

Correcting the final exam

Exercice n°1 : (9 marks)

A car manufacturer wants to study the relationship between a car's fuel consumption (Y) and its speed (X). The 10 measurements are given in the table below :

x_i	50	60	80	90	95	100	110	120	130	140
y_i	3	4	5	5.5	6	6.5	7.5	8.5	10	11.5

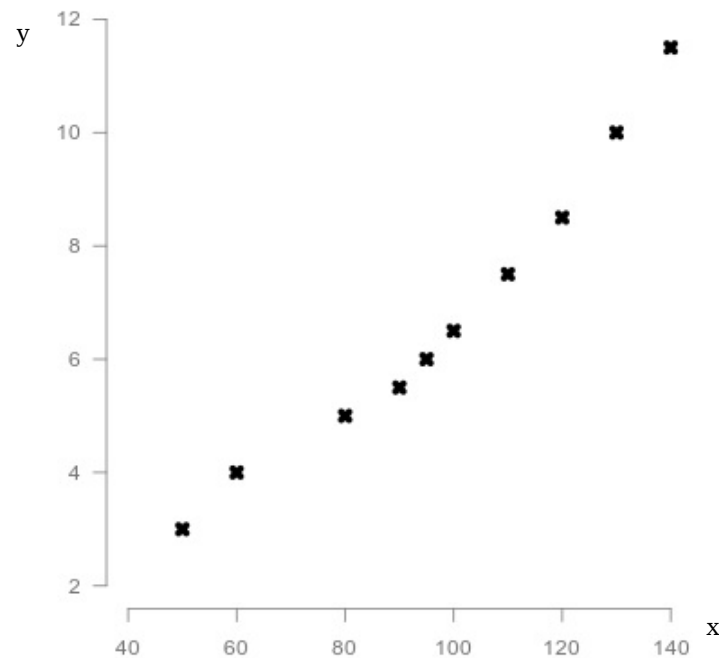
- a) Construct a scatterplot for the data .
b) What can we say about the relationship between X and Y ?
c) Let $Z = \ln Y$. Complete the table

x_i	50	60	80	90	95	100	110	120	130	140
Z_i										

- d) Determine the equation of the regression line from z to x using the method of least squares .
e) Deduce the equation of y in terms of x.
f) Estimate the fuel consumption of a vehicle travelling at 150 km/h ?

Solution :

- a) Scatterplot for the data : (1 mark)



The relationship between a car's fuel consumption (Y) and its speed (X)

b) **X and Y do not have a linear relationship (or X and Y have a weak linear relationship).** (0.5 mark)

c) Let $Z = \ln Y$. Complete the table

x_i	50	60	80	90	95	100	110	120	130	140
Z_i	1.10	1.39	1.61	1.70	1.79	1.87	2.01	2.14	2.30	2.44

(2.5 mark)

d) Equation of the regression line $z = ax + b$ using the method of least squares :

$$\bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i = 97.5 \quad (0.5 \text{ mark})$$

$$\bar{z} = \frac{1}{10} \sum_{i=1}^{10} z_i = 1.84 \quad (0.5 \text{ mark})$$

$$V(x) = \frac{1}{10} \sum_{i=1}^{10} x_i^2 - \bar{x}^2 = 756.25 \quad (0.5 \text{ mark})$$

$$\text{Cov}(x, z) = \frac{1}{10} \sum_{i=1}^{10} x_i z_i - \bar{x} \bar{z} = 11.12 \quad (0.5 \text{ mark})$$

$$\text{then } a = \frac{11.12}{756.25} = 0.014 \text{ and } B = \bar{z} - A\bar{x} = 1.84 - 0.014 \times 97.5 = 0.48$$

$$\text{So, } (D) : z = 0.48 + 0.014x \quad (1 \text{ mark})$$

e) We have $z = \ln y = 0.48 + 0.014x$, so $y = e^{0.48 + 0.014x} = 0.61e^{0.014x} \quad (1 \text{ mark})$

f) For $x = 150 \text{ km/h}$ then $y = 0.61e^{0.014 \times 150} = 13.2 \text{ litre} \quad (1 \text{ mark})$

Exercise n°2 (7 marks)

A box contains three balls numbered 2, 3 and 5. Two balls are drawn in successively with the drawn ball being returned to the box. Let X be the sum of the numbers obtained.

- Find the sample space Ω of this random experiment.
- Find the probability distribution of X , its expectation $E(X)$ and its variance $V(X)$.
- Let $Y = 3X + 10$. Deduce its expectation $E(Y)$ and its variance $V(Y)$.

Solution :

a) The sample space :

$$\Omega = \{(2; 2); (2; 3); (2; 5); (3; 2); (3; 3); (3; 5); (5; 2); (5; 3); (5; 5)\}$$

(1 mark)

b) We have

$$X : \Omega \longrightarrow \mathbb{R}$$

$$(i, j) \mapsto X(i, j) = i + j$$

$$\text{Therefore, } X(\Omega) = \{4; 5; 6; 7; 8; 10\}. \quad (1 \text{ mark})$$

The probability distribution of X :

X	4	5	6	7	8	10
$P(X = x_i)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(2 mark)

The expectation :

$$\begin{aligned}
E(X) &= \sum_{i=1}^6 P(X = x_i) \times x_i \quad (0.25 \text{ mark}) \\
&= \frac{1}{9} \times 4 + \frac{2}{9} \times 5 + \frac{1}{9} \times 6 + \frac{2}{9} \times 7 + \frac{2}{9} \times 8 + \frac{1}{9} \times 10 \quad (0.25 \text{ mark}) \\
&= \frac{20}{3} \\
&\simeq 6.67 \quad (0.25 \text{ mark})
\end{aligned}$$

The variance :

$$\begin{aligned}
V(X) &= \sum_{i=1}^6 P(X = x_i) x_i^2 - E(X)^2 \quad (0.25 \text{ mark}) \\
&= \frac{1}{9} \times 4^2 + \frac{2}{9} \times 5^2 + \frac{1}{9} \times 6^2 + \frac{2}{9} \times 7^2 + \frac{2}{9} \times 8^2 + \frac{1}{9} \times 10^2 - \left(\frac{20}{3}\right)^2 \quad (0.25 \text{ mark}) \\
&= \frac{28}{9} \\
&\simeq 3.11 \quad (0.25 \text{ mark})
\end{aligned}$$

c) Let $Y = 3X + 10$.

The expectation

$$\begin{aligned}
E(Y) &= E(3X + 10) \quad (0.25 \text{ mark}) \\
&= 3E(X) + 10 \\
&= 3 \times \frac{20}{3} + 10 \quad (0.25 \text{ mark}) \\
&= 30 \quad (0.25 \text{ mark})
\end{aligned}$$

The variance

$$\begin{aligned}
V(Y) &= V(3X + 10) \quad (0.25 \text{ mark}) \\
&= 3^2 V(X) \quad (0.25 \text{ mark}) \\
&= 9 \times \frac{28}{9} \\
&= 28 \quad (0.25 \text{ mark})
\end{aligned}$$

Exercise n°3 (4 marks)

Two machines A and B respectively produce 65% and 35% of the total number of parts manufactured in a factory. The percentages of defective parts produced by these machines are 3% and 2% respectively. If an part selected at random, find the probability :

- that a part is defective.
- that a defective part comes from A.
- that a non-defective part comes from B.

Solution :

Let be the following events A : “the part was produced by machine A”, so $P(A) = 0.65$.
 B : “the part was produced by machine B”, so $P(B) = 0.35$.

d = defective part. $\bar{d}$ = non-defective part. then $P(d/A) = 0.03$ and $P(d/B) = 0.02$

a) we're looking for :

$$\begin{aligned}P(d) &= P(d \cap A) + P(d \cap B) \quad (0.5 \text{ mark}) \\&= P(A)P(d/A) + P(B)P(d/B) \quad (0.5 \text{ mark}) \\&= 0.65 \times 0.03 + 0.35 \times 0.02 \quad (0.25 \text{ mark}) \\&= 0,0265 \quad (0.25 \text{ mark})\end{aligned}$$

b) we're looking for :

$$\begin{aligned}P(A/d) &= \frac{P(A \cap d)}{P(d)} \quad (0.25 \text{ mark}) \\&= \frac{P(A)P(d/A)}{P(d)} \quad (0.25 \text{ mark}) \\&= \frac{0.65 \times 0.03}{0.0265} \quad (0.25 \text{ mark}) \\&\simeq 0.736 \quad (0.25 \text{ mark})\end{aligned}$$

c) we're looking for :

$$\begin{aligned}P(B/\bar{d}) &= \frac{P(B \cap \bar{d})}{P(\bar{d})} \quad (0.5 \text{ mark}) \\&= \frac{P(B)P(\bar{d}/B)}{1 - P(d)} \quad (0.5 \text{ mark}) \\&= \frac{0.35 \times 0.98}{1 - 0.0265} \quad (0.25 \text{ mark}) \\&\simeq 0,35 \quad (0.25 \text{ mark})\end{aligned}$$

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Good luck