3. 3. Formulation of the plate element for the study of plane elasticity problems:

Types of plane elasticity problems

Plane stresses: *the stresses are in a plane:*

If the plane is x-y, the stresses are a function of (x, y) *:*

$$\sigma_{zz} = \sigma_{yz} = \sigma_{xz} = 0 \quad or\ in\ \ engineering\ notations\!: \ \sigma_z = \tau_{yz} = \tau_{xz} = 0$$

Which gives in deformations:

$$\varepsilon_{yz} = \varepsilon_{xz} = 0, \varepsilon_{zz} = -\frac{\upsilon}{E}(\sigma_{xx} + \sigma_{zz}) \quad or\ in\ \ engineering\ notations\!: \ \varepsilon_{yz} = \varepsilon_{xz} = 0,$$

$$and\ \ \sigma_z = -\frac{\upsilon}{E}(\sigma_x + \sigma_z)$$

So, the displacement field is: $\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix}, w = 0$

Plane deformations: *the deformations are in a plane:*

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$$and\ \ \sigma_z = -\frac{\upsilon}{E}(\sigma_x + \sigma_z)$$

So, the displacement field is: $\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix}, w = 0$

The stresses are :

$$\sigma_{yz} = \sigma_{xz} = 0, \sigma_{zz} = \lambda(\varepsilon_{xx} + \varepsilon_{zz}) \ or\ in\ \ engineering\ notations\!: \ \tau_{yz} = \tau_{xz} = 0,$$

et $\sigma_z = \lambda(\varepsilon_x + \varepsilon_z)$ *is a function of de* (x, y) where

$$\lambda = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)}$$

3.3.1. Formulation of the triangular element

3.3.1.1. Basic equations:

- ***Displacements field***

$$U = [N]\{d\}; U = \begin{Bmatrix} u \\ v \end{Bmatrix}; \{d\} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}; [N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix}$$

- ***deformations field:***

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix}$$

$$\varepsilon = [B]\{d\} \rightarrow \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{2\Delta} \begin{bmatrix} y_2 - y_3 & 0 & y_3 - y_1 & 0 & y_1 - y_2 & 0 \\ 0 & x_3 - x_2 & 0 & x_1 - x_3 & 0 & x_2 - x_1 \\ x_3 - x_2 & y_2 - y_3 & x_1 - x_3 & y_3 - y_1 & x_2 - x_1 & y_1 - y_2 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix}$$

✓ ***Stresses field:***

In the 3D case we have:

$$\{\sigma\} = [C]\{\varepsilon\} \ \ ou \ \ \{\varepsilon\} = [C]^{-1}\{\sigma\} \ \text{où}: \ \{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \sigma_{z} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} \ \ and \ \{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \varepsilon_{z} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$

$$[C] = \frac{E}{(1-2v)(1+v)} \begin{bmatrix} 1-v & v & v & 0 & 0 & 0 \\ v & 1-v & v & 0 & 0 & 0 \\ v & v & 1-v & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-2v)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-2v)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-2v)/2 \end{bmatrix}$$

In the plane stresses case we have

$$\sigma_{zz} = 0 \ \rightarrow \frac{\lambda}{v}\big(v(+\varepsilon_{yy}) + (1-v)\varepsilon_{zz}\big) = 0 \rightarrow \varepsilon_{zz} = \frac{-v}{1-v}(\varepsilon_{xx} + \varepsilon_{yy})$$

$$\sigma_{xx} = \frac{\lambda}{v}\left((1-v)\varepsilon_{xx} + v\varepsilon_{yy} + v\frac{-v}{1-v}(\varepsilon_{xx} + \varepsilon_{yy}\right)$$

$$\sigma_{xx} = \frac{E}{1-v^2}\varepsilon_{xx} + \frac{vE}{1-v^2}\varepsilon_{yy}; \quad \sigma_{yy} = \frac{vE}{1-v^2}\varepsilon_{xx} + \frac{E}{1-v^2}\varepsilon_{yy}; \sigma_{xy} = \frac{E}{2(1+v)}2\varepsilon_{xy}$$

$$\{\sigma\} = [C]\{\varepsilon\} \rightarrow \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \dfrac{1-v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \dfrac{1-v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{Bmatrix}$$

In the plane deformations case, we have:

$$\{\sigma\} = [C]\{\varepsilon\} \rightarrow \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \dfrac{1-2v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \dfrac{1-2v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{Bmatrix}$$

3.3.1.2. Variational formulation:

$$\boldsymbol{\pi = U + P}$$

$$\boldsymbol{U} = \int_V \frac{1}{2}\boldsymbol{\varepsilon^T \sigma dV} = \frac{1}{2}\int_V ([\boldsymbol{B}]\{\boldsymbol{d}\})^T[\boldsymbol{C}][\boldsymbol{B}]\{\boldsymbol{d}\}\boldsymbol{dv} = \frac{1}{2}\{\boldsymbol{d}\}^T\left(\int_V [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\boldsymbol{dv}\right)\{\boldsymbol{d}\} = \frac{1}{2}\{\boldsymbol{d}\}^T[\boldsymbol{k}]\{\boldsymbol{d}\}$$

$$P = -\left(\{\boldsymbol{d}\}^T\{\boldsymbol{f_n}\} + \int_\Gamma U^T\{T\}\,ds + \int_V U^T\{X\}\,dv\right) = -\left(\{\boldsymbol{d}\}^T\{\boldsymbol{f_n}\} + \int_\Gamma U^T\{T\}\,ds + \int_V U^T\{X\}\,dv\right)$$

$$\boldsymbol{P} = -\left(\{\boldsymbol{d}\}^T\{\boldsymbol{f_n}\} + \int_\Gamma ([N]\{d\})^T\{T\}\,ds + \int_V ([N]\{d\})^T\{X\}\,dv\right)$$

$$\boldsymbol{P} = -\{\boldsymbol{d}\}^T\left(\{\boldsymbol{f_n}\} + \int_\Gamma [N]^T\{T\}\,ds + \int_V [N]^T\{X\}\,dv\right) = -\{\boldsymbol{d}\}^T\{\boldsymbol{f}\}$$

$$\boldsymbol{\pi} = \frac{1}{2}\{\boldsymbol{d}\}^T\left(\int_V [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\boldsymbol{dv}\right)\{\boldsymbol{d}\} - \{\boldsymbol{d}\}^T\{\boldsymbol{f}\}$$

The minimization of the potential energy which is expressed by $\frac{\partial \pi}{\partial d_i} = 0$ gives :

$$\left(\int_V [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\boldsymbol{dv}\right)\{\boldsymbol{d}\} - \{\boldsymbol{f}\} = \boldsymbol{0}$$

$$\{\boldsymbol{f}\} = [\boldsymbol{k}]\{\boldsymbol{d}\} \quad \textit{avec} \quad [\boldsymbol{k}] = \int_V [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\boldsymbol{dv}$$

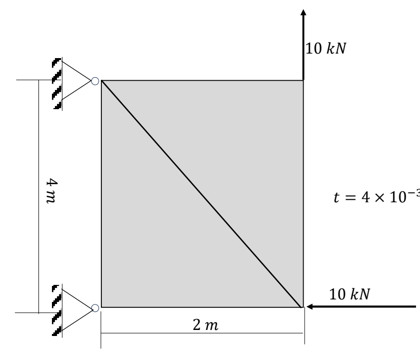
As $[\boldsymbol{B}]$ and $[\boldsymbol{C}]$ are constants :

$$[\boldsymbol{k}] = \int_V [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\boldsymbol{dv} = [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\int_V \boldsymbol{dv}$$

For a plate of constant thickness t : $\int_V \boldsymbol{dv} = \boldsymbol{\Delta t}$

$$[\boldsymbol{k}] = [\boldsymbol{B}]^T[\boldsymbol{C}][\boldsymbol{B}]\Delta \boldsymbol{t}$$

3.3.1.3. Example:



$t = 4 \times 10^{-3}\ m\ ;\ \nu = 0.3, E = 2 \times 10^8\ kN/m^2$

Element 1 :

$\boldsymbol{E = 2 \times 10^8\ kN/m^2\ ;\ \nu = 0.3; (x_1, y_1) = (0,0);\ (x_2, y_2) = (2,0); (x_3, y_3) = (0,4), t = 0.004;\ \Delta = 4}$

$$[\boldsymbol{B}]^1 = \begin{bmatrix} -0.50 & 0.00 & 0.50 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.25 & 0.00 & 0.00 & 0.00 & 0.25 \\ -0.25 & -0.50 & 0.00 & 0.50 & 0.25 & 0.00 \end{bmatrix}$$

$$[\boldsymbol{C}]^1 = \begin{bmatrix} 219780219.78 & 65934065.93 & 0.00 \\ 65934065.93 & 219780219.78 & 0.00 \\ 0.00 & 0.00 & 76923076.92 \end{bmatrix}$$

$$[\boldsymbol{k}]^1 = \begin{bmatrix} 956043.96 & 285714.29 & -879120.88 & -153846.15 & -76923.08 & -131868.13 \\ 285714.29 & 527472.53 & -131868.13 & -307692.31 & -153846.15 & -219780.22 \\ -879120.88 & -131868.13 & 879120.88 & 0.0000000 & 0.0000000 & 131868.13 \\ -153846.15 & -307692.31 & 0.0000000 & 307692.31 & 153846.15 & 0.0000000 \\ -76923.08 & -153846.15 & 0.0000000 & 153846.15 & 76923.08 & 0.0000000 \\ -131868.13 & -219780.22 & 131868.13 & 0.0000000 & 0.0000000 & 219780.22 \end{bmatrix}$$

Element 2 :

$$E = 2 \times 10^8 \frac{kN}{m^2};\ \nu = 0.3\ ;(x_1, y_1) = (2, 0)\ ;\ (x_2, y_2) = (2, 4)\ ;(x_3, y_3) = (0, 4)\ ;\ \ t = 0.004\ ;\ \Delta = 4$$

$$[B]^2 = \begin{bmatrix} 0.00 & 0.00 & 0.50 & 0.00 & -0.50 & 0.00 \\ 0.00 & -0.25 & 0.00 & 0.25 & 0.00 & 0.00 \\ -0.25 & 0.00 & 0.25 & 0.50 & 0.00 & -0.50 \end{bmatrix}$$

$$[C]^2 = \begin{bmatrix} 219780219.78 & 65934065.93 & 0.00 \\ 65934065.93 & 219780219.78 & 0.00 \\ 0.00 & 0.00 & 76923076.92 \end{bmatrix}$$

$$[k]^2 = \begin{bmatrix} 76923.08 & 0.0000000 & -76923.08 & -153846.15 & 0.0000000 & 153846.15 \\ 0.0000000 & 219780.22 & -131868.13 & -219780.22 & 131868.13 & 0.0000000 \\ -76923.08 & -131868.13 & 956043.96 & 285714.29 & -879120.88 & -153846.15 \\ -153846.15 & -219780.22 & 285714.29 & 527472.53 & -131868.13 & -307692.31 \\ 0.0000000 & 131868.13 & -879120.88 & -131868.13 & 879120.88 & 0.0000000 \\ 153846.15 & 0.0000000 & -153846.15 & -307692.31 & 0.0000000 & 307692.31 \end{bmatrix}$$

Boundary conditions:

$$D_1 = D_2 = D_5 = D_6 = 0$$

$$[k]^* = \begin{bmatrix} 956043.96 & 0.0000000 & -76923.08 & -153846.15 \\ 0.0000000 & 527472.53 & -131868.13 & -219780.22 \\ -76923.08 & -131868.13 & 956043.96 & 285714.29 \\ -153846.15 & -219780.22 & 285714.29 & 527472.53 \end{bmatrix}$$

$$\{F\}^* = \begin{Bmatrix} -10 \\ 0 \\ 0 \\ 10 \end{Bmatrix}$$

$$\begin{Bmatrix} -10 \\ 0 \\ 0 \\ 10 \end{Bmatrix} = \begin{bmatrix} 956043.96 & 0.0000000 & -76923.08 & -153846.15 \\ 0.0000000 & 527472.53 & -131868.13 & -219780.22 \\ -76923.08 & -131868.13 & 956043.96 & 285714.29 \\ -153846.15 & -219780.22 & 285714.29 & 527472.53 \end{bmatrix} \begin{Bmatrix} D_3 \\ D_4 \\ D_7 \\ D_8 \end{Bmatrix}$$

$$\begin{Bmatrix} D_3 \\ D_4 \\ D_7 \\ D_8 \end{Bmatrix} = \begin{Bmatrix} -7.143 \\ 8.315 \\ -6.570 \\ 23.898 \end{Bmatrix} \times \mathbf{10^{-6}}$$

Stresses in the elements:

Element 1 : $\{d\}^1 = \begin{Bmatrix} 0 \\ 0 \\ -7.143 \\ 8.315 \\ 0 \\ 0 \end{Bmatrix}$

$$\sigma^1 = [C]^1[B]^1\{d\}^1$$

$$\sigma^1 = \begin{bmatrix} 219780219.78 & 65934065.93 & 0.00 \\ 65934065.93 & 219780219.78 & 0.00 \\ 0.00 & 0.00 & 76923076.92 \end{bmatrix} \begin{bmatrix} -0.50 & 0.00 & 0.50 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.25 & 0.00 & 0.00 & 0.00 & 0.25 \\ -0.25 & -0.50 & 0.00 & 0.50 & 0.25 & 0.00 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -7.143 \\ 8.315 \\ 0 \\ 0 \end{Bmatrix} 10^{-6}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^1 = \begin{Bmatrix} -784.94 \\ -235.48 \\ 319.81 \end{Bmatrix}$$

Element 2 : $\{d\}^1 = \begin{Bmatrix} -7.143 \\ 8.315 \\ -6.570 \\ 23.898 \\ 0 \\ 0 \end{Bmatrix} \times 10^{-6}$

$$\sigma^2 = [C]^2[B]^2\{d\}^2$$

$$\sigma^2 = \begin{bmatrix} 219780219.78 & 65934065.93 & 0.00 \\ 65934065.93 & 219780219.78 & 0.00 \\ 0.00 & 0.00 & 76923076.92 \end{bmatrix} \begin{bmatrix} 0.00 & 0.00 & 0.50 & 0.00 & -0.50 & 0.00 \\ 0.00 & -0.25 & 0.00 & 0.25 & 0.00 & 0.00 \\ -0.25 & 0.00 & 0.25 & 0.50 & 0.00 & -0.50 \end{bmatrix} \begin{Bmatrix} -7.143 \\ 8.315 \\ -6.570 \\ 23.898 \\ 0 \\ 0 \end{Bmatrix} 10^{-6}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^2 = \begin{Bmatrix} -465.09 \\ 639.63 \\ 930.18 \end{Bmatrix}$$

3.3.2 ***Formulation of the rectangular element***

$$U = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_2 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix}, \quad avec \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{ab} \begin{bmatrix} -b+y & 0 & b-y & 0 & y & 0 & -y & 0 \\ 0 & -a+x & 0 & -x & 0 & x & 0 & a-x \\ -a+x & -b+y & -x & b-y & x & y & a-x & -y \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix}$$

$$\{\sigma\} = [C]\{\varepsilon\}$$

In plane stresses $\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix}$; $\begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{Bmatrix}$

In plane stresses:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & \frac{1-2\upsilon}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix};\quad \begin{Bmatrix} \sigma_{x} \\ \sigma_{y} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & \frac{1-2\upsilon}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{Bmatrix}$$

For a plate with constant thickness t:

$$[\boldsymbol{k}] = \boldsymbol{t} \int_0^{\boldsymbol{a}} \int_0^{\boldsymbol{b}} [\boldsymbol{B}]^{\boldsymbol{T}} [\boldsymbol{C}][\boldsymbol{B}] \boldsymbol{dxdy}$$