Mohamed Boudiaf University - M'sila
Faculty of Technology
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Module: Probabilities-Statistics

Chapter 1: Basic definitions-One variable statistical series

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1 Introduction

Statistics is a branch of science dealing with collecting, organizing, summarizing, analysing and making decisions from data.
Statistics is divided into two main areas, which are descriptive and inferential statistics:
1-Descriptive statistics are procedures used to summarize, organize, and make sense of a set of scores or observations.
2-Inferential statistics deals with methods that use sample results, to help in estimation or make decisions about the population.

2 Basic Concepts in Statistics

We will begin by defining the terms used in statistics .

2.1 Population-Sample-Variables-Measurement

Definition 1. *A **population** is the set of all elements (observations), items, or objects that bring them a common recipe and at least one that will be studied their properties for a particular goalt.*
*The components of the population are called **individuals** or **statistical unity**.*

Remark 1. *Note that a population can be a collection of any things, like Ipad set, Books, animals or inanimate, therefore it does not necessary deal with people.*

Definition 2. *A **Sample** is the subset of the population.*
*When the sample consists of the entire population,it is called a **census**.*

Example 1. *Population: Students at the University of M'sila.*
Statistical units: Students.
Sample: Students in the second year of technology.

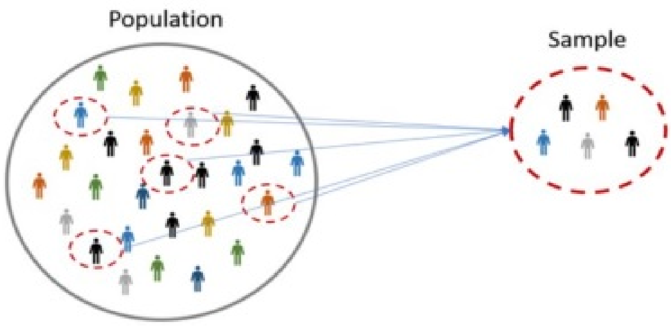


Figure 1: Population and Sample

Definition 3. *A **variable** is a characteristic under study that takes different values for different elements.*

Example 2. *Civil Status. Place of residence Age. Socio-professional categories. Eye colour. Number of languages spoken. Height. Temperature. Nationality. Age. Field of study..*

Definition 4. ***Modalities** are the possible values or categories that a variable can take.*

Example 3. *1- The variable is "marital status".*
The modalities are "single, married, divorced, Widowed".

2-The variable is "socio-professional categories".
The modalities are "Employees, workers, retirees,...".
3-For the variable "Gender," modalities are: Male, Female.

2.1.1 Types of Statistical Variables

In statistics, we have two types of variables according to their elements; first type is called **quantitative variable** and the second one is called **qualitative variable.**

Definition 5. ***Qualitative variable** (or categorical data) gives us names or labels that are not numbers representing the observations. It can be of one of the following types:*

a) Nominal: the modalities are not ordered, for example: eye colour.

b) Ordinal: the modalities are ordered, for example: BAC mention (passable, fairly good, good).

Quantitative variable:

> **Definition 6.** *Quantitative variable gives us numbers representing counts or modalities. It can be of one of the following types:*

a)Discrete variables assume values that can be counted.

For example: number of children per family.

b) Continuous variables assume all values between any two specific values, i.e. they take all values in an interval.

For example: height, weight.

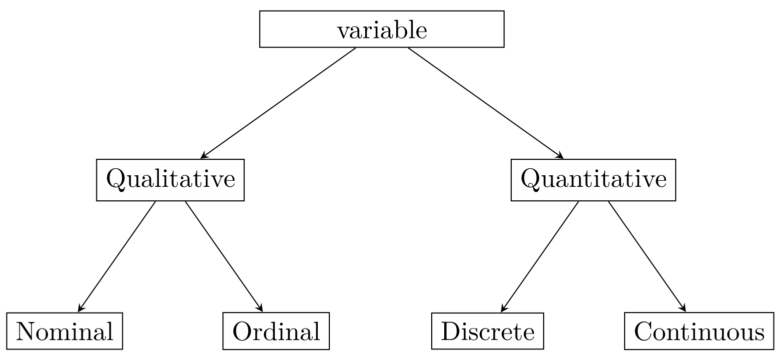


Figure 2: Types of variables

3 Single-variable statistical series

In this section we will learn how to organize and display the Qualitative and Quantitative data.
A statistical series is the sequence of observations of one (or more) variable(s), taken from the individuals in a population.
The values of the variable X are denoted $x_1, ..., x_i..., x_N$.

3.1 Raw Data

Definition 7. *Data recorded in the sequence in which they are collected and before they are processed or ranked are called* ***raw data****. Consider the following three examples to discuss the concept of raw data.*

Example 4. *Suppose that a sample of 50 second year technology students at the University of M'sila were selected and these students were asked about their degree of satisfaction with of the exam results. The answers of these students are recorded below where*

(v) means very highly satisfied,
(s) means somewhat satisfied, and
(n) means not satisfied.

n	n	n	v	s	n	n	n	v	v	n	s	v	v	v	n	n	s	n	s	v	n	s	v	v
v	s	v	n	n	v	s	n	v	v	v	v	s	s	v	v	n	s	s	v	v	v	n	s	n

Example 5. *We are interested in the age of each of the 50 employees in a company. We have the following raw data: 36; 30; 30; 56; 58; 47; 30; 45; 47; 18; 47; 33; 26; 51; 41; 33; 45; 39; 36; 41; 51; 21; 33; 30; 18; 56; 24; 26; 41; 26; 37; 26; 33; 39; 51; 56; 33; 24; 51; 37; 24; 37; 41; 41; 45; 33; 45; 33; 30; 37.*

Example 6. *In a city, 45 families were surveyed for the number of cell phones they used. Prepare a discrete frequency array based on their replies as recorded below. 1; 3; 2 ;2; 2; 2; 1; 2; 1; 2; 2 ;3; 3; 3; 3; 3; 3; 2 ;3 ;2 ;2 ;6; 1; 6; 2; 1; ;5 1; 5; 3; 2; 4; 2; 7; 4; 2; 4; 3; 4; 2; 0; 3; 1; 4; 3.*

3.2 Organize and graph Qualitative data

In this section we will learn how to organize and display the Qualitative and Quantitative data.

Definition 8. ***The total frequency*** *noted N, is the number of individuals that make up the population.* $card(\Omega) = N$.

Definition 9. *A* ***frequency (or count)*** *of a modalitiy is the number of times a data value occurs. noted* n_i

Definition 10. ***A relative frequency*** *of a modalitiy noted* $f_i = \dfrac{n_i}{N}$

Definition 11. *The percentage of a modalitiy is* $p_i = fi \times 100$

Properties 1) $\Sigma n_i = N$
2) $\Sigma f_i = 1$
3) $0 f_i \leqslant 1$

3.2.1 Organize Qualitative data

In this section we will study some methods that used to organize qualitative data set.

a) Frequency table

Definition 12. *A frequency table for qualitative data lists all categories , names or labels and the number of elements that belong to each of the categories, names or labels.*

Example 7. *Refer to example 4 we can view the frequency table as follow:*

Variable	*Frequency* n_i
v	20
s	12
n	18
$Sum = \Sigma$	50

b)Relative Frequency and Percentage Distributions

Example 8. *Let's take the previous example. Applying the definition of relative frequency and the percentage of each category we get the following table:*

Variable	*Frequency* n_i	*Relative Frequency* f_i	*Percentage* $p_i\%$
v	20	*0.40*	*40*
s	12	*0.24*	*24*
n	18	*0.36*	*36*
$Sum = \Sigma$	50	*1*	*100*

3.2.2 Graphical Presentation of Qualitative Data

There are many types of graphs that are used to display qualitative data; in this part we will study and graph two of such graphs which they are commonly used to display the qualitative data, these graphs are the **Bar chart** and the **Pie chart**.

1)Bar Graph (Chart)

Definition 13. *A graph made of bars whose heights represent of respective categories is called a* ***bar graph****.*

Construct a Bar Graph (Chart)

1. Represent the categories on the horizontal axis (All categories are represented by intervals of the same width).
2. Mark the frequencies on the vertical axis.
3. Draw one bar for each category .

Example 9. *Refer to the example 7, we construct bar graph*

2)Pie Chart

Definition 14. *A circle divided into portions that represents the relative frequencies or percentages of a population or a sample of different categories is called a pie chart.*

Construct a Pie Graph (Chart)

1. Draw a circle. 2. Find the central angle for each category by the following equation: $\alpha_i = f_i 360^0$.
3. Draw sectors corresponding to the angles that obtained in step 2.

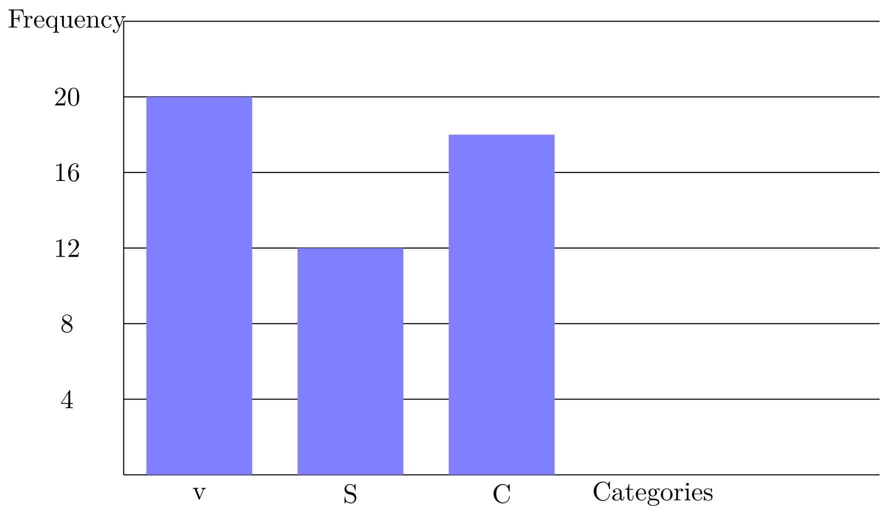


Figure 3: Bar graph of satisfaction with exam results

Example 10.

Let's take example 7,

Variable	Frequency n_i	Relative Frequency f_i	angle α_i
v	20	0.40	144
s	12	0.24	86.4
n	18	0.36	129.6
$Sum = \Sigma$	50	1	360

3.3 Organizing and Graphical Presentation Quantitative Data

In this section we will study some methods that used to organize quantitative data set.

Definition 15. *The ascending cumulative frequency ($N_{i\nearrow}$) of a value x_i can be found by adding all the frequencies less than x_i.*

Definition 16. *The descending cumulative frequency ($N_{i\searrow}$) of a value x_i indicates the frequency of values that are greater than or equal to x_i.*

3.3.1 Organize Quantitative data

a) Frequency Table:

Definition 17. *A frequency table for quantitative data is an effective way to summarize or organize a dataset. It's usually composed of two columns:*

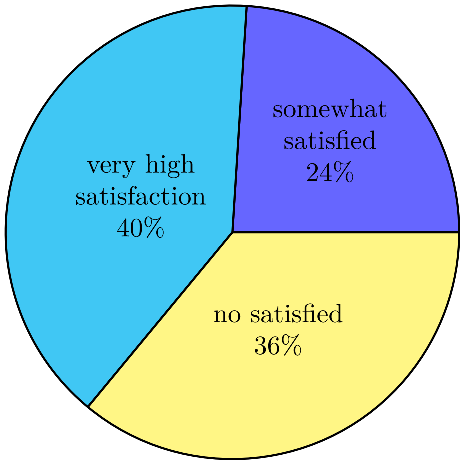


Figure 4: Pie chart of satisfaction with exam results

The values or class intervals.
Their frequencies

For a discrete quantitative variable, we take the following example

Example 11. *Refer to example 6, we can view the frequency table as follow::*

modalities x_i	*Frequency* n_i
0	1
1	7
2	15
3	12
4	5
5	2
6	2
7	1
$Sum = \Sigma$	45

Example 12. *Refer to the previous example,*

modalities x_i	*Frequency* n_i	$N_i\nearrow$	$N_i\searrow$
0	1	*1*	*45*
1	7	*8*	*44*
2	15	*23*	*37*
3	12	*35*	*22*
4	5	*40*	*10*
5	2	*42*	*5*
6	2	*44*	*3*
7	1	*45*	*1*
$Sum = \Sigma$	45	/	/

For a continuous quantitative variable ; we need to apply the following steps.

Construct a Frequency Distribution Table:

If a number of classes k are not given, we follow the steps below:
1-Calculate the range:$w = x_{max} - x_{min}$
2-The number of classes, k, is calculated using one of two formulas :
Sturge's rule $k = 1+3.322LogN \in N$, Yule's rule $k = 2.5(N)^{\frac{1}{4}}$, with $5 \leq k \leq 15$.
3-Determine the Width of classse $L = \dfrac{w}{k}$.
4-Find the class limits: $[x_{min}, x_{min} + L[$,.......,$[x_{max} - L, x_{max}]$
-Calculate the Midpoints of each classse,$m_i = \dfrac{Lowerlimit + Upperlimit}{2}$
6-Count the number of data entries for each class, and record the number in the row of the table for that class.

Class	Midpoint	Frequency n_i
$[x_{min}, x_{min} + L[$	m_1	n_1
$\vdots$	$\vdots$	$\vdots$
$[x_{max} - L, x_{max}]$	m_k	n_k
Sum	/	N

Example 13. *Refer to example 5, we can view the frequency table as follow::*
Step1 *Calculate the range:*$w = x_{max} - x_{min} = 58 - 18 = 40$
Step2 *The number of classes* $k = 1 + 3.322Log50 = 6.6 \simeq 7$

Step3 *the Width of classse* $L = \dfrac{w}{N_c} = \dfrac{40}{7} \simeq 5.7 \simeq 6.$

Class	*Frequency* n_i	*Midpoint*
$[18; 24[$	3	21
$[24; 30[$	*7*	*27*
$[30; 36[$	12	33
$[36; 42[$	*13*	39
$[42; 48[$	7	45
$[48; 54[$	4	51
$[54; 60]$	4	57
Sum	*50*	/

3.3.2 Graphical Presentation of Quantitative Data

1)Case of a Discrete quantitative data :

Bar chart This graph has two axes, a horizontal axis representing the values of the variable, and a vertical axis representing the frequencies or relative frequencies. Each value is associated with a segment whose height is proportional to the frequencies or the frequency of this modality.

Example 14.

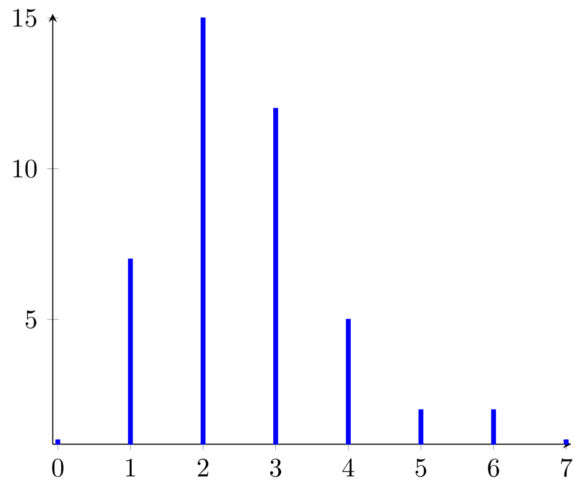


Figure 5: Bar chart of the number of cell phones

2)Case of a Continuous quantitative data :

Histogram A histogram of grouped data in a frequency distribution table with equal class widths is a graph in which class boundaries are marked on the horizontal axis and the frequencies, relative frequencies, or percentages are marked on a vertical axis.

Remark 2.

Histograms in case of unequal classes widths: we calculate the adjusted frequency h_i using the formula,

$$h_i = \frac{a}{a_i} \times n_i$$

were, $a = PGCD(a_1, ...a_k)$

Example 15. *Refer to example 5,*

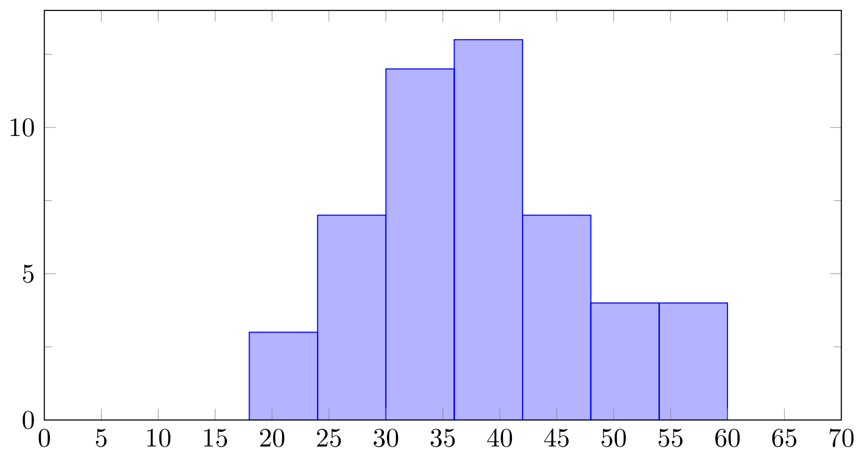


Figure 6: Histogram of the age of the employees

4 Measures of Central Tendency

A measure of central tendency is very important tool that refer to the centre of a histogram or a frequency distribution curve. In This section we will discuss three measures of central tendency and learn how to calculate it. Such measures are the mean, the median, and the mode for the two cases (continuos and discete quantitative variables).

4.1 Arithmetic mean

Let the following statistical series $x_1, ..., x_i..., x_N$.

Definition 18. *The arithmetic mean (or the average) is the sum of all the statistical data divided by the number of data items :*

$$\bar{x} = \frac{x_1 + x_2 + ... + x_N}{N} \tag{1}$$

Example 16. *The numbers of children in 8 families are 0, 0, 1, 1, 1, 2, 3, 4. The mean is*

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{0+0+1+1+1+2+3+4}{8} = \frac{12}{8} = 1.5$$

Definition 19.

(Weighted arithmetic mean) If a value x_i is observed n_i times, formula (1) becomes:

$$\bar{x} = \Sigma_{i=1}^{k} \frac{n_i x_i}{N} = \Sigma_{i=1}^{k} f_i x_i \tag{2}$$

Example 17. *Consider the table:*

x_i	$y\ n_i$
0	2
1	3
2	1
3	1
4	1
$Sum = \Sigma$	8

The mean is

$$\bar{x} = \Sigma_{i=1}^{k} \frac{n_i x_i}{N} = \frac{3 \times 0 + 2 \times 1 + 1 \times 2 + 1 \times 3 + 1 \times 4}{8} = \frac{12}{8} = 1.5$$

Remark 3. . *In the continuous case, we choose the value xi equal to the centre of the of the corresponding class ci , i.e.* $\bar{c} = \Sigma_{i=1}^{k} \frac{n_i c_i}{N} = \Sigma_{i=1}^{k} f_i c_i$

4.1.1 Properties of the mean

i) The sum of the deviations from the mean is 0 :
$\Sigma_{i=1}^{k} n_i (x_i - \bar{x}) = 0$

ii) Linearity property: If
$\forall i \in \{1, 2, ..., k\}, a, b \in \mathbb{R}$ we have: $y_i = a x_i + b$, then :$\bar{y} = a\bar{x} + b$.

iii) Property of partial means: Consider two statistical series with respective means $\bar{x_1}$ and $\bar{x_2}$ and respective numbers N_1 and N_2.
The mean of the two series is
$\bar{x} = \dfrac{N_1 \bar{x_1} + N_2 \bar{x_2}}{N_1 + N_2}$

Proof. See WS. □

4.2 The Mode

Definition 20. *The mode (Mo) is the value that occurs most often in a data set.*

Example 18. *Refer to example 5,*

Variable	*Frequency* n_i
v	20
s	12
n	18
$Sum = \Sigma$	50

The mode $Mo = v$.

Example 19. *Refer to example 12,*

modalities x_i	*Frequency* n_i
0	1
1	7
2	15
3	12
4	5
5	2
6	2
7	1
$Sum = \Sigma$	45

The mode $Mo = 2$.

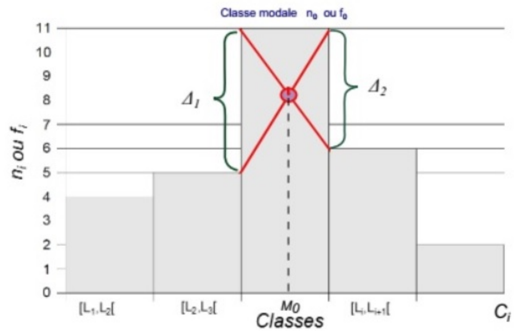
Remark 4. *1) The mode can be calculated for all types of quantitative and qualitative variable.*
2) The mode is not necessarily unique.
3) For a series divided into classes, we speak of a modal class.

4.2.1 The mode for a continuous quantitative variable

can be calculated by the following formula :

$$Mo = L_{i-1} + a_i \frac{\Delta_1}{\Delta_1 + \Delta_2} \tag{3}$$

with
L_{i-1} : The lower bound of the modal class.
a_i :is the class width of the mode class.
Δ_1 is the deference between the frequency of the mode class and the frequency of the previous class.
Δ_2 is the deference between the frequency of the mode class and the frequency of the next class.



Example 20. *In example 13 :*

Class	*Frequency* n_i	*Midpoint*
$[18; 24[$	3	21
$[24; 30[$	*7*	*27*
$[30; 36[$	12	33
$[36; 42[$	*13*	39
$[42; 48[$	7	45
$[48; 54[$	4	51
$[54; 60]$	4	57
Sum	*50*	/

The modal class $= [36; 42[$. *Therefore,* $Mo = 36 + 6\dfrac{13-12}{(13-12)+(13-7)} \simeq 36.8$

4.3 The median

Definition 21. *The median* (Me) *is the value of the middle term in a data set that has been ranked in increasing or decreasing order.*

The Median for Ungrouped Data Note that, to find the median of a given data we need the following three steps
1. Rank the given data sets (in increasing or decreasing order)

2. Find the middle term for the ranked data set that obtained in step 1.
3. The value of this term represents the median.

The median of the ranked data $x_1, ..., x_i..., x_N$. is given by

$$Mo = \begin{cases} x_{\frac{n+1}{2}}, & \text{if } n \text{ is odd} \\ \dfrac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}, & \text{if } n \text{ is even} \end{cases}$$

Example 21. *Find the median for the data set:*
312, 257, 421, 289, 526, 374, 497.
Solution: First, the data set after we have ranked in increasing order is:
257 ,289 ,312, 374, 421, 497, 526.
$N = 7$ *is odd, then* $Mo = x_4 = 374$

Example 22. *Find the median for the data set:*
8, 12, 7, 17, 14, 45, 10, 13, 17, 13, 9, 11
Solution: First, we rank the data in increasing order:
7, 8 ,9 ,10, 11, 12, 13, 13, 14, 17, 17, 45.
$N = 12$ *is even, then* $Mo = \dfrac{x_6 + x_7}{2} = \dfrac{12 + 13}{2} = 12.5$

The Median for grouped Data Suppose that we have a frequency distribution table with k classes, then one calculate the median of this grouped data by the following relation:

$$Me = L_i + \frac{\left(\frac{N}{2} - N_i\right)}{N_{i+1} - N_i}\left(L_{i+1} - L_i\right)$$

where, l_i is the lower bound of the median class.
l_{i+1} is the upper bound of the median class.
N_{i+1} is the ascending cumulative frequency of the median class.
N_i is the ascending cumulative frequency of the previous class.

5 Measures of Variation

The usual dispersion characteristics are range, variance and standard deviation.

5.1 Range

Definition 22. *The range for ungrouped data is defined by: Range = Largest value – Smallest value*

Example 23. *Find the range for the data set: 40, 10, 20, 30, 35, 40, 50, 60.*
Solution:
The largest value is 60, and the smallest value is 10. Therefore
Range = Largest value – Smallest value = 60 – 10 = 50

5.2 The Variance and Standard Deviation

Definition 23. *The variance is the quantity:*

$$V(x) = \Sigma_{i=1}^{k} \frac{n_i(x_i - \bar{x})^2}{N} = \Sigma_{i=1}^{k} f_i(x_i - \bar{x})^2 \tag{4}$$

Definition 24. *Standard deviation The standard deviation is the quantity*

$$\sigma_x = \sqrt{V(x)} \tag{5}$$

Properties

1) $V(x) = \Sigma_{i=1}^{k} \frac{n_i x_i^2}{N} - \bar{x}^2 = \Sigma_{i=1}^{k} f_i x_i^2 - \bar{x}^2$

2) If $\forall i \in \{1, 2, ..., k\}, a, b \in \mathbb{R}$ we have: $y_i = ax_i + b$, $a, b \in R$, then :
$V(y) = a^2 V x$,
$\sigma_y = |a| \sigma_x$.

3) $V(x) \geq 0$.

5.3 Coefficient of Variation

Definition 25. *The coefficient of variation (C_v) is a relative measure of the dispersion of the data around the mean. The coefficient of variation is given by*

$$C_v = \frac{\sigma}{\bar{x}} \tag{6}$$

Properties 1) The greater the coefficient of variation, the greater the dispersion.
2) Generally expressed as a percentage. Without unit.
3) Used to compare 2 statistical series.

6 Measures of shape

The measure of central tendency and measure of dispersion can describe the distribution but they are not sufficient to describe the nature of the distribution. For this purpose, we use other two statistical measures that compare the shape tothe normal curve called **Skewness and Kurtosis**.
Skewness and Kurtosis are the two important characteristics of distribution that are studied in descriptive statistics .
The distribution shape of a qualitative data cannot be described as the data are not numeric.

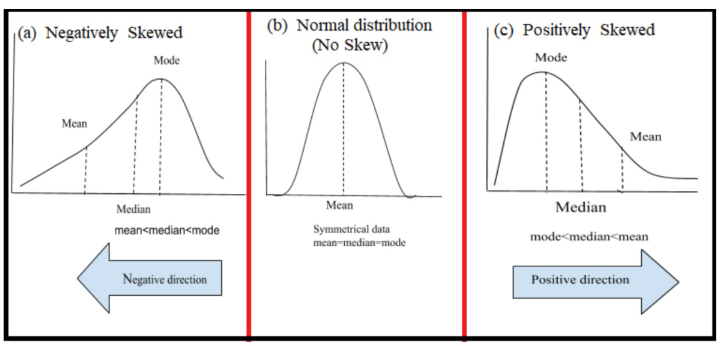
Skewness

Definition 26. *Skewness is a statistical number that tells us if a distribution is symmetric or not. A distribution is symmetric if the right side of the distribution is similar to the left side of the distribution.*
If a distribution is symmetric, then the Skewness value is 0. i.e. If a distribution is Symmetric (normal distribution): Me= $\bar{x}$*= Mo, (Skewness value is 0)*
If Skewness is greater than 0, then it is called right-skewed or that the right tail is longer than the left tail.
If Skewness is less than 0, then it is called left-skewed or that the left tail is longer than the right tail.

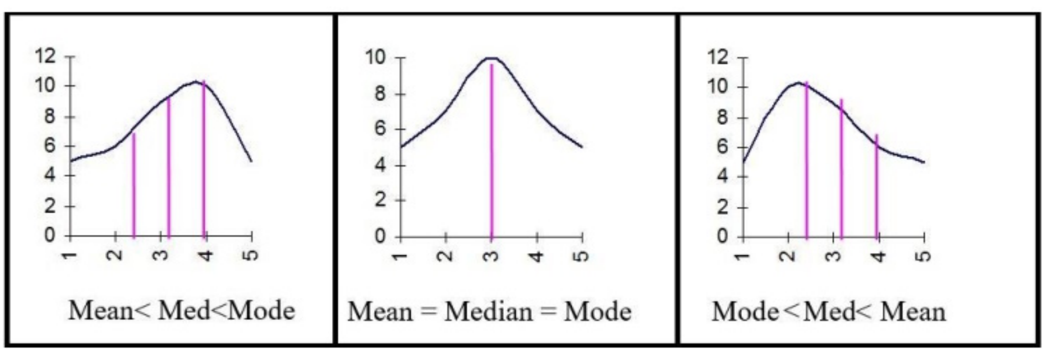
The Formula of Skewness is:

$$Skewness = \frac{\Sigma n_i (x_i - \bar{x})^3}{N\sigma^3}$$

Example 24. *For example, the symmetrical and skewed distributions are shown by curves as:*



And other example,



Symmetry: mean = median = mode
Asymmetry on the right: mode < median < mean

Left asymmetry: mean < median < mode

6.0.1 How to Calculate the Skewness Coefficient?

Various methods can calculate skewness, with Pearson's coefficient being the most commonly used method.

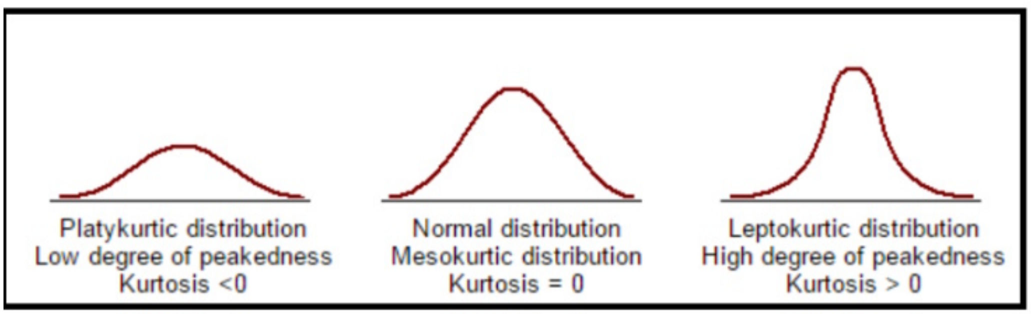
Pearson's first coefficient of skewness

6.1 Kurtosis

Definition 27. *Kurtosis is a statistical number that tells us if a distribution is taller or shorter than a normal distribution.*
If a distribution is similar to the normal distribution, the Kurtosis value is 0.
If Kurtosis is greater than 0, then it has a higher peak compared to the normal distribution.
If Kurtosis is less than 0, then it is flatter than a normal distribution.

There are three types of distributions:
Leptokurtic: Sharply peaked with fat tails, and less variable.
Mesokurtic: Medium peaked
Platykurtic: Flattest peak and highly dispersed.

For example, The different types of Kurtosis:



The Formula of Kurtosis is:

$$Kurtosis = \frac{\Sigma n_i (x_i - \bar{x})^4}{N\sigma^4} - 3$$