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Module: Numerical Methods

Worksheet n°2

Polynomial Interpolation

Exercise n°1: Vandermonde Matrix & Cramer's Rule

Use the **Vandermonde** matrix to obtain the second-degree polynomial passing through the following points:

1.

x_i	1	2	5
y_i	0	-9	18

2. Points: $(0, 3)$, $(2, 7)$, and $(6, 45)$.

Exercise n°2: Identification vs Lagrange

Given the following interpolation points: $(0, 1), (1, 3), (2, 7)$.

1. Determine the degree n of the interpolation polynomial P knowing that $n + 1 = N$ (where N is the number of points).
2. Find the interpolation polynomial passing through the three points using:
   (a) The identification method.
   (b) The Lagrange polynomial method.

Exercise n°3: Intuitive Interpolation

1. Construct the Lagrange polynomial P that interpolates the points: $(-1, e), (0, 1)$ and $(1, e)$.
2. Without performing any calculations, provide the expression of the Lagrange polynomial Q which interpolates the points: $(-1, -1), (0, 0)$ and $(1, -1)$. Justify your answer.

Exercise n°4: Error Upper Bound

Let $f(x) = \dfrac{1}{1+x}$.

1. Determine the Lagrange interpolation polynomial $P_2(x)$ for $f(x)$ at nodes $x_0 = 0, x_1 = 1, x_2 = 3$.
2. Estimate the value of $f(1.5)$ using $P_2(x)$.
3. Calculate the theoretical upper bound for the error committed at $x = 1.5$ using the formula:

$$|R_n(x)| \leq \frac{\max |f^{(n+1)}(\xi)|}{(n+1)!} \left| \prod_{i=0}^{n} (x - x_i) \right|$$

Exercise n°5: Newton Forward Differences

Consider the points shown in the following table (equally spaced nodes):

x_i	-1	0	1	2
y_i	2	5	4	5

1. Construct the **finite difference table** (Δy_i) for these points.
2. Deduce the equation of Newton's polynomial of degree three $P_3(x)$ passing through all the points.

Exercise n°6: Newton Divided Differences

Consider the points shown in the following table (non-equally spaced nodes):

x_i	0	2	3	5
$f(x_i)$	-1	2	9	87

1. Construct the **divided differences table** ($f[x_i, \ldots, x_k]$).
2. Find the equation of the parabola passing through the first three points.
3. Deduce the equation of Newton's polynomial of degree three passing through all the points.

Exercise n°7: Theoretical Bound vs Actual Error

Let $f(x) = \ln(x)$ defined on $[1, 2]$.

1. Determine the linear Lagrange polynomial $P_1(x)$ using nodes $x_0 = 1$ and $x_1 = 2$.
2. Calculate the **actual error** $|f(1.5) - P_1(1.5)|$.
3. Calculate the theoretical error bound at $x = 1.5$ and compare it with the actual error.