Chapter 2: Finite elements in one dimension

Introduction

In this chapter, we focus on structures composed of elements whose behavior is governed by differential equations (equation with a single variable). This includes systems of springs, planar or spatial trusses, continuous beams, grids of beams, and planar or spatial frames.

Other examples: flow through pipes, seepage through narrow, confined soil mass; heat conduction across a fire wall; flow analysis in channels …

For each of these cases, we will first formulate the element in a coordinate system specific to the element called a local coordinate system (i.e., write the relationships between its nodal forces and its nodal displacements or its degrees of freedom in the form of a system of linear equations).

$$\{\boldsymbol{f}\}^{e} = [\boldsymbol{k}]^{e}\{\boldsymbol{d}\}^{e} \qquad \text{where:}$$

$\{f\}^e$ is the element nodal forces vector, including the equivalent nodal forces for the loads between nodes,

$[k]^e$ is a square matrix that depends on the geometric and physical characteristics of the element and is called the stiffness matrix of the element, and

$\{d\}^e$ is the element nodal displacements vector or the element degrees of freedom vector.

Next, we will focus on the complete structure to form - by assembling its elements in a coordinate system chosen for the entire structure, called the global coordinate system - a global system:

$$\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{D}\} \quad Where:$$

$\{F\}$ is the nodal forces vector of the structure, including the equivalent nodal forces for the loads between nodes of its elements,

$[K]$ is the global stiffness matrix of the entire structure and

$\{D\}$ is the vector of nodal displacements or the vector of degrees of freedom of the structure.

For these cases, the elements used are straight elements. The common element is the two-node element (linear element), but we can also use the three-node element (quadratic element) or the four-node element (cubic element).