UNIVERSITE MOHAMED BOUDIAF - M'SILA
FACULTE DE TECHNOLOGIE
DEPARTEMENT DE GENIE CIVIL

FILIERE : Génie Civil
SPECIALITE : Master Géotechnique (M1/S1)
COURS : Mécanique des milieux continus

INTERROGATION N°1

(Date : 30/11/2025 – Durée : 1 heure 30)

Ex. 1 (12 points) Considérons les vecteurs $\bar{U} = {}^t\{5 \quad 5 \quad 0\}$ et $\bar{V} = {}^t\{5 \quad -5 \quad 0\}$ définis dans le repère orthonormé $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$. On demande de calculer les opérations suivantes et de commenter éventuellement les résultats correspondants :

- Les normes $r_{\bar{U}}$ et $r_{\bar{V}}$
- Le produit scalaire $\bar{U} \cdot \bar{V}$
- Le produit vectoriel $\bar{W} = \bar{U} \wedge \bar{V}$
- Le produit mixte $(\bar{U}, \bar{V}, \bar{W})$
- Le produit tensoriel $\bar{\bar{T}} = \bar{U} \otimes \bar{V}$
- Le déterminant du tenseur $\bar{\bar{T}}$
- La trace du tenseur $\bar{\bar{T}}$
- La transposée du tenseur $\bar{\bar{T}}$
- La partie symétrique $\bar{\bar{T}}_s$
- La partie antisymétrique $\bar{\bar{T}}_a$

Ex. 2 (8 points) Soit le tenseur suivant :

$$\bar{\bar{M}} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 3 \\ 2 & 1 & -1 \end{pmatrix}$$

On demande de calculer :

- $Tr(\bar{\bar{M}}) = M_{pp} = ?$
- $[Tr(\bar{\bar{M}})]^2 = M_{pp} M_{qq} = ?$
- $Tr(\bar{\bar{M}}^2) = M_{pq} M_{qp} = ?$
- $Tr(\bar{\bar{M}}\ {}^t\bar{\bar{M}}) = M_{pq} M_{pq} = ?$

N.B. : Seules les notes de cours sont autorisées. Ne pas oublier d'encadrer les résultats.

Bonne chance

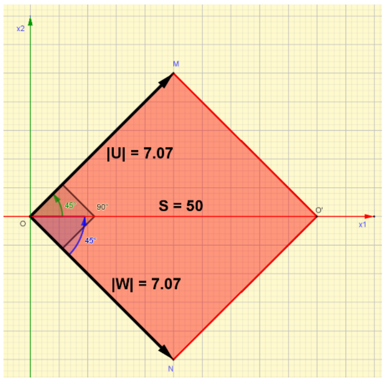
Prof. Mohamed Khemissa

Mécanique des milieux continus – Interro. N° : 1 (*30/11/2025*)

Corrigé

Exercice 1

Les vecteurs considérés sont :



► $\bar{U} = {}^t\{5 \quad 5 \quad 0\} = \begin{Bmatrix} 5 \\ 5 \\ 0 \end{Bmatrix} \qquad / \quad (\bar{x}_1, \bar{x}_2, \bar{x}_3)$

► $\bar{V} = {}^t\{5 \quad -5 \quad 0\} = \begin{Bmatrix} 5 \\ -5 \\ 0 \end{Bmatrix} \qquad / \quad (\bar{x}_1, \bar{x}_2, \bar{x}_3)$

- **<u>Normes</u> :**

Norme du vecteur $\bar{A}$: $\boldsymbol{r = \|\bar{A}\| = \sqrt{A_p A_p} \quad | \quad p = 1, 2, 3}$

$\Rightarrow \quad r_{\bar{U}} = \|\bar{U}\| = \sqrt{U_1^2 + U_2^2 + U_3^2} = \sqrt{5^2 + 5^2 + 0^2} = 5\sqrt{2} \qquad$ #1

$\Rightarrow \quad r_{\bar{V}} = \|\bar{V}\| = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{5^2 + (-5)^2 + 0^2} = 5\sqrt{2} \qquad$ #1

<u>Remarque</u> :

$r_{\bar{U}} = r_{\bar{V}} = 5\sqrt{2} \quad \Rightarrow$ Les deux vecteurs sont orthogonaux et de même longueur.

- **Produit scalaire : $\boldsymbol{\bar{U} \cdot \bar{V}}$**

$\boldsymbol{\bar{U} \cdot \bar{V} = U_p V_p} \quad \Rightarrow \quad \bar{U} \cdot \bar{V} = U_1 V_1 + U_2 V_2 + U_3 V_3 = 5 \times 5 + 5 \times (-5) + 0 \times 0 = 0 \qquad$ #1

<u>Vérification 1</u> :

$\boldsymbol{\bar{U} \cdot \bar{V} = \frac{1}{4}(\|\bar{U} + \bar{V}\|^2 - \|\bar{U} - \bar{V}\|^2)} \quad | \quad \bar{U} + \bar{V} = {}^t\{10 \quad 0 \quad 0\}$ et $\bar{U} - \bar{V} = {}^t\{0 \quad 10 \quad 0\}$

$\Rightarrow \quad \bar{U} \cdot \bar{V} = \frac{1}{4} \times \left[\left(\sqrt{10^2 + 0^2 + 0^2}\right)^2 - \left(\sqrt{0^2 + 10^2 + 0^2}\right)^2\right] = \frac{1}{4}(100 - 100) = 0$ *Ok.*

<u>Vérification 2</u> :

$\boldsymbol{\bar{U} \cdot \bar{V} = \|\bar{U}\| \times \|\bar{V}\| \times \cos(\alpha)} \quad | \quad \alpha = Angle(\bar{U}, \bar{V}) \quad \& \quad \bar{U} \perp \bar{V} \quad \Rightarrow \quad \alpha = \pi/2$

$\Rightarrow \quad \cos(\alpha) = \frac{\bar{U} \cdot \bar{V}}{\|\bar{U}\| \times \|\bar{V}\|} = \frac{0}{5\sqrt{2} \times 5\sqrt{2}} = 0 \quad \Rightarrow \quad \alpha = \pi/2$ *Ok.*

- **Produit vectoriel : $\boldsymbol{\bar{W} = \bar{U} \wedge \bar{V}}$**

$$\boldsymbol{\bar{W} = \bar{U} \wedge \bar{V} = \begin{vmatrix} \bar{x}_1 & \bar{x}_2 & \bar{x}_3 \\ U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \end{vmatrix} = \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix} = {}^t\{W_1 \quad W_2 \quad W_3\}} \qquad / \quad (\bar{x}_1, \bar{x}_2, \bar{x}_3)$$

$$\Rightarrow \quad \bar{W} = \bar{U} \wedge \bar{V} = \begin{vmatrix} \bar{x}_1 & \bar{x}_2 & \bar{x}_3 \\ 5 & 5 & 0 \\ 5 & -5 & 0 \end{vmatrix} = \begin{vmatrix} 5 & 0 \\ -5 & 0 \end{vmatrix} \bar{x}_1 - \begin{vmatrix} 5 & 0 \\ 5 & 0 \end{vmatrix} \bar{x}_2 + \begin{vmatrix} 5 & 5 \\ 5 & -5 \end{vmatrix} \bar{x}_3$$

$$= 0\bar{x}_1 + 0\bar{x}_2 - 50\bar{x}_3$$

$$\Rightarrow \quad \bar{W} = \begin{Bmatrix} 0 \\ 0 \\ -50 \end{Bmatrix} = {}^t\{0 \quad 0 \quad -50\} \; / \; (\bar{x}_1, \bar{x}_2, \bar{x}_3)$$ #1

Vérification :

$$\overline{\boldsymbol{W}} = \overline{\boldsymbol{U}} \wedge \overline{\boldsymbol{V}}$$

$$\Rightarrow \quad \boldsymbol{W_i = \epsilon_{ijk} U_j V_k} \quad | \quad \boldsymbol{\epsilon_{ijk}} = \begin{cases} \boldsymbol{+1 \;\; si\; permutation\; paire : 1,2,3 \;/\; 2,3,1 \;/\; 3,1,2} \\ \boldsymbol{-1 \;\; si\; permutation\; impaire : 1,3,2 \;/\; 2,1,3 \;/\; 3,2,1} \end{cases}$$

$$\Rightarrow \begin{cases} W_1 = \epsilon_{123} U_2 V_3 + \epsilon_{132} U_3 V_2 = +1 \times 5 \times 0 - 1 \times 0 \times (-5) = 0 \\ W_2 = \epsilon_{231} U_3 V_1 + \epsilon_{213} U_1 V_3 = +1 \times 0 \times 5 - 1 \times 5 \times 0 = 0 \\ W_3 = \epsilon_{312} U_1 V_2 + \epsilon_{321} U_2 V_1 = +1 \times 5 \times (-5) - 1 \times 5 \times 5 = -50 \end{cases} \qquad \textit{Ok.}$$

Symbole de permutation
$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = +1$
$\epsilon_{132} = \epsilon_{213} = \epsilon_{321} = -1$

- **Produit mixte : $(\overline{\boldsymbol{U}}, \overline{\boldsymbol{V}}, \overline{\boldsymbol{W}})$**

$$(\overline{\boldsymbol{U}}, \overline{\boldsymbol{V}}, \overline{\boldsymbol{W}}) = \begin{vmatrix} \boldsymbol{U_1} & \boldsymbol{U_2} & \boldsymbol{U_3} \\ \boldsymbol{V_1} & \boldsymbol{V_2} & \boldsymbol{V_3} \\ \boldsymbol{W_1} & \boldsymbol{W_2} & \boldsymbol{W_3} \end{vmatrix}$$

$$\Rightarrow \quad (\overline{U}, \overline{V}, \overline{W}) = \begin{vmatrix} 5 & 5 & 0 \\ 5 & -5 & 0 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= 5 \times \begin{vmatrix} -5 & 0 \\ 0 & -50 \end{vmatrix} - 5 \times \begin{vmatrix} 5 & 0 \\ 0 & -50 \end{vmatrix} + 0 \times \begin{vmatrix} 5 & -5 \\ 0 & 0 \end{vmatrix}$$

$$= 5 \times (-5) \times (-50) - 5 \times 5 \times (-50) + 0$$

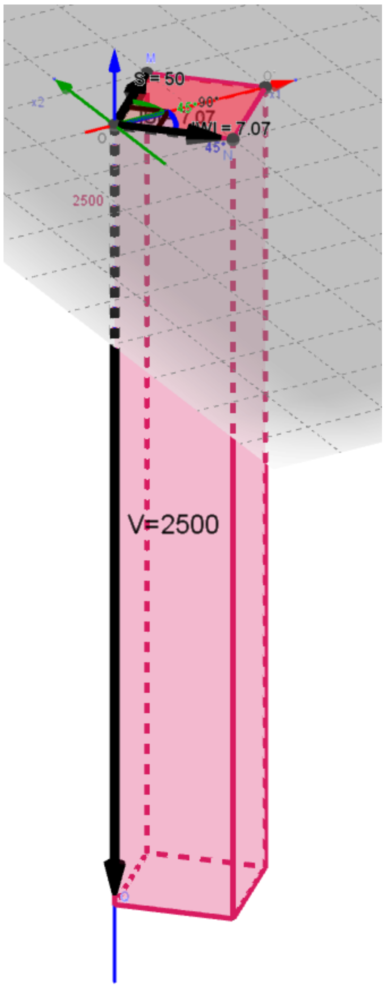
$$= 2500 \qquad \#1$$

Vérification : $\boldsymbol{V} = |(\overline{\boldsymbol{U}}, \overline{\boldsymbol{V}}, \overline{\boldsymbol{W}})| = \boldsymbol{S \times h}$

$$h = |W_3| = 50$$

$$S = \|\overline{V}\| \times \|\overline{W}\| \times sin(\alpha) = 5\sqrt{2} \times 5\sqrt{2} \times sin\left(\frac{\pi}{2}\right) = 50$$

$$\Rightarrow \quad V = 50 \times 50 = 2500 \qquad \textit{Ok.}$$



- **Produit tensoriel : $\overline{\overline{\boldsymbol{T}}} = \overline{\boldsymbol{U}} \otimes \overline{\boldsymbol{V}}$**

$$\overline{\overline{\boldsymbol{T}}} = \overline{\boldsymbol{U}} \otimes \overline{\boldsymbol{V}} \quad \Rightarrow \quad \overline{\overline{\boldsymbol{T}}} = [\boldsymbol{t_{ij}}] = [\boldsymbol{U_i V_j}] = \begin{bmatrix} \boldsymbol{U_1V_1} & \boldsymbol{U_1V_2} & \boldsymbol{U_1V_3} \\ \boldsymbol{U_2V_1} & \boldsymbol{U_2V_2} & \boldsymbol{U_2V_3} \\ \boldsymbol{U_3V_1} & \boldsymbol{U_3V_2} & \boldsymbol{U_3V_3} \end{bmatrix}$$

$$\Rightarrow \quad \overline{\overline{T}} = \begin{bmatrix} 5 \times 5 & 5 \times (-5) & 5 \times 0 \\ 5 \times 5 & 5 \times (-5) & 5 \times 0 \\ 0 \times 5 & 0 \times (-5) & 0 \times 0 \end{bmatrix}$$

$$= 25 \times \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \#1$$

- **Trace du tenseur $\overline{\overline{\boldsymbol{T}}}$**

$$\boldsymbol{Tr(\overline{\overline{T}}) = t_{ij}} \quad \Rightarrow \quad \boldsymbol{t_{ii} = t_{11} + t_{22} + t_{33}}$$

$$\Rightarrow \quad t_{ii} = 25 \times (1 - 1 + 0) = 0 \qquad \#1$$

$\Rightarrow$ C’est un tenseur déviatorique #1

- **Déterminant du tenseur $\bar{\bar{T}}$**

$$\boldsymbol{det(\bar{\bar{T}}) = |t_{ij}| = \begin{vmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{vmatrix}}$$

$$\Rightarrow \quad det(\bar{\bar{T}}) = 25 \times \begin{vmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 25 \times \left[1 \times \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - (-1) \times \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}\right] = 0 \qquad \#1$$

<u>Vérification</u> :

$$\boldsymbol{det(\bar{\bar{T}}) = \frac{1}{6}\epsilon_{ijk}\epsilon_{pqr}t_{ip}t_{jq}t_{kr}}$$

$$\Rightarrow \quad det(\bar{\bar{T}}) = \frac{1}{6}\begin{pmatrix} \epsilon_{123}\epsilon_{123}t_{11}t_{22}t_{33} + \epsilon_{123}\epsilon_{231}t_{12}t_{23}t_{31} + \epsilon_{123}\epsilon_{312}t_{13}t_{21}t_{32} \\ + \\ \epsilon_{123}\epsilon_{132}t_{11}t_{23}t_{32} + \epsilon_{123}\epsilon_{213}t_{12}t_{21}t_{33} + \epsilon_{123}\epsilon_{321}t_{13}t_{22}t_{31} \\ + \\ \epsilon_{231}\epsilon_{123}t_{21}t_{32}t_{13} + \epsilon_{231}\epsilon_{231}t_{22}t_{33}t_{11} + \epsilon_{231}\epsilon_{312}t_{23}t_{31}t_{12} \\ + \\ \epsilon_{231}\epsilon_{132}t_{21}t_{33}t_{12} + \epsilon_{231}\epsilon_{213}t_{22}t_{31}t_{13} + \epsilon_{231}\epsilon_{321}t_{23}t_{32}t_{11} \\ + \\ \epsilon_{312}\epsilon_{123}t_{31}t_{12}t_{23} + \epsilon_{312}\epsilon_{231}t_{32}t_{13}t_{21} + \epsilon_{312}\epsilon_{312}t_{33}t_{11}t_{22} \\ + \\ \epsilon_{312}\epsilon_{132}t_{31}t_{13}t_{22} + \epsilon_{312}\epsilon_{213}t_{32}t_{11}t_{23} + \epsilon_{312}\epsilon_{321}t_{33}t_{12}t_{21} \\ + \\ \epsilon_{132}\epsilon_{123}t_{11}t_{32}t_{23} + \epsilon_{132}\epsilon_{231}t_{12}t_{33}t_{21} + \epsilon_{132}\epsilon_{312}t_{13}t_{31}t_{22} \\ + \\ \epsilon_{132}\epsilon_{132}t_{11}t_{33}t_{22} + \epsilon_{132}\epsilon_{213}t_{12}t_{31}t_{23} + \epsilon_{132}\epsilon_{321}t_{13}t_{32}t_{21} \\ + \\ \epsilon_{213}\epsilon_{123}t_{21}t_{12}t_{33} + \epsilon_{213}\epsilon_{231}t_{22}t_{13}t_{31} + \epsilon_{213}\epsilon_{312}t_{23}t_{11}t_{32} \\ + \\ \epsilon_{213}\epsilon_{132}t_{21}t_{13}t_{32} + \epsilon_{213}\epsilon_{213}t_{22}t_{11}t_{33} + \epsilon_{213}\epsilon_{321}t_{23}t_{12}t_{31} \\ + \\ \epsilon_{321}\epsilon_{123}t_{31}t_{22}t_{13} + \epsilon_{321}\epsilon_{231}t_{32}t_{23}t_{11} + \epsilon_{321}\epsilon_{312}t_{33}t_{21}t_{12} \\ + \\ \epsilon_{321}\epsilon_{132}t_{31}t_{23}t_{12} + \epsilon_{321}\epsilon_{213}t_{32}t_{21}t_{13} + \epsilon_{321}\epsilon_{321}t_{33}t_{22}t_{11} \end{pmatrix}$$

$$\Rightarrow \quad det(\bar{\bar{T}}) = \frac{25}{6}\begin{pmatrix} (+1)\times(+1)\times 1\times(-1)\times 0 + (+1)\times(+1)\times(-1)\times 0\times 0 + (+1)\times(+1)\times 0\times 1\times 0 \\ + \\ (+1)\times(-1)\times 1\times 0\times 0 + (+1)\times(-1)\times(-1)\times 1\times 0 + (+1)\times(-1)\times 0\times(-1)\times 0 \\ + \\ (+1)\times(+1)\times 1\times 0\times 0 + (+1)\times(+1)\times(-1)\times 0\times 1 + (+1)\times(+1)\times 0\times 0\times(-1) \\ + \\ (+1)\times(-1)\times 1\times 0\times(-1) + (+1)\times(-1)\times(-1)\times 0\times 0 + (+1)\times(-1)\times 0\times 0\times 1 \\ + \\ (+1)\times(+1)\times 0\times(-1)\times 0 + (+1)\times 0\times 0\times 0\times 1 + (+1)\times(+1)\times 0\times 1\times(-1) \\ + \\ (+1)\times(-1)\times 0\times 0\times(-1) + (+1)\times(-1)\times 0\times 1\times 0 + (+1)\times(-1)\times 0\times(-1)\times 1 \\ + \\ (-1)\times(+1)\times 1\times 0\times 0 + (-1)\times(+1)\times(-1)\times 0\times 1 + (-1)\times(+1)\times 0\times 0\times(-1) \\ + \\ (-1)\times(-1)\times 1\times 0\times(-1) + (-1)\times(-1)\times(-1)\times 0\times 0 + (-1)\times(-1)\times 0\times 0\times 1 \\ + \\ (-1)\times(+1)\times 1\times(-1)\times 0 + (-1)\times(+1)\times(-1)\times 0\times 0 + (-1)\times(+1)\times 0\times 1\times 0 \\ + \\ (-1)\times(+1)\times 1\times 0\times 0 + (-1)\times(+1)\times(-1)\times 1\times 0 + (-1)\times(+1)\times 0\times(-1)\times 0 \\ + \\ (-1)\times(+1)\times 0\times(-1)\times 0 + (-1)\times(-1)\times 0\times 0\times 1 + (-1)\times(-1)\times 0\times 1\times(-1) \\ + \\ (-1)\times(-1)\times 0\times 0\times(-1) + (-1)\times(-1)\times 0\times 1\times 0 + (-1)\times(-1)\times 0\times(-1)\times 1 \end{pmatrix}$$

$$\Rightarrow \quad det(\bar{\bar{T}}) = 0 \qquad \textit{Ok.}$$

- **Transposée du tenseur $\bar{\bar{T}}$**

$$\bar{\bar{\boldsymbol{T}}} = [\boldsymbol{t_{ij}}] \quad \Rightarrow \quad {}^t\bar{\bar{\boldsymbol{T}}} = [\boldsymbol{t_{ji}}]$$

$$\Rightarrow \quad \bar{\bar{T}} = [t_{ij}] = 25 \times \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad {}^t\bar{\bar{T}} = [t_{ji}] = 25 \times \begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \#1$$

- **Partie symétrique $\bar{\bar{T}}_s$ du tenseur $\bar{\bar{T}}$**

$$\bar{\bar{\boldsymbol{T}}}_s = \frac{1}{2}\left(\bar{\bar{\boldsymbol{T}}} + {}^t\bar{\bar{\boldsymbol{T}}}\right) \quad \Rightarrow \quad (\boldsymbol{t_s})_{ij} = \frac{1}{2}\left(\boldsymbol{t_{ij}} + \boldsymbol{t_{ji}}\right)$$

$$\Rightarrow \quad \bar{\bar{T}}_s = \frac{25}{2} \times \left[\begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] = 25 \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \#1$$

- **Partie antisymétrique $\bar{\bar{T}}_a$ du tenseur $\bar{\bar{T}}$**

$$\bar{\bar{\boldsymbol{T}}}_a = \frac{1}{2}\left(\bar{\bar{\boldsymbol{T}}} - {}^t\bar{\bar{\boldsymbol{T}}}\right) \quad \Rightarrow \quad (\boldsymbol{t_a})_{ij} = \frac{1}{2}\left(\boldsymbol{t_{ij}} - \boldsymbol{t_{ji}}\right)$$

$$\Rightarrow \quad \bar{\bar{T}}_a = \frac{25}{2} \times \left[\begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] = 25 \times \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \#1$$

Exercice 2

- $\boldsymbol{Tr\left(\bar{\bar{M}}\right) = M_{pp}}$

$$\Rightarrow \quad Tr\left(\bar{\bar{M}}\right) = M_{pp} = M_{11} + M_{22} + M_{33} = 1 + 2 - 1 = 2 \qquad \#2$$

- $\boldsymbol{\left[Tr\left(\bar{\bar{M}}\right)\right]^2 = M_{pp}M_{qq}}$

$$\Rightarrow \quad \left[Tr\left(\bar{\bar{M}}\right)\right]^2 = M_{pp}M_{qq} = (M_{11} + M_{22} + M_{33})(M_{11} + M_{22} + M_{33})$$

$$\Rightarrow \quad \left[Tr\left(\bar{\bar{M}}\right)\right]^2 = (1 + 2 - 1) \times (1 + 2 - 1) = 2 \times 2 = 4 \qquad \#2$$

Vérification :

$$Tr\left(\bar{\bar{M}}\right) = 2 \quad \Rightarrow \quad \left[Tr\left(\bar{\bar{M}}\right)\right]^2 = 2^2 = 4 \qquad \textit{Ok.}$$

- $\boldsymbol{Tr\left(\bar{\bar{M}}^2\right) = M_{pq}M_{qp}}$

$$\Rightarrow \quad Tr\left(\bar{\bar{M}}^2\right) = M_{pq}M_{qp} = M_{11}^2 + M_{22}^2 + M_{33}^2 + 2(M_{12}M_{21} + M_{23}M_{32} + M_{13}M_{31})$$

$$\Rightarrow \quad Tr\left(\bar{\bar{M}}^2\right) = M_{pq}M_{qp} = 1^2 + 2^2 + (-1)^2 + 2 \times [0 \times (-1) + 3 \times 1 + 0 \times 2] = 12 \qquad \#2$$

Vérification :

$$\bar{\bar{M}} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 3 \\ 2 & 1 & -1 \end{pmatrix} \quad \Rightarrow \quad \bar{\bar{M}}^2 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 3 \\ 2 & 1 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 3 \\ 2 & 1 & -1 \end{pmatrix}$$

$$\Rightarrow \quad \bar{\bar{M}}^2 = \begin{pmatrix} 1 \times 1 + 0 \times (-1) + 0 \times 2 & 1 \times 0 + 0 \times 2 + 0 \times 1 & 1 \times 0 + 0 \times 3 + 0 \times (-1) \\ (-1) \times 1 + 2 \times (-1) + 3 \times 2 & (-1) \times 0 + 2 \times 2 + 3 \times 1 & (-1) \times 0 + 2 \times 3 + 3 \times (-1) \\ 2 \times 1 + 1 \times (-1) + (-1) \times 2 & 2 \times 0 + 1 \times 2 + (-1) \times 1 & 2 \times 0 + 1 \times 3 + (-1) \times (-1) \end{pmatrix}$$

$$\Rightarrow \quad \bar{\bar{M}}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 7 & 3 \\ -1 & 1 & 4 \end{pmatrix} \quad \Rightarrow \quad Tr(\bar{\bar{M}}^2) = (\bar{\bar{M}}^2)_{ii} = 1 + 7 + 4 = 12 \qquad Ok.$$

- $\boldsymbol{Tr(\bar{\bar{M}}\ {}^t\bar{\bar{M}}) = M_{pq}M_{pq} = Tr({}^t\bar{\bar{M}}\bar{\bar{M}})}$

$$\Rightarrow \quad Tr(\bar{\bar{M}}\ {}^t\bar{\bar{M}}) = M_{pq}M_{pq} = M_{11}M_{11} + M_{12}M_{12} + M_{13}M_{13} + M_{21}M_{21} + M_{22}M_{22} + M_{23}M_{23} + \\ + M_{31}M_{31} + M_{32}M_{32} + M_{33}M_{33}$$

$$\Rightarrow \quad Tr(\bar{\bar{M}}\ {}^t\bar{\bar{M}}) = M_{pq}M_{pq} = 1 \times 1 + 0 \times 0 + 0 \times 0 + (-1) \times (-1) + 2 \times 2 + 3 \times 3 + \\ + 2 \times 2 + 1 \times 1 + (-1) \times (-1) = 21 \qquad \#2$$

Vérification :

$$\bar{\bar{M}} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 3 \\ 2 & 1 & -1 \end{pmatrix} \quad \Rightarrow \quad {}^t\bar{\bar{M}} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix}$$

$$\Rightarrow \quad \bar{\bar{M}}\ {}^t\bar{\bar{M}} = \begin{pmatrix} 1 \times 1 + 0 \times 0 + 0 \times 0 & 1 \times (-1) + 0 \times 2 + 0 \times 3 & 1 \times 2 + 0 \times 1 + 0 \times (-1) \\ (-1) \times 1 + 2 \times 0 + 3 \times 0 & (-1) \times (-1) + 2 \times 2 + 3 \times 3 & (-1) \times 2 + 2 \times 1 + 3 \times (-1) \\ 2 \times 1 + 1 \times 0 + (-1) \times 0 & 2 \times (-1) + 1 \times 2 + (-1) \times 3 & 2 \times 2 + 1 \times 1 + (-1) \times (-1) \end{pmatrix}$$

$$\Rightarrow \quad \bar{\bar{M}}\ {}^t\bar{\bar{M}} = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 14 & -3 \\ 2 & -3 & 6 \end{pmatrix}$$

$$\Rightarrow \quad Tr(\bar{\bar{M}}\ {}^t\bar{\bar{M}}) = (\bar{\bar{M}}\ {}^t\bar{\bar{M}})_{ii} = 1 + 14 + 6 = 21 \qquad Ok.$$