Tutorials 01

Exercise: Electrical Network Analysis with Linear Algebra and Probability

An electrical engineer analyzes a network with 3 buses (nodes) connected together. The voltages at the buses are given by the vector:
$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where V_i is the voltage at bus i in volts. The network admittance matrix (relating currents and voltages) is:

$$\mathbf{Y} = \begin{bmatrix} 10 & -5 & -5 \\ -5 & 10 & -5 \\ -5 & -5 & 10 \end{bmatrix} \text{ S (siemens)}$$

1. Compute the current vector $\mathbf{I} = \mathbf{YV}$ for $\mathbf{V} = \begin{bmatrix} 230 \\ 220 \\ 210 \end{bmatrix}$ V.
2. Compute the 2-norm (Euclidean norm) of $\mathbf{V}$ and interpret its meaning.
3. Suppose V_1 is a random variable following a normal distribution $V_1 \sim \mathcal{N}(230, 4^2)$ V, $V_2 \sim \mathcal{N}(220, 5^2)$ V, and $V_3 \sim \mathcal{N}(210, 3^2)$ V.

   a) Compute the expected value and variance of the current $I_1 = 10V_1 - 5V_2 - 5V_3$.

   b) What is the probability that $I_1 > 15$ A?

Exercise: Predicting Power Loss in a Transmission Line

Problem Statement:

An engineer wants to estimate the **power loss** P_{loss} **(in watts)** in a transmission line as a function of the **line current** I **(in amps)** using a **linear model:** $P_{loss} = \theta_0 + \theta_1 I$

Given data (measured in the field):

Current I (A)	Power Loss P_{loss} (W)
10	120
20	190
30	250
40	310
50	380

Tasks:

1. Formulate the **linear regression cost function** for this dataset.
2. Compute the **optimal parameters** θ_0, θ_1 analytically using the **normal equation**.
3. Predict the power loss when $I = 35$ A.
4. Implement the regression using **Scikit-learn** in Python.

Exo 03 (devoir)

Consider a simple electrical circuit with three resistors in a mesh configuration, where the goal is to find the current in one branch. The circuit can be modeled by Kirchhoff's Voltage Law (KVL), leading to the matrix equation $\mathbf{AI} = \mathbf{V}$, where:

- $\mathbf{I} = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$ is the vector of mesh currents (in amperes, A).
- $\mathbf{V} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$ is the voltage source vector (in volts, V).
- The resistance matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix}$, where R_1, R_2, R_3 are the resistances (in ohms, Ω).

Assume the resistors have nominal values but with tolerances: R_1 and R_3 are fixed at 5 Ω each, while R_2 is a random variable uniformly distributed between 8 Ω and 12 Ω (due to manufacturing variation). This uniform distribution models uncertainty in component values, common in electrical engineering for reliability analysis in AI systems.

1. **Linear Algebra Part:** Solve for the current vector $\mathbf{I}$ in terms of R_2 (i.e., find I_1 and I_2 using matrix inversion or solving the system). Then, compute the L2 norm (Euclidean norm) of $\mathbf{I}$, which represents the root-mean-square (RMS) current magnitude, useful for power dissipation calculations: $\|\mathbf{I}\|_2 = \sqrt{I_1^2 + I_2^2}$.
2. **Probability Part:** Since $R_2 \sim \mathcal{U}(8, 12)$ (uniform distribution), find the expectation $\mathbb{E}[I_1]$ and variance $\mathrm{Var}(I_1)$ of the current I_1 (the primary branch current). Recall that for a uniform random variable $X \sim \mathcal{U}(a, b), \mathbb{E}[X] = \frac{a+b}{2}$ and $\mathrm{Var}(X) = \frac{(b-a)^2}{12}$. However, since I_1 is a function of R_2, you'll need to derive these from the expression for $I_1(R_2)$.

Exo 04

🔧 Physical context (DC motor)

In a **DC motor**, the **electromagnetic torque** T depends mainly on:

- $x_1 = I$: armature current (A)
- $x_2 = \omega$: angular speed (rad/s)

For a limited operating region, the torque can be approximated by a **linear model**:

$$\boxed{T = \theta_0 + \theta_1 I + \theta_2 \omega}$$

This type of approximation is widely used in:

- motor identification
- control design
- AI-based modeling of electromechanical systems

📊 Experimental data

Measurements obtained from a DC motor test bench:

Test	Current I (A)	Speed ω (rad/s)	Torque T (N·m)
1	1	50	1.6
2	2	60	2.3
3	3	70	3.0
4	4	80	3.7

❓ Questions

1. Write the regression model in **matrix form**
2. Estimate the parameters $\theta_0, \theta_1, \theta_2$ using the **normal equation**
3. Give the **identified motor torque model**
4. Predict the torque for:
   - $I = 5$ A
   - $\omega = 90$ rad/s
5. Implement the solution in **Python**