.

Exercise 2: Matrices and Convergence Criteria

Let A be the following matrix:

$$A = \begin{pmatrix} 10 & 2 & -2 \\ 1 & 10 & 1 \\ 2 & 2 & 10 \end{pmatrix}$$

a) Decompose $A = D - E - F$ where:

- $D =$ diagonal part
- $-E =$ strictly lower triangular part
- $-F =$ strictly upper triangular part

b) Determine the **Jacobi iteration matrix**:

$$J = D^{-1}(E + F)$$

c) Determine the **Gauss-Seidel iteration matrix**:

$$G = (D - E)^{-1}F$$

▷ *Hint: First compute $(D-E)^{-1}$ using the formula for 3×3 matrix inverse or row reduction*

d) Calculate the **infinity norm** for both matrices:

$$\|M\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |m_{ij}|$$

e) Based on the calculated norms, what can you conclude about the **convergence** of both methods?

*Exercise 3: Convergence Study with Norms

$$A = \begin{pmatrix} 6 & -1 & 1 \\ -2 & 7 & -1 \\ 1 & -1 & 8 \end{pmatrix}$$

1. Verify the strict diagonal dominance condition.
2. Derive the Jacobi iteration matrix J.
3. Compute the infinity norm $\|J\|_\infty$.
4. Does the condition $\|T\| < 1$ hold? What can you conclude about convergence?

Exercise 4: Successive Over-Relaxation (SOR) Method

Apply the **SOR method** with relaxation factor $\omega = 1.1$ to:

$$\begin{cases} 10x_1 + 2x_2 - x_3 = 27 \\ -3x_1 - 6x_2 + 2x_3 = -61.5 \\ x_1 + x_2 + 5x_3 = -21.5 \end{cases}$$

a) Verify that the matrix has a strictly dominant diagonal.

b) Write the SOR iteration formula:

$$x_i^{(k+1)} = (1-\omega)x_i^{(k)} + \frac{\omega}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right)$$

Compute $x^{(1)}$ and $x^{(2)}$ starting from:

$$x^{(0)} = (0, 0, 0)^T$$

c) Discuss:

- The effect of $\omega > 1$
- What happens when $\omega = 1$
- What happens if $\omega \geq 2$

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