Mohamed Boudiaf University - M'sila **University Year : 2025/2026**
Faculty of Technology **19/01/2026 (8h30-10h30)**
Department of Civil Engineering **Duration : 2h**
Module : Probability and Statistics

Final Exam

Course Questions :(4 marks)

Let Ω be a sample space and P a probability an Ω.

a) State the axioms of probability.

b) Let $A, B \subset \Omega$. Show that : $P\left(B \cap \bar{A}\right) = P\left(B\right) - P\left(A \cap B\right)$

Exercise n°1 : (08 marks)

In a soil mechanics study, the settlement rate $S(t)$ of a clay layer under a foundation was measured over 28 days :

Time t (days)	4	8	12	16	20	24	28
Settlement Rate $S(t)$ (mm/day)	12.00	8.94	6.66	4.97	3.70	2.76	2.06

a) Draw the scatter plot (t, S) in an orthogonal frame :
Horizontal axis : 1 cm $\rightarrow$ 4 days. Vertical axis : 1 cm $\rightarrow$ 1 mm/day.

b) Is the relationship between S and t linear ? Justify.

c) Let $Z = \ln(S)$. Complete the table (round results to 3 decimal places) :

Time t (days)	4	8	12	16	20	24	28
$Z = \ln(S)$	2.485	2.191					

d) Find the equation of the regression line from z to t, using the method of least squares.

e) Deduce the equation of the settlement rate S in terms of t (in the form $S = Ae^{Bt}$).

f) Estimate the settlement rate S when $t = 40$ days.

Exercise n°2 (5 marks)

In a structural health monitoring study, let X be the number of micro-cracks detected in a 1-meter section of a concrete beam. The probability distribution function (PDF) of X is given by the following table :

x_i	0	1	2	3	4
$P(X = x_i)$	0.4	0.3	c	0.1	0.05

1. Determine the value of the constant c.
2. Determine the expression of the Cumulative Distribution Function (CDF), $F(x)$.
3. Calculate the probability that at least 2 cracks are detected in the section.
4. Calculate the expected number of cracks $E(X)$ and the variance $V(X)$.
5. If the repair cost is given by $C = 50X + 10$ dollars, calculate $E(C)$.

Exercise n°3 (3 marks)

A survey of soil samples from a highway project gives the following information :
40% of samples have High Clay Content (H), 18% of samples have Low Load-Bearing Capacity (L) and 10% of samples have both High Clay and Low Load-Bearing Capacity.

Express the percentage of samples :

a) Having high clay content or low load-bearing capacity.

b) Not having high clay content.

c) Having neither high clay content nor low load-bearing capacity.

d) Having low load-bearing capacity but not high clay content

Good Luck