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**Module: Numerical Methods**

# Worksheet n°3

## Direct method for solving systems of linear equations

## Exercice n°1 :

Consider the following system of linear equations :

$$(S)\begin{cases} 2x_1 + x_2 - x_3 = 3 \\ x_1 + 3x_2 + 2x_3 = 5 \\ 3x_1 - 2x_2 + 4x_3 = 10 \end{cases}$$

**1)** Write the system (S) in augmented matrix form $\left[A \vdots b\right]$.

**2)** Determine an augmented upper triangular matrix $\tilde{A}$ by Gauss's Elimination method.

**3)** Solve the system obtained.

**4)** Deduce the determinant of A.

## *Exercise n°2:

Using the Gauss elimination method: Calculate

$$\Delta = \begin{vmatrix} 1 & x_0^1 & x_0^2 \\ 1 & x_1^1 & x_1^2 \\ 1 & x_2^1 & x_2^2 \end{vmatrix}$$

## Exercise n°3:

Let the following system of linear equations be given:

$$\begin{cases} x_1 + x_2 + x_3 = 1, \\ 2x_1 + 3x_2 + 5x_3 = 3, \\ 4x_1 + 5x_2 + 9x_3 = 6. \end{cases}$$

**(a)** Write the system in matrix form $AX = b$.

**(b)** Show that the matrix $A$ admits a unique LU-decomposition,

$$A = LU,$$

where $L$ is a lower triangular matrix with unit diagonal entries, and $U$ is an upper triangular matrix.

**(c)** Determine explicitly the matrices $L$ and $U$, then solve the system.

**(d)** Deduce the determinant of $A$.

# Exercise n°4:

Consider the system $(S)$ defined by:

$$(S) \begin{cases} x_1 + 2x_2 + 3x_3 = -13, \\ 2x_1 + 5x_2 + 10x_3 = 12, \\ 3x_1 + 10x_2 + 29x_3 = -3. \end{cases}$$

**(i)** Write the system in the form $AX = b$.

**(ii)** Show that the matrix $A$ is symmetric and positive definite.

**(iii)** Compute the Cholesky factorization $A = MM^T$.

**(iv)** Solve the system $(S)$ using Choleskys method.

**(v)** Deduce the determinant of $A$.

# Exercise n°5:

Let be the tridiagonal matrix A defined by

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ -4 & 5 & 2 & 0 \\ 0 & -3 & -1 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

(i) Show that the matrix $A$ admits a unique decomposition $A = LU$, where $L$ is a lower bidiagonal matrix with unit diagonal entries ($l_{ii} = 1$), and $U$ is an upper bidiagonal matrix.

(ii) Determine explicitly the matrices $L$ and $U$.

(iii) Using the $LU$ decomposition of $A$, solve the system $AX = b$ for

$$b = \begin{bmatrix} -4 \\ 36 \\ -27 \\ 12 \end{bmatrix}.$$