

Directed Work Series n°5

(Conditional probabilities)

***Exercise n°1 :**

In a tutorial group, 15% of Maths 1 marks were insufficient, 25% of Physics 1 marks were insufficient and 10% of students had insufficient marks in both modules.

- a) A student has an insufficient mark in physics 1. Calculate the probability that he also has a mark that is insufficient in Maths 1.
- b) A student has a mark that is insufficient in Maths 1. Calculate the probability that he also has a mark that is insufficient in Physics 1.

Exercise n°2 : A coin and a dice are tossed. Consider the following events .:

$A = \text{"Get heads"} ; B = \text{"Get a number } < 3 \text{"}$.

- 1) Determine the universe Ω of this random experiment.
- 2) Calculate $P(A)$, $P(B)$, $P(A \cap B)$, and $P_B(A)$.
- 3) Are events A and B independent? (justify your answer).
- 4) Are events A and B incompatible? (justify your answer).

Exercise n°3 :

Three machines, A, B, and C, produce 50%, 30%, and 20% of the total number of parts manufactured in a factory, respectively. The percentages of defective parts produced by these machines are 3%, 4%, and 5%. Take a part at random and calculate the probability :

- a) a part is defective.
- b) a defective part comes from A.
- c) a non-defective part comes from C.

Exercise n°4 :

Let $A, B, C \subset \Omega$, such that $P(B) > 0$. Show that

- a) $P_B(\phi) = 0 ; P_B(\Omega) = 1$.
- b) $P(\bar{A}/B) = 1 - P_B(A)$
- c) $P_B(C \cup A) = P_B(C) + P_B(A) - P_B(C \cap A)$

Exercise n°5 :

Let $A, B \subset \Omega$, . suppose that A and B independent. Show that

- a) $\bar{A}$ and B are independent.
- b) A and $\bar{B}$ are independent.
- c) $\bar{A}$ and $\bar{B}$ are independent.