Solutions to the Directed Works Series n°4

Exercice n°1 :

A, B and $A \cup B$ are three events with probabilities 0.4, 0.5 and 0.6.

Calculate the probability of the events :
$\bar{A}$, $\bar{B}$, $A \cap B$, $\bar{A} \cap B$, $A \cap \bar{B}$, $\bar{A} \cap \bar{B}$, $\bar{A} \cup B$, , $\bar{A} \cup \bar{B}$.

Solution :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.6 = 0.4 + 0.5 - P(A \cap B)$$
$$P(A \cap B) = 0.3$$

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Step 2 : Complements

$$P(\bar{A}) = 1 - P(A) = 0.6$$

$$P(\bar{B}) = 1 - P(B) = 0.5$$

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Step 3 : Intersections

$$P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.5 - 0.3 = 0.2$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.4 - 0.3 = 0.1$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - 0.6 = 0.4$$

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Step 4 : Unions

$$P(\bar{A} \cup B) = 1 - P(A \cap \bar{B}) = 1 - 0.1 = 0.9$$

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B) = 1 - 0.3 = 0.7$$

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Exercice n°2 :

A box contains 15 light bulbs, which 5 are defective. We take out 6 bulbs at random. Find the probability in

a) No bulb is defective ;

b) One bulb is defective ;

c) Two bulbs are defective.

d) all 3 bulbs are defective ;

e) At least one bulb is defective.

f) At least two bulbs are defective.

Solution :

Exercise 2 :

A box contains 15 light bulbs, of which 5 are defective and 10 are good. We draw 6 bulbs at random without replacement.

$$|\Omega| = \binom{15}{6} = 5005.$$

a) No bulb is defective

$$P(0d) = \frac{\binom{5}{0}\binom{10}{6}}{\binom{15}{6}} = \frac{210}{5005} \approx 0.04198 \simeq 4.20\%.$$

b) One bulb is defective

$$P(1d) = \frac{\binom{5}{1}\binom{10}{5}}{\binom{15}{6}} = \frac{1260}{5005} \approx 0.25175 \simeq 25.18\%.$$

c) Two bulbs are defective

$$P(2d) = \frac{\binom{5}{2}\binom{10}{4}}{\binom{15}{6}} = \frac{2100}{5005} \approx 0.41958 \simeq 41.96\%.$$

d) All 3 bulbs are defective (exactly 3 defectives among the 6)

$$P(3d) = \frac{\binom{5}{3}\binom{10}{3}}{\binom{15}{6}} = \frac{1200}{5005} \approx 0.23976 \simeq 23.98\%.$$

e) At least 1 bulb is defective

$$\begin{aligned} P(\text{ 0d}) + P(1\text{ d})) + P(2\text{ d})) + P(3\text{ d})) + P(4\text{ d})) + P(5\text{ d})) &= 1 \Leftrightarrow \\ P(1\text{ d})) + P(2\text{ d})) + P(3\text{ d})) + P(4\text{ d})) + P(5\text{ d})) &= 1 - P(0\text{ d}) \Leftrightarrow \\ P(\text{ at least 1 bulb } \ is defective) &= 1 - P(0\text{ d}) \\ &= 1 - \frac{210}{5005} \\ &= \frac{4795}{5005} \simeq 0.9580 = 95.80\% \end{aligned}$$

f) At least two bulbs are defective

$$\begin{aligned} P(\text{ at least 2 bulbs } \ is defective) &= 1 - P(0\text{ d}) - P(1\text{ d}) \\ &= 1 - \frac{210}{5005} - \frac{1260}{5005} \\ &= \frac{3535}{5005} \simeq 0.7063 = 70.63\% \end{aligned}$$

***Exercice n°3 :** An urn contains 12 balls : 3 red, 4 blue and 5 yellow. 3 balls are drawn simultaneously. Calculate the probability of the following events :

a) A="all three balls are red" ;

b) B="one ball of each colour is drawn" ;

c) C="none of the three balls is red" ;

d) D="at least one of the three balls is red" ;

e) E="at least one of the three balls is blue" ;

f) F="at most one of the three balls is blue" ;

Solution :

An urn contains 12 balls : 3 red (R), 4 blue (B), and 5 yellow (Y). Three balls are drawn simultaneously (without replacement). The total number of outcomes is

$$\binom{12}{3} = 220.$$

a) A = "all three balls are red"

$$P(A) = \frac{\binom{3}{3}}{\binom{12}{3}} = \frac{1}{220} \approx 0.004545 \;\Rightarrow\; 0.4545\%.$$

b) B = "one ball of each colour is drawn"

$$P(B) = \frac{\binom{3}{1}\binom{4}{1}\binom{5}{1}}{\binom{12}{3}} = \frac{3 \cdot 4 \cdot 5}{220} = \frac{60}{220} = \frac{3}{11} \approx 0.272727 \;\Rightarrow\; 27.2727\%.$$

c) C = "none of the three balls is red"

$$P(C) = \frac{\binom{4+5}{3}}{\binom{12}{3}} = \frac{\binom{9}{3}}{220} = \frac{84}{220} \approx 0.381818 \;\Rightarrow\; 38.1818\%.$$

d) D = "at least one of the three balls is red"

$$P(D) = 1 - P(\text{no red}) = 1 - \frac{84}{220} = \frac{136}{220} \approx 0.618182 \;\Rightarrow\; 61.8182\%.$$

e) E = "at least one of the three balls is blue"

$$P(E) = 1 - P(\text{no blue}) = 1 - \frac{\binom{3+5}{3}}{\binom{12}{3}} = 1 - \frac{\binom{8}{3}}{220} = 1 - \frac{56}{220} = \frac{164}{220} \approx 0.745455 \;\Rightarrow\; 74.5455\%.$$

f) F = "at most one of the three balls is blue"

$$P(F) = P(0\text{ blue}) + P(1\text{ blue}) = \frac{\binom{8}{3}}{220} + \frac{\binom{4}{1}\binom{8}{2}}{220} = \frac{56}{220} + \frac{4 \cdot 28}{220} = \frac{56+112}{220} = \frac{168}{220} \approx 0.763636 \;\Rightarrow\; 76.3636\%.$$

Exercice n°4 : A survey gives the following information : 35 % of people go to cinema C, 12 % to museum M and 6 % to both. Express the percentage of people :

a) going to the cinema or museum.

b not going to the cinema.

c) going neither to the cinema nor to the museum.

d going to the cinema but not to the museum.

Solution :

A survey gives : $P(C) = 35\%$, $P(M) = 12\%$, and $P(C \cap M) = 6\%$.

a) Percentage going to the cinema *or* museum :

$$P(C \cup M) = P(C) + P(M) - P(C \cap M) = 35\% + 12\% - 6\% = 41\%.$$

b) Percentage not going to the cinema :

$$P(\overline{C}) = 1 - P(C) = 65\%.$$

c) Percentage going neither to the cinema nor to the museum :

$$P(\overline{C} \cap \overline{M}) = 1 - P(C \cup M) = 1 - 41\% = 59\%.$$

d) Percentage going to the cinema but not to the museum :

$$P(C \cap \overline{M}) = P(C) - P(C \cap M) = 35\% - 6\% = 29\%.$$

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