

Final Exam

Exercise n°1 : (7 marks)

Consider the function $f(x)$ defined by : $f(x) = \exp(x) - x - 5$

1. Show that $f(x) = 0$ has a unique solution α in the interval $[1; 2]$.
2. Verify that the function $g(x) = \ln(x + 5)$ satisfies the conditions of the fixed point theorem in the interval $[1; 2]$.
3. Calculate the root α with $x_0 = 2$ and the precision $\epsilon = 5 \times 10^{-3}$ (take 4 digits after the decimal point).

Exercise n°2 (9 marks)

Consider the following linear system (S) :

$$(S) \begin{cases} x_1 - 3x_2 = 28 \\ -3x_1 + 13x_2 - 8x_3 = 96 \\ -8x_2 + 41x_3 = -985 \end{cases}$$

- i) Write the system (S) as $AX = b$.
- ii) Verify that the matrix A is symmetric positive definite.
- iii) Calculate the Cholesky factorisation of $A = MM^t$.
- iv) Solve system (S) using Cholesky's factorisation.
- v) Deduce the determinant of A.

Exercise n°3 (4 marks)

Consider the points given in the table below :

x_i	0	2	3	5
$f(x_i)$	-7	-13	5	143

- a) Construct the divided differences table.
- b) Find the equation of Newton's polynomial passing the first three points.
- c) Deduce the equation of Newton's polynomial of degree three passing through all the points.