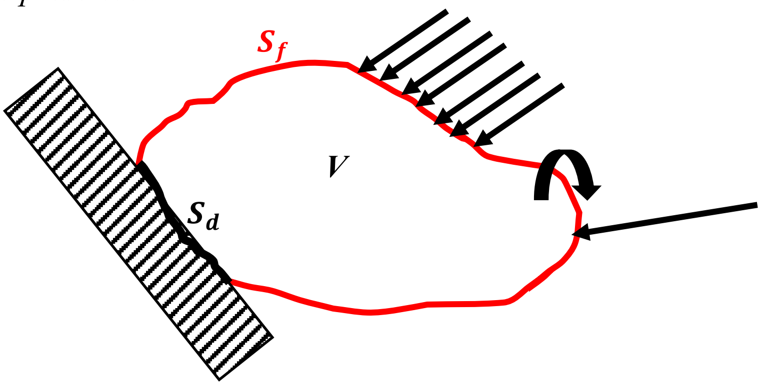
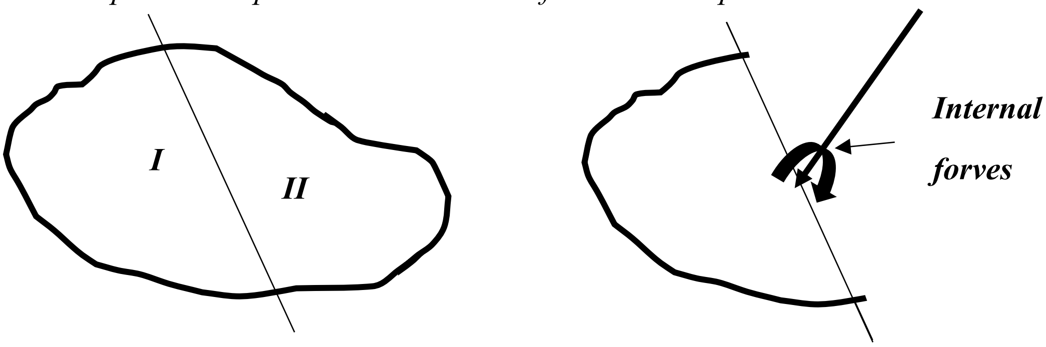
Appendix 1: Reminder of the Equations of Equilibrium of an Elastic Solid

1. Stresses and Strains:

Let's consider a continuous elastic solid with a volume $\boldsymbol{V}$ *and a surface* $\boldsymbol{S}$ *. The surface* $\boldsymbol{S}$ *can be divided into two parts: A part* $\boldsymbol{S_F}$ *on which external forces are known (volume forces or body forces distributed within the volume* $\boldsymbol{f_b}$ *, and surface forces* $\boldsymbol{f_s}$ *acting on parts of or the entire surface), and a part* $\boldsymbol{S_d}$*, on which displacements are prescribed.*

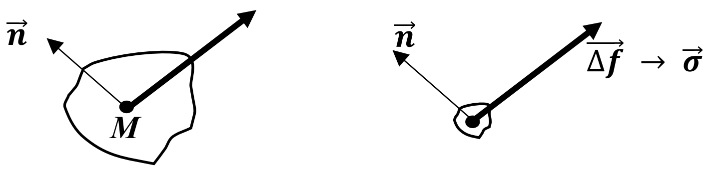


The internal forces, which arise due to external actions, can be identified by conceptually isolating a part of the body. The forces that must be applied on the plane of the imagined cut to maintain the equilibrium of this isolated part correspond to these internal forces at that plane.



For a surface element $\Delta \boldsymbol{s}$ *with normal* $\vec{\boldsymbol{n}}$ *, around a point* $\boldsymbol{m}$ *on this plane, an internal force* $\overrightarrow{\Delta \boldsymbol{f}}$ *acts.,*

The vector $\vec{\boldsymbol{\sigma}} = \lim\limits_{\Delta s \to 0} \frac{\overrightarrow{\Delta f}}{\Delta s}$ *is called the stress vector at point* $\boldsymbol{m}$ *on the facet with normal* $\vec{\boldsymbol{n}}$*.*



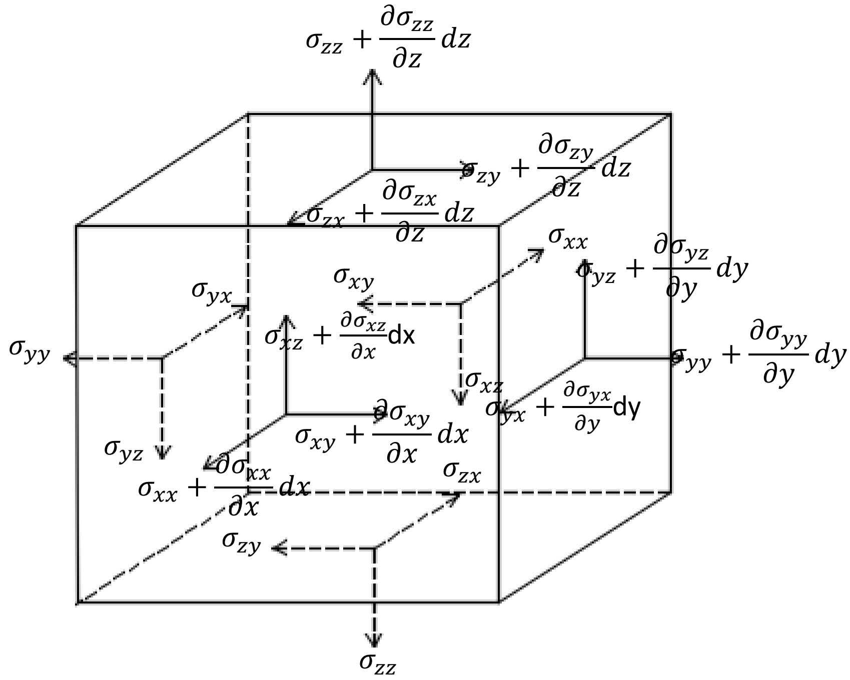
The ***state of stress*** *at a point is defined by the set of stress vectors acting at that point on three orthogonal facets.*

Each stress vector can be decomposed into a component normal to the facet and two components tangential to the facet. Each component is assigned two indices: the first indicates the normal to the facet on which the stress acts, and the second indicates the direction of the stress.

These vectors are grouped into a mathematical entity called a ***tensor****. If we consider facets parallel to the coordinate planes* x, y and z*, the stress tensor then takes the following form:*

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \boldsymbol{\sigma_{xx}} & \boldsymbol{\sigma_{yx}} & \boldsymbol{\sigma_{zx}} \\ \boldsymbol{\sigma_{xy}} & \boldsymbol{\sigma_{yy}} & \boldsymbol{\sigma_{zy}} \\ \boldsymbol{\sigma_{xz}} & \boldsymbol{\sigma_{yz}} & \boldsymbol{\sigma_{zz}} \end{bmatrix}$$

The ***sign convention*** *is such that the stress directions, shown in the figure below, are considered positive directions.*



By taking the moments of forces around the central axes of the cube in the equilibrium state, we demonstrate the Maxwell-Betti parity law or shear equivalence relations:

$$\sum M_{/GX} = 0 \ \rightarrow (\sigma_{yz} + \frac{\partial \sigma_{yz}}{\partial y} dy + \sigma_{yz}) \frac{dxdzdy}{2} - (\sigma_{zy} + \frac{\partial \sigma_{zy}}{\partial z} dz + \sigma_{zy}) \frac{dxdzdy}{2} = 0$$

By neglecting the terms $\frac{\partial \sigma_{yz}}{\partial y} dy$ *and* $\frac{\partial \sigma_{zy}}{\partial z} dz$ *compared with* σ_{yz} *et* σ_{zy} *the previous equation reduces to*

$$2\sigma_{yz} - 2\sigma_{zy} = 0 \ \rightarrow \boldsymbol{\sigma_{yz} = \sigma_{zy}}$$

Similarly, the balance of moments around Gy and Gz gives:

$$\boldsymbol{\sigma_{xy} = \sigma_{yx}}, \ \boldsymbol{\sigma_{xz} = \sigma_{zx}}$$

The stress tensor then takes the form:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \boldsymbol{\sigma_{xx}} & \boldsymbol{\sigma_{xy}} & \boldsymbol{\sigma_{xz}} \\ \boldsymbol{\sigma_{xy}} & \boldsymbol{\sigma_{yy}} & \boldsymbol{\sigma_{yz}} \\ \boldsymbol{\sigma_{xz}} & \boldsymbol{\sigma_{yz}} & \boldsymbol{\sigma_{zz}} \end{bmatrix}$$

Which can be presented by a vector: $\boldsymbol{\sigma} = [\boldsymbol{\sigma_{xx}} \quad \boldsymbol{\sigma_{yy}} \quad \boldsymbol{\sigma_{zz}} \quad \boldsymbol{\sigma_{yz}} \quad \boldsymbol{\sigma_{xz}} \quad \boldsymbol{\sigma_{xy}}]^{\boldsymbol{T}}$

Therefore, there are six independent stress components at a point in 3D solids.

Another notation (engineering notation) is used:

$$[\boldsymbol{\sigma}] = \begin{bmatrix} \boldsymbol{\sigma_x} & \boldsymbol{\tau_{xy}} & \boldsymbol{\tau_{xz}} \\ \boldsymbol{\tau_{xy}} & \boldsymbol{\sigma_y} & \boldsymbol{\tau_{yz}} \\ \boldsymbol{\tau_{xz}} & \boldsymbol{\tau_{yz}} & \boldsymbol{\sigma_z} \end{bmatrix}$$

$$\boldsymbol{\sigma} = [\boldsymbol{\sigma_x} \quad \boldsymbol{\sigma_y} \quad \boldsymbol{\sigma_z} \quad \boldsymbol{\tau_{yz}} \quad \boldsymbol{\tau_{xz}} \quad \boldsymbol{\tau_{xy}}]^{\boldsymbol{T}}$$

In response to the stresses produced, the points of the body undergo displacements whose components in the x, y and z directions are noted u, v and w.

The rotations produced around the $\boldsymbol{x}, \boldsymbol{y}$ ***and*** $\boldsymbol{z}$ ***directions can be determined from the displacements by computing the rotational of the displacement vector:*** $\overrightarrow{rot}(U) = \begin{Bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} = \begin{vmatrix} i & j & k \\ \frac{\partial}{x} & \frac{\partial}{v} & \frac{\partial}{z} \\ u & v & w \end{vmatrix}$

$$\boldsymbol{\theta_x} = \frac{\boldsymbol{\partial w}}{\boldsymbol{\partial v}} - \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial z}}\ ; \qquad \boldsymbol{\theta_y} = \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial z}} - \frac{\boldsymbol{\partial w}}{\boldsymbol{\partial x}}\ ; \qquad \boldsymbol{\theta_z} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial x}} - \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial y}}$$

Displacements can cause variations in the dimensions and/or shape of the solid body. The unit values of these variations are called deformations.

Let's suppose two points that, before deformation, occupy the positions:

$$M(x, y, z) \qquad \textit{and} \qquad N\,(x + dx, y + dy\,, z + dz)$$

After deformation they will occupy the positions:

$$M'\,(x + u, y + v\ et\ z + w) \qquad \textit{and} \qquad N'(x + dx + u + du, y + dy + v + dv\ et\ z + dz + w + dw)$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz;\ dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz; dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

The component in the direction x of $\overrightarrow{MN}$ *is* $\boldsymbol{dx}$

The component in the direction x of $\overrightarrow{M'N'}$ *is* $\left(x + dx + u + \frac{\partial u}{\partial x}dx\right) - (x + u) = \boldsymbol{dx} + \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial x}}\boldsymbol{dx}$

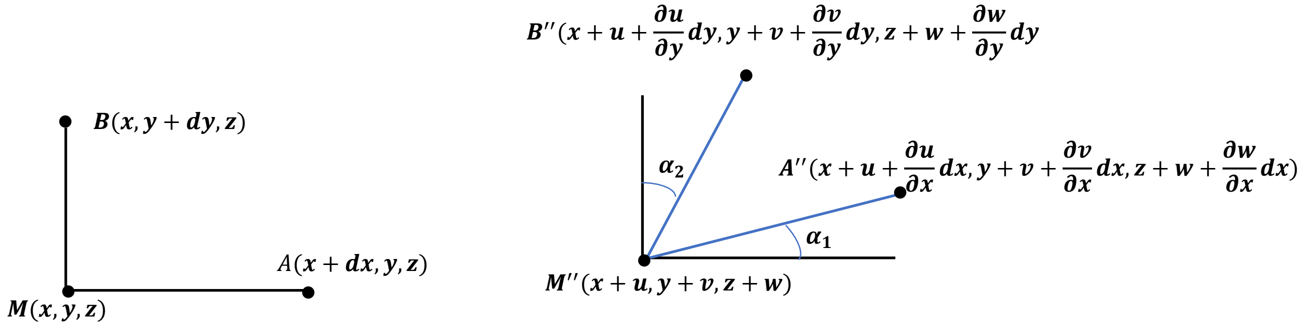
The unit length variation (or linear deformation) in the $\boldsymbol{x}$ *direction is therefore:*

$$\boldsymbol{\varepsilon_x} = \frac{\left(dx + \frac{\partial u}{\partial x}dx\right) - dx}{dx} = \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial x}}$$

Likewise :

$$\boldsymbol{\varepsilon_y} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial y}}; \quad \boldsymbol{\varepsilon_z} = \frac{\boldsymbol{\partial w}}{\boldsymbol{\partial z}}$$

Let's now consider a rectangular facet with sides dx *and* dy *and let's examine the angular variation undergone by one of its right angles.*



Here M'', A'', B'' *are the projections on the plane xy of the points* M', A', B' *positions of the points* M, A, B *after deformation.*

By using: $\alpha(in\, rd) = tg(\alpha)$ *when* α *is very small, we will have:*

$$\boldsymbol{\alpha_1} = tg(\alpha_1) = \frac{\left(\boldsymbol{y + v + \frac{\partial v}{\partial x}dx}\right) - \boldsymbol{y + v}}{dx} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial x}}$$

$$\boldsymbol{\alpha_2} = tg(\alpha_2) = \frac{\left(\boldsymbol{x + u + \frac{\partial u}{\partial y}dy}\right) - (\boldsymbol{x + u})}{dy} = \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial y}}$$

The unit angular variation (or angular deformation) in the **xy** *plane is:* $\boldsymbol{\alpha_1 + \alpha_2}$ *noted in index notation* $\boldsymbol{2\varepsilon_{xy}}$

$$\boldsymbol{2\varepsilon_{xy}} = \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial y}} + \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial x}}$$

Similarly, the unit angular variations in the other two planes are:

$$\boldsymbol{2\varepsilon_{yz}} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial z}} + \frac{\boldsymbol{\partial w}}{\boldsymbol{\partial y}} \qquad ; \qquad \boldsymbol{2\varepsilon_{xz}} = \frac{\boldsymbol{\partial u}}{\boldsymbol{\partial z}} + \frac{\boldsymbol{\partial w}}{\boldsymbol{\partial x}}$$

So, corresponding to the six stress components, there are six strain components at any point in a solid, which can also be written in a similar vector form:

$$\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon_{xx}} \quad \boldsymbol{\varepsilon_{yy}} \quad \boldsymbol{\varepsilon_{zz}} \quad \boldsymbol{\varepsilon_{yz}} \quad \boldsymbol{\varepsilon_{xz}} \quad \boldsymbol{\varepsilon_{xy}}]^{\boldsymbol{T}}$$

The strain tensor then takes the form:

$$[\boldsymbol{\varepsilon}] = \begin{bmatrix} \boldsymbol{\varepsilon_x} & \boldsymbol{\varepsilon_{xy}} & \boldsymbol{\varepsilon_{xz}} \\ \boldsymbol{\varepsilon_{xy}} & \boldsymbol{\varepsilon_y} & \boldsymbol{\varepsilon_{yz}} \\ \boldsymbol{\varepsilon_{xz}} & \boldsymbol{\varepsilon_{yz}} & \boldsymbol{\varepsilon_z} \end{bmatrix}$$

In engineer notation:

$$[\boldsymbol{\varepsilon}] = \begin{bmatrix} \boldsymbol{\varepsilon_x} & \frac{1}{2}\boldsymbol{\gamma_{xy}} & \frac{1}{2}\boldsymbol{\gamma_{xz}} \\ \frac{1}{2}\boldsymbol{\gamma_{xy}} & \boldsymbol{\varepsilon_y} & \frac{1}{2}\boldsymbol{\gamma_{yz}} \\ \frac{1}{2}\boldsymbol{\gamma_{xz}} & \frac{1}{2}\boldsymbol{\gamma_{yz}} & \boldsymbol{\varepsilon_z} \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon_x} & \boldsymbol{\varepsilon_y} & \boldsymbol{\varepsilon_z} & \frac{1}{2}\boldsymbol{\gamma_{yz}} & \frac{1}{2}\boldsymbol{\gamma_{xz}} & \frac{1}{2}\boldsymbol{\gamma_{xy}} \end{bmatrix}^T$$

The six strain-displacement relations can be rewritten in the following matrix form:

$$\boldsymbol{\varepsilon = LU}$$

where: $\boldsymbol{U}$ *is the displacement vector*

$$\boldsymbol{U} = \begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \\ \boldsymbol{w} \end{Bmatrix}$$

And $\boldsymbol{L}$ *is a matrix of partial differentiation operators:*

$$\boldsymbol{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}$$

2. Constitutive equations or behavior laws:

The constitutive equation gives the relationship between stress and strain in the material of a solid. (Hooke's law in the elastic domain).

The tensile test of a homogeneous and isotropic prismatic bar whose longitudinal direction is noted for example x showed that under a longitudinal tensile stress σ_x *the bar undergoes a unit elongation in the elastic domain* $\boldsymbol{\varepsilon_x}$ *of value proportional to* $\boldsymbol{\sigma_x}$*:* $\boldsymbol{\varepsilon_x = \frac{\sigma_x}{E}}$ *;* $\boldsymbol{E}$ *is an intrinsic characteristic of the material determined from the tensile test and called longitudinal modulus of elasticity or Young's modulus.*

At the same time the bar undergoes a unit shortening in the transverse direction of a value proportional to the unit elongation $\boldsymbol{\varepsilon_x}$*:* $\boldsymbol{\varepsilon_y = \varepsilon_z = -\upsilon\varepsilon_x}$*, where* $\boldsymbol{\upsilon}$ *is a characteristic of the material called Poisson's ratio.*

The shear test shows that the shear stress $\boldsymbol{\sigma_{xy}}$*or* $\boldsymbol{\tau_{xy}}$ *produces a unit angular variation in the* ***x-y*** *plane of a value proportional to it,* $\boldsymbol{2\varepsilon_{xy} = \gamma_{xy} = \frac{2\sigma_{xy}}{G} = \frac{\tau_{xy}}{G}}$

Where $\boldsymbol{G}$ *is a characteristic of the material called transverse shear modulus or shear modulus. It is shown that:*

$$\boldsymbol{G = \frac{E}{2(1+v)}}$$

For isotropic materials we can draw up the table below for the case of a body subjected to the complete stress tensor:

	σ_{xx} or σ_x	σ_{yy} or σ_y	σ_{zz} or σ_z	σ_{xy}or τ_{xy}	σ_{yz}or τ_{yz}	σ_{xz}or τ_{xz}
ε_{xx} or ε_x	$1/E$	$-v/E$	$-v/E$			
ε_{yy} or ε_y	$-v/E$	$1/E$	$-v/E$			
ε_{zz} or ε_z	$-v/E$	$-v/E$	$1/E$			
$2\varepsilon_{xy}$or γ_{xy}				$1/G$		
$2\varepsilon_{yz}$or γ_{yz}					$1/G$	
$2\varepsilon_{xz}$or γ_{xz}						$1/G$

Which gives:

$$\boldsymbol{\varepsilon = [D]\sigma}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -v & -v & 0 & 0 & 0 \\ -v & 1 & -v & 0 & 0 & 0 \\ -v & -v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+v) \end{bmatrix}\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix}$$

Or :

$$\boldsymbol{\sigma = [C]\varepsilon}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1-2\nu)(1+\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-2\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-2\nu)/2 \end{bmatrix}\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix}$$

For anisotropic materials, the generalized Hooke's law takes the following matrix form where the constants C_{ij} are determined from tests:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{361} \\ & & & c_{44} & c_{45} & c_{46} \\ & Sym & & & c_{55} & c_{56} \\ & & & & & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix}$$

3. Dynamic equilibrium equations

To formulate the dynamic equilibrium equations, let's consider an infinitesimal cube of solid.

The equilibrium of forces in the x direction gives:

$$\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx\right) dydz - \sigma_{xx} dydz + \left(\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial y}\right) dxdz - \sigma_{yx} dxdz + \left(\sigma_{zx} + \frac{\partial \sigma_{zx}}{\partial z} dz\right) dxdy - \sigma_{zx} dxdy + f_x dxdydz = \rho \ddot{u} dxdydz$$

where $\boldsymbol{\rho \ddot{u} dxdydz}$ is the inertial force of the elementary volume $\boldsymbol{dxdydz}$.

By dividing on $\boldsymbol{dxdydz}$, the previous equation becomes one of the equilibrium equations, written as

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + f_x = \rho \ddot{u}$$

Similarly, the balance of forces in the y and z directions gives the other two equilibrium equations:

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + f_y = \rho \ddot{v}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + f_z = \rho \ddot{w}$$

This set of three equilibrium equations can be written in matrix form:

$$\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial\partial}{\partial x} & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} + \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix} = \rho \begin{Bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{w} \end{Bmatrix}$$

$$L^T \sigma + f_b = \rho \ddot{U}$$

where $\boldsymbol{f_b} = \begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix}$ is the vector of external volume forces in the x, y and z directions

In terms of displacements this equation is written: $\boldsymbol{L^T CLU + f_b = \rho \ddot{U}}$

If the loads applied to the solid are static, then this equation takes the following form:

$$\boldsymbol{L^T CLU + f_b = 0}$$

4. Boundary conditions

There are two types of boundary conditions: displacement boundary conditions (essential conditions) and stress conditions (natural conditions).

The displacement boundary condition is written as:

$$\boldsymbol{u} = \boldsymbol{\bar{u}} \text{ ; and/or } \boldsymbol{v} = \boldsymbol{\bar{v}} \text{ ; ; and/or } \boldsymbol{w} = \boldsymbol{\bar{w}}$$

where: $\boldsymbol{\bar{u}}$; $\boldsymbol{\bar{v}}$; *et* $\boldsymbol{\bar{w}}$ *are the prescribed values for the components* $\boldsymbol{u}$; $\boldsymbol{v}$ *et* $\boldsymbol{w}$

For most real cases, the displacement boundary conditions describe the supports and therefore the prescribed displacement values are often zero.

Stress or force boundary conditions are often written as

$$[\boldsymbol{\sigma}]\vec{\boldsymbol{n}} = \boldsymbol{\bar{t}},$$

where $\vec{\boldsymbol{n}}$ *is the unit vector of the normal,* $\vec{\boldsymbol{n}} = \lfloor \boldsymbol{l} \quad \boldsymbol{m} \quad \boldsymbol{n} \rfloor^{\boldsymbol{T}}$, *in explicit form:*

$$\boldsymbol{l\sigma_{xx} + m\sigma_{xy} + n\sigma_{xz} = t_x}$$

$$\boldsymbol{l\sigma_{xy} + m\sigma_{yy} + n\sigma_{yz} = t_y}$$

$$l\boldsymbol{\sigma_{xz} + m\sigma_{yz} + n\sigma_{zz} = t_z}$$

Which can be written in the form:

$$\begin{bmatrix} \boldsymbol{l} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{n} & \boldsymbol{m} \\ \boldsymbol{0} & \boldsymbol{m} & \boldsymbol{0} & \boldsymbol{n} & \boldsymbol{0} & \boldsymbol{l} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{n} & \boldsymbol{m} & \boldsymbol{l} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\sigma_{xx}} \\ \boldsymbol{\sigma_{yy}} \\ \boldsymbol{\sigma_{zz}} \\ \boldsymbol{\sigma_{yz}} \\ \boldsymbol{\sigma_{xz}} \\ \boldsymbol{\sigma_{xy}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{t_x} \\ \boldsymbol{t_y} \\ \boldsymbol{t_z} \end{Bmatrix}$$

Or in condensed form:

$$[\boldsymbol{n}]\boldsymbol{\sigma} = \boldsymbol{\bar{t}}$$

Boundary conditions are usually ***"complementary"****: when displacements are predefined on a part of the boundary, forces are usually unknown there and vice versa.*

Theoretically, these equations can be applied to all types of structures such as trusses, beams, frames, plates and shells. However, treating all structural components as 3D solids makes the calculation very expensive, and sometimes practically impossible. Therefore, theories to take the geometric advantage of different types of solids and structural components have been developed. Applying these theories in an appropriate way can reduce the analysis and computational effort. A brief description of these theories is given for each element studied.