Mohamed Boudiaf University – M'Sila

Faculty of Technology | Department of Civil Engineering

Module: Numerical Methods | Academic Year: 2025/2026

Date: 13/05/2026 | From 8:00 to 10:00

FINAL EXAM

Theoretical Questions(3 marks)

1. State the Newton–Raphson iterative formula.
2. Define a matrix with a strictly dominant diagonal. $A = (a_{ij})_{n\times n}$.
3. Give two sufficient conditions for convergence of Jacobi and Gauss–Seidel methods.

Exercise 1 (5 marks)
Consider the function $f(x) = x^3 - 2x - 2$.

1. Graphically determine an interval [a,b] of length 1 containing the root α, and justify using the Intermediate Value Theorem.
2. Show that Newton–Raphson method can be applied on $[1, 2]$.
3. Approximate the root α using Newton–Raphson method with precision $\varepsilon = 10^{-3}$.

Exercise 2 (8 marks)
Consider the system:

$$\begin{cases} 7x_1 - 3x_2 + x_3 = -8 \\ -14x_1 + 5x_2 - 3x_3 = 13 \\ 7x_1 - 4x_2 + 4x_3 = -3 \end{cases}$$

Assume that the matrix A admits an LU decomposition without pivoting: $A = LU$, where L is a unit lower triangular matrix and U is upper triangular.

1. Write the system in matrix form $AX = b$.
2. Determine the matrices L and U.
3. Solve the system using LU decomposition.
4. Deduce $\det(A)$.

Exercise 3 (4 marks)
Let

$$A = \begin{pmatrix} 5 & 1 & 1 \\ 1 & 6 & 1 \\ 1 & 1 & 7 \end{pmatrix}$$

1. Write $A = D - E - F$, where D is diagonal, $-E$ strictly lower triangular, and $-F$ strictly upper triangular.
2. Determine the Jacobi iteration matrix $J = D^{-1}(E + F)$.
3. Compute
$$\|J\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |j_{ij}|$$
.
4. Using the obtained result, conclude whether the Jacobi method converges.

Module Manager: A. Merini
Good Luck