Chapitre 3 :

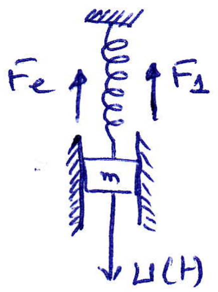
Analyse des Systèmes Echantillonnés dans l’espace d’état

Chapitre 03 Analyse des systèmes Echantillonnés dans l'espace d'état

→ Rappel sur l'espace d'état continue :

Exemple 01 :

Soit le système mécanique suivant :



$$u(t) \longrightarrow \boxed{\text{système}} \longrightarrow x(t) = y(t)$$

$$\sum F_i = m\ddot{x}$$

$$u(t) - Fe - F_1 = m\ddot{x}$$

$$u(t) - f\dot{x} - Kx = m\ddot{x} \quad \text{avec} \quad \begin{cases} f : \text{coef de frottement} \\ K : \text{Const du ressort} \end{cases}$$

$$m\ddot{x} + f\dot{x} + Kx = u(t) \quad \text{Eq dif du 2}^{\text{è}} \text{ ordre}$$

- Choix des variables d'état ⇒ 2 variables d'état $x(t)$, $\dot{x}(t)$

$$\begin{matrix} x_1 = x \\ x_2 = \dot{x} \end{matrix} \Rightarrow \begin{cases} \dot{x}_1 = \dot{x} = x_2 \\ \dot{x}_2 = \ddot{x} = -\frac{K}{m}x_1 - \frac{f}{m}x_2 + \frac{1}{m}u(t) \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \overbrace{\begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{f}{m} \end{pmatrix}}^{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \overbrace{\begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}}^{B} u$$

$$y = \underbrace{\begin{pmatrix} 1 & 0 \end{pmatrix}}_{C} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

A: Matrice d'evaluation ou dynamique

B: Matrice de commande

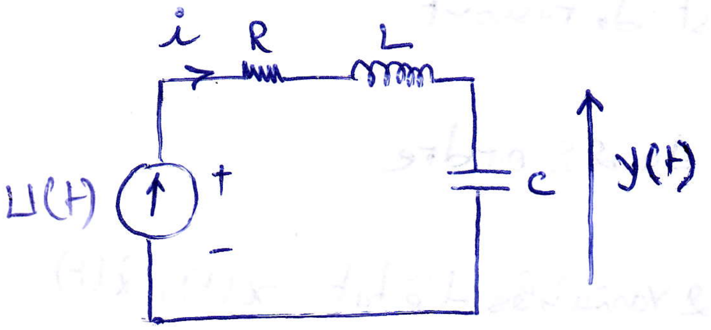
C: Matrice d'observation

D'1 façon générale la représentation d'état d'1 syst d'ordre n est données par:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + \underbrace{D}_{\text{matrice de transfert direct}}u \end{cases}$$

<u>Exemple 02</u>:

Circuit RLC:



$$u(t) = Ri + L\frac{di}{dt} + y \text{ ----- } (*)$$

$$y(t) = \frac{1}{C}\int i(t)\,dt$$

$$\dot{y}(t) = \frac{1}{C} i(t) \Rightarrow i(t) = \dot{y}C$$

$$\frac{di(t)}{dt} = \ddot{y}C$$

On ramplace dans (*)

$$u = RC\dot{y} + LC\ddot{y} + y \Rightarrow \boxed{\ddot{y} + \frac{R}{L}\dot{y} + \frac{1}{LC}y = \frac{1}{LC}u(t)}$$

$$x_1 = y(t) \quad x_2 = \dot{y}(t) \Rightarrow \begin{cases} \dot{x}_1 = \dot{y}(t) \\ \dot{x}_2 = \ddot{y}(t) \end{cases}$$

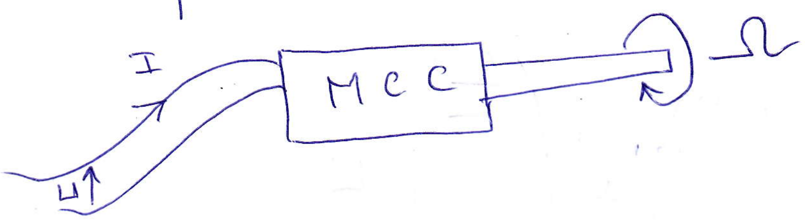
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{-1}{LC} x_1 - \frac{R}{L} x_2 + \frac{1}{LC} u(t) \end{cases}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{-1}{LC} & -\frac{R}{L} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{LC} \end{pmatrix} u$$

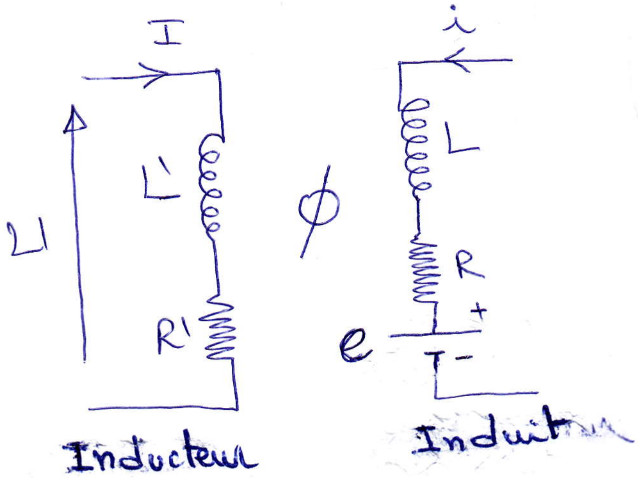
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Exemple 03 =

La représentation d'état d'1 MCC



$$\begin{cases} \dot{X} = AX + Bu \\ y = CX + Du \end{cases}$$



I : courant Inducteur

u : tension d'entrée

Ω : vitesse

i : courant induit

ϕ = flux

$e = K_m \phi \Omega$

C_m : Couple mécanique

$C_m = K_m \phi i$

Eq Electrique

$$U = Ri + L\frac{di}{dt} + \underbrace{K_m \phi}\Omega$$

$$U = Ri + L\frac{di}{dt} + K\Omega$$

On prend (i, Ω) comme variable d'état =

$$\boxed{\frac{di(t)}{dt} = -\frac{R}{L}i(t) - \frac{K}{L}\Omega + \frac{1}{L}U}$$

Eq Mécanique

$$\sum M = J\dot{\Omega}$$

$$J\dot{\Omega} = C_m - f\Omega$$

$$J\dot{\Omega} = K\,i - f\Omega$$

$$\boxed{\frac{d\Omega}{dt} = \frac{K}{J}i - \frac{f}{J}\Omega}$$

$$\begin{pmatrix} \frac{di(t)}{dt} \\ \frac{d\Omega(t)}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{K}{L} \\ \frac{K}{J} & -\frac{f}{J} \end{pmatrix} \begin{pmatrix} i(t) \\ \Omega(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} U$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} i(t) \\ \Omega(t) \end{pmatrix}$$

<u>Exemple 04</u> =

Soit l'eq diff suivante =

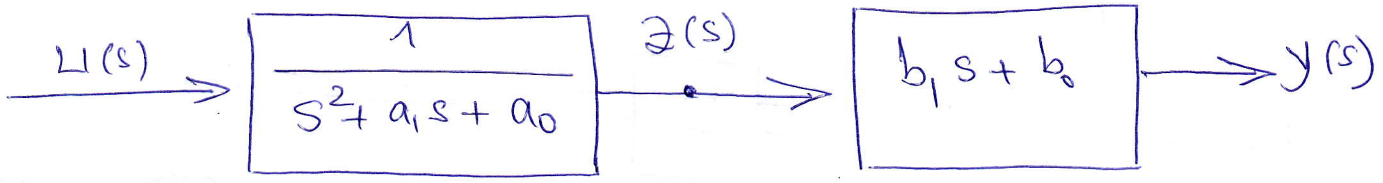
$$\ddot{y} + a_1\dot{y} + a_0 y = b_1\dot{U} + b_0 U \quad \text{------} \quad (*)$$

(64)

$$\mathcal{L}(*) \Rightarrow s^2 y(s) + a_1 s y(s) + a_0 y(s) = b_1 s u(s) + b_0 u(s)$$

$$y(s)\left[s^2 + a_1 s + a_0\right] = u(s)\left[b_1 s + b_0\right]$$

$$\frac{y(s)}{u(s)} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \quad \text{possant } z(s) = \frac{u(s)}{s^2 + a_1 s + a_0}$$



$$\begin{cases} z(s) = \dfrac{u(s)}{s^2 + a_1 s + a_0} \\ \\ y(s) = z(s)\left(b_1 s + b_0\right) \end{cases}$$

$\mathcal{L}^{-1}$

$$\begin{cases} \ddot{z}(t) + a_1 \dot{z}(t) + a_0 z(t) = u(t) \\ \\ y(t) = b_1 \dot{z}(t) + b_0 z(t) \end{cases}$$

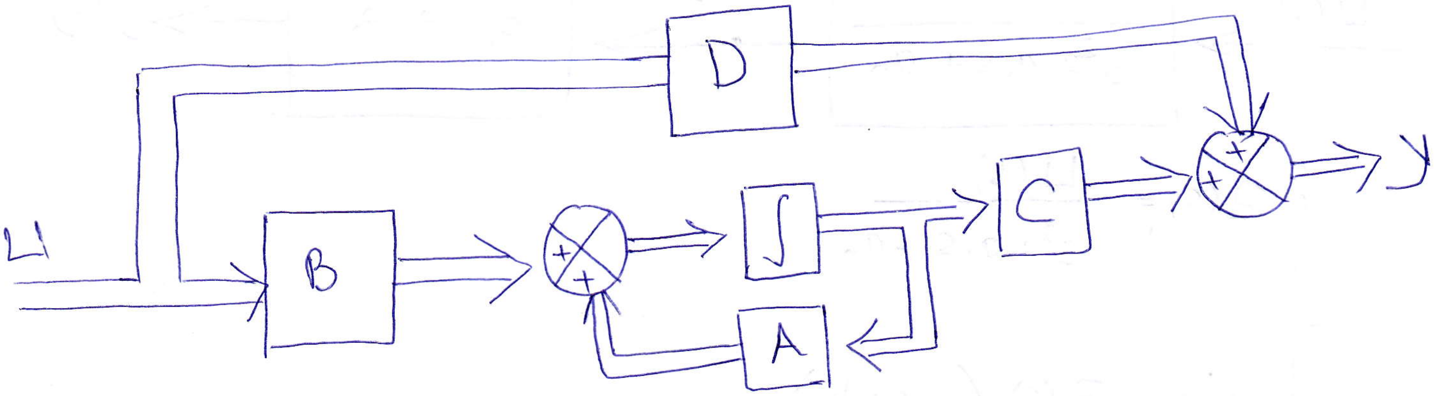
$$\begin{cases} x_1 = z(t) \\ \\ x_2 = \dot{z}(t) \end{cases} \rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \\ \dot{x}_2 = -a_0 x_1 - a_1 x_2 + u(t) \end{cases}$$

$$y(t) = b_0 x_1 + b_1 x_2 \qquad \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_0 & -a_1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} b_0 & b_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Diagramme de simulation d'un représentation continue

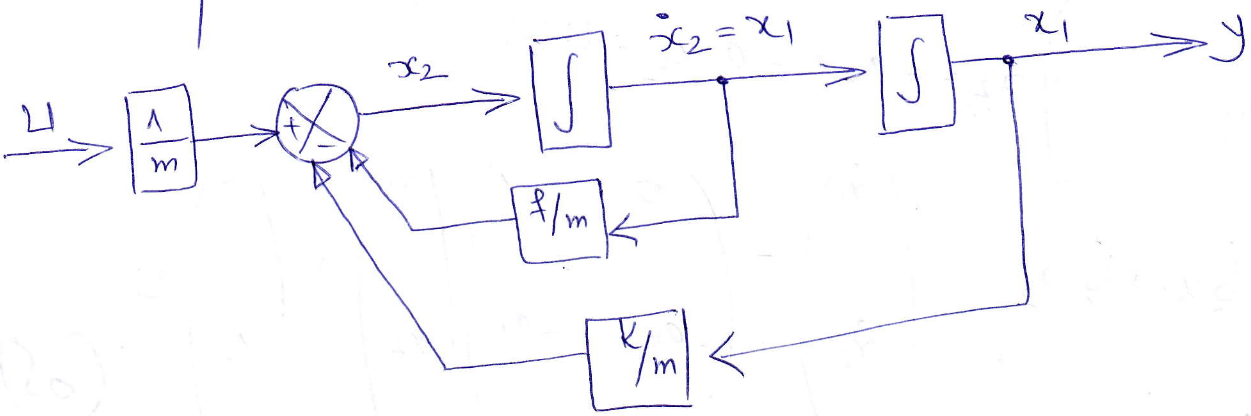
$$\begin{cases} \dot{X} = AX + BU \\ y = CX + DU \end{cases}$$



Exemple =

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{f}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} U \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{K}{m} x_1 - \frac{f}{m} x_2 + \frac{U}{m} \end{cases}$$



Représentation d'état des syst discrets

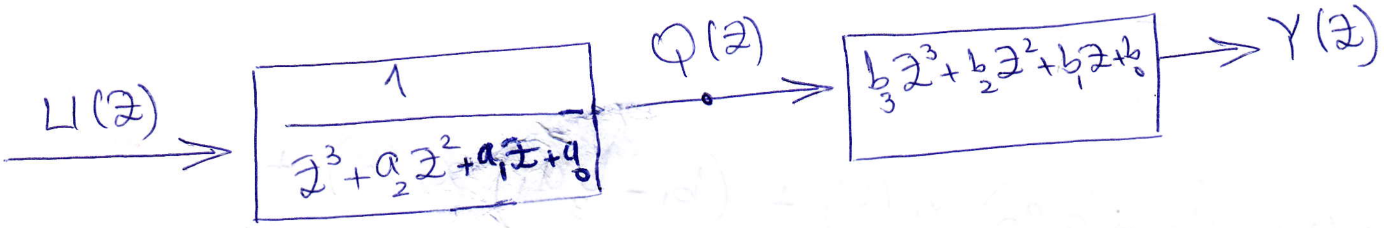
Exemple : soit l'eq au diff

$$y(k+3) + a_2\, y(k+2) + a_1\, y(k+1) + a_0\, y(k) = b_3\, u(k+3) + b_2\, u(k+2) + b_1\, u(k+1) + b_0\, u(k)$$

Variable Auxiliaire :

appliquant la T.Z on trouve

$$\left[z^3 + a_2 z^2 + a_1 z + a_0\right] Y(z) = \left[b_3 z^3 + b_2 z^2 + b_1 z + b_0\right] U(z)$$



$$\begin{cases} Q(z) = \dfrac{U(z)}{z^3 + a_2 z^2 + a_1 z + a_0} \\[2ex] Y(z) = \left(b_3 z^3 + b_2 z^2 + b_1 z + b_0\right) Q(z) \end{cases}$$

$$z^{-1} \longrightarrow \begin{cases} q(k+3) + a_2\, q(k+2) + a_1\, q(k+1) + a_0\, q(k) = u(k) \\[1ex] y(k) = b_3\, q(k+3) + b_2\, q(k+2) + b_1\, q(k+1) + b_0\, q(k) \end{cases}$$

$$\begin{cases} x_1(k) = q(k) \\ x_2(k) = q(k+1) \\ x_3(k) = q(k+2) \end{cases} \Rightarrow \begin{cases} x_1(k+1) = q(k+1) \\ x_2(k+1) = q(k+2) \\ x_3(k+1) = q(k+3) \end{cases}$$

$$\Rightarrow \begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = x_3(k) \\ x_3(k+1) = u(k) - a_0 x_1(k) - a_1 x_2(k) - a_2 x_3(k) \end{cases}$$

$$y(k) = b_0 x_1(k) + b_1 x_2(k) + b_2 x_3(k) + b_3\left[-a_0 x_1(k) - a_1 x_2(k) - a_2 x_3(k) + u(k)\right]$$

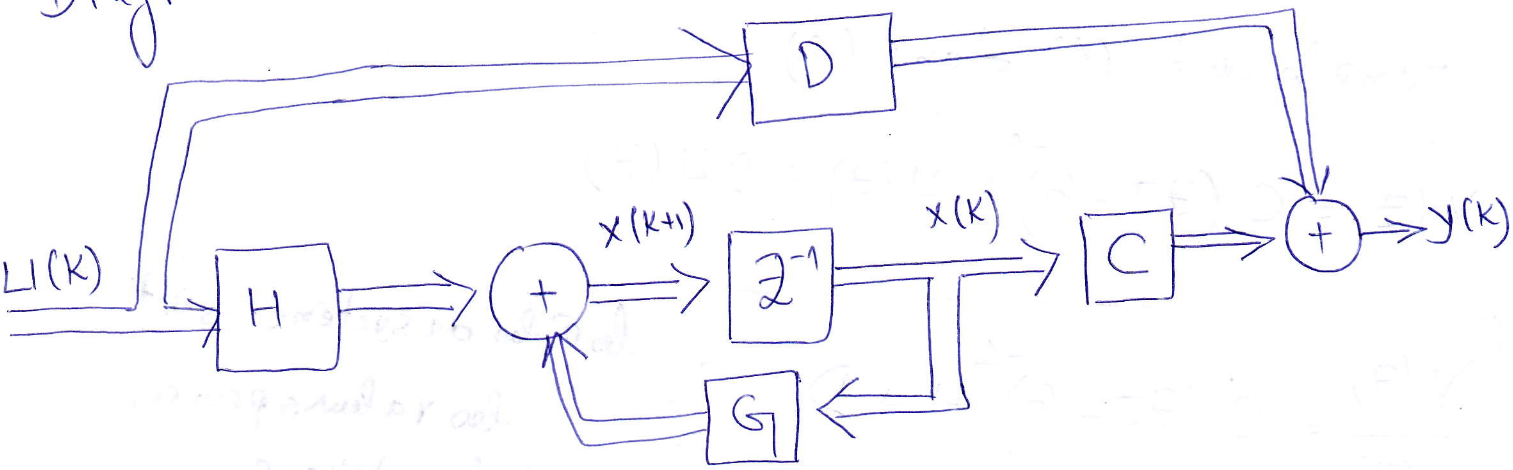
$$y(k) = (b_0 - b_3 a_0)\, x_1(k) + (b_1 - b_3 a_1)\, x_2(k) + (b_2 - b_3 a_2)\, x_3(k) + b_3 u(k)$$

$$y(k) = \underbrace{\begin{pmatrix} b_0 - b_3 a_0 & b_1 - b_3 a_1 & b_2 - b_3 a_2 \end{pmatrix}}_{C} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix} + \underbrace{(b_3)}_{D}\, u(k)$$

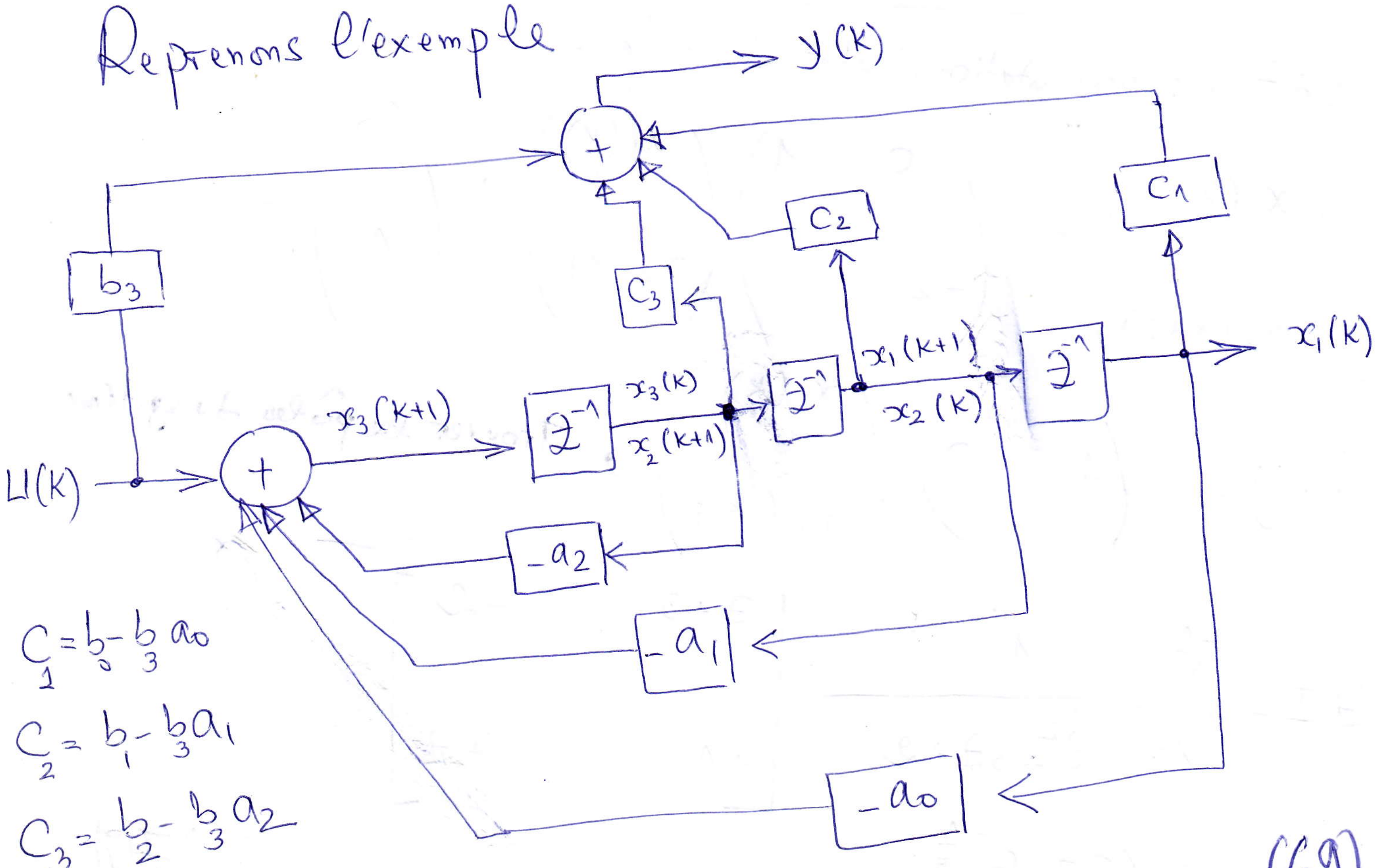
$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{pmatrix}}_{A \equiv G} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{B \equiv H} u(k)$$

$$\begin{cases} X(k+1) = G\,X(k) + H\,u(k) \\ y(k) = C\,X(k) + D\,u(k) \end{cases}$$

Diagramme de simulation



Reprenons l'exemple



$C_1 = b_0 - b_3 a_0$

$C_2 = b_1 - b_3 a_1$

$C_3 = b_2 - b_3 a_2$