

Worksheet n° 1

Solving Non-linear Equations $f(x) = 0$ and Error Analysis

Exercise n° 1:

All digits in the following numbers are **correct**. Calculate the absolute and relative errors:

a) 0.1342 b) 7.480 c) 0.34235×10^{-3} d) 7.000

Exercise n° 2:

Let $x = 0.1256$ be given with an error of 0.5%.

- 1) Give the absolute error of x .
- 2) Determine the exact number of significant digits of this number.
- 3) Round the result to the last csd .

Exercise n° 3:

Let: $x = 2.5 \pm 0.01$; $y = 1.2 \pm 0.02$; $z = 3.2 \pm 0.03$; $t = 5.1 \pm 0.01$.

- 1) Verify that all digits of x, y, z , and t are **correct**.
- 2) Let $S = x^2 + y^2 + z^2 + t^2$.
 - a) Determine the number of **CSD** of S .
 - b) Round the result to the last **CSD** and give the final result.

***Exercise n° 4:**

Let: $x = 5.27 \pm 0.01$; $y = 28.61 \pm 0.04$; $z = 15.80 \pm 0.02$.

Calculate for $f = \frac{x^2 y^{1/3}}{z^3}$:

- a) The value of f and its relative and absolute errors.
- b) Give the final result of f in the form $f^* \pm \Delta f$.

Exercise n° 5:

Consider the equation $f(x) = x^3 - x - 3 = 0$.

- 1) Graphically find an interval $[a, b]$ of length 1 containing the root α .
 - 2) Using the bisection method with precision $\varepsilon = 10^{-2}$:
 - a) Calculate the number of iterations needed.
 - b) Find the root α .

Exercise n° 6:

Calculate $\sqrt{3}$ with a precision of $\varepsilon = 10^{-2}$ using the bisection method (Hint: use $**f(x) = x^2 - 3$).

Exercise n° 7:

Consider $f(x) = \exp(x)x^2 = 0$. 1) Show that $f(x) = 0$ has a unique solution $\alpha \in [1, 2]$.

- 2) Consider the fixed-point forms: $g_1(x) = e^x/2$ and $g_2(x) = \ln(x + 2)$.
 - a) Which function satisfies the fixed-point conditions? Justify.
 - b) Starting with $x_0 = 1.5$, calculate the number of iterations for $\varepsilon = 10^{-3}$

c) Calculate the root α .

Exercise n° 8:

Let $f(x) = x \ln x - 3 = 0$.

- 1) Show that the root lies in $[2, 3]$.
- 2) Verify that the function satisfies the necessary conditions for the Newton-Raphson method to converge in the interval $[2, 3]$.
- 3) Calculate the root using the Newton-Raphson method with $\varepsilon = 10^{-3}$.