

LAB WORK No. 2

Time Domain Analysis of First and Second Order Continuous Linear Systems

I. OBJECTIVE

The aim of this lab work is to study and identify the transient and steady-state regimes in the step response of single-input single-output (SISO) continuous linear systems of 1st and 2nd order. We will study both open-loop and closed-loop control cases.

II. THEORETICAL BACKGROUND

II.1 First Order System

A first-order system is governed by a first-order differential equation of the form:

$$T \cdot dy(t)/dt + y(t) = k \cdot x(t)$$

where $x(t)$ is the system input and $y(t)$ its output.

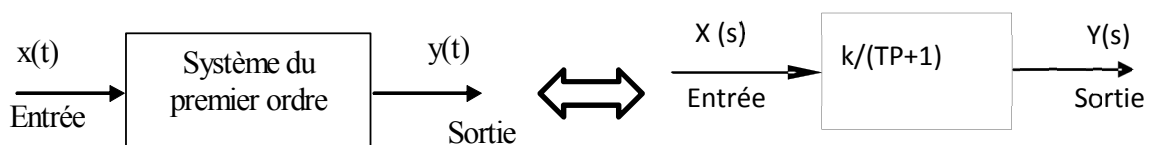


Figure 1.2. Block Diagram

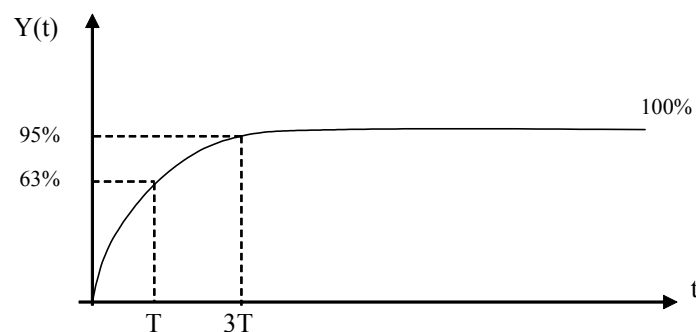


Figure 1.3 Time Response (Step Response)

With K: The static gain (steady state)

T: The system time constant

The transfer function obtained by Laplace Transform:

$$G(s) = Y(s)/X(s) = k/(T \cdot s + 1)$$

The response to a step input of amplitude E for such a system will be given in the time

domain by the solution: $y(t) = kE(1 - e^{-\frac{t}{T}})$ where the final value of the steady state is equal to k times the input. At $t=T$, the system response reaches 63% of its final value and 95% after $tr=3T$, the latter being called the 5% settling time.

II.2 Second Order System

The normalized equation:

$$d^2y(t)/dt^2 + 2\xi\omega_0 \cdot dy(t)/dt + \omega_0^2 \cdot y(t) = k \cdot \omega_0^2 \cdot x(t)$$

Transfer function:

$$G(s) = Y(s)/X(s) = k \cdot \omega_0^2 / (s^2 + 2\xi\omega_0 \cdot s + \omega_0^2) \quad \text{Where:}$$

- ω_0 = undamped natural frequency
- ξ = damping ratio
- ω_d = damped natural frequency $\omega_d = \omega_0 \sqrt{1 - \xi^2}$

which is obtained when $|\xi| < 1$. The roots of the denominator are therefore given by:

$$s_{1,2} = -\xi\omega_0 \pm j\omega_0 \sqrt{1 - \xi^2}$$

In this case, the system response to a unit step input with $k=1$ is given by the

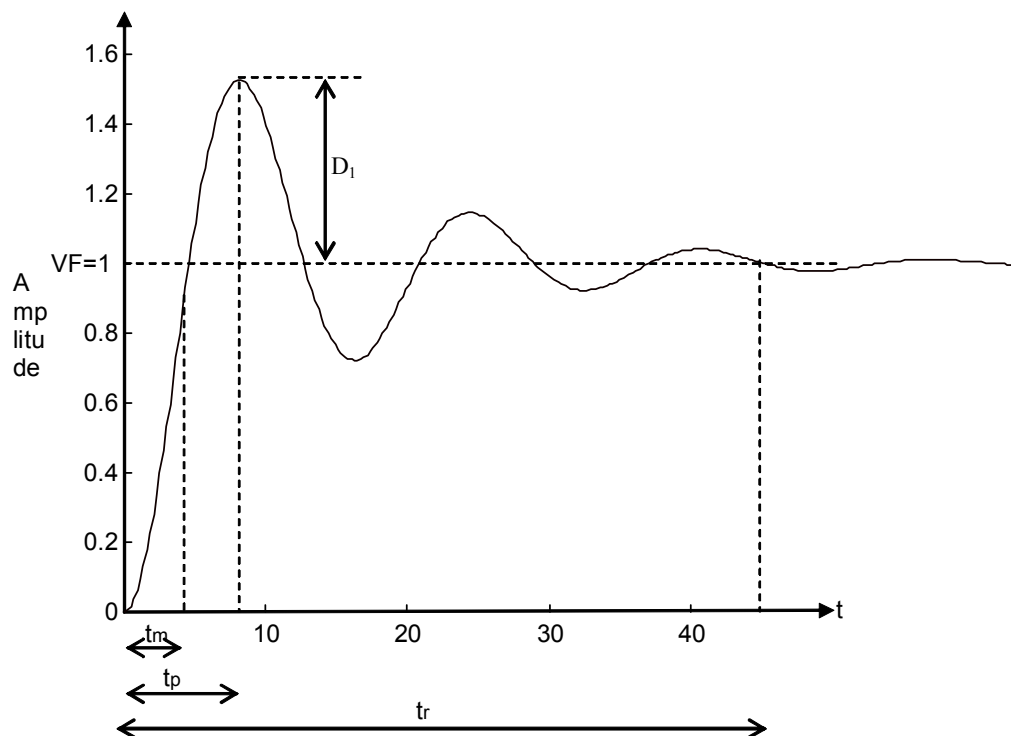
$$\text{relation: } y(t) = 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\xi\omega_0 t} \sin(\omega_0 \sqrt{1 - \xi^2} t + \phi)$$

where $\phi = \cos^{-1} \xi$

The system response is characterized by the following parameters:

- **tm: Rise Time** - characterizes the time taken for the response to increase from 0% to 100% of its final value (in the case of an underdamped oscillatory response).
- **V_F: Final Value** - the steady-state output value obtained as $t \rightarrow \infty$.
- **D₁: First Maximum Overshoot** - the first peak value exceeding the final steady-state value.

- **tp: Peak Time** - defined as the time required for the output to reach its first overshoot peak.
- **t_r: Settling Time** - defined as the time required for the output to reach and remain within a specified tolerance band around the final value ($\pm 10\%$, $\pm 5\%$, or $\pm 2\%$).



III. PREPARATORY WORK

- **Preparatory Work**

For the first-order system controlled in closed-loop with unit feedback and a gain k in the forward path:

1. Provide the closed-loop transfer function of the system
2. Express the new time constant T' as a function of T and k . Deduce the response time t'_r
3. Express the static gain k' and the steady-state error $E'(\infty)$ as a function of k

For the second-order system placed in a unit feedback control loop (closed-loop control):

1. Provide the closed-loop transfer function of the system
2. Put the obtained transfer function into standard form
3. Express the new time constant T' as a function of T and k . Deduce the system response time t'_r (CL)
4. Express the static gain k' and the steady-state error $E'(\infty)$ as a function of k

IV. SIMULATION

1. CASE OF A FIRST-ORDER SYSTEM

a) Open-loop study (O.L)

1. Implement the corresponding setup using SIMULINK
2. Apply a step signal to the system input
3. Plot the open-loop system response by completing Tables 1 and 2 for:

a) Open-Loop Study

Table 1: $k=1$

τ	0.5	1.0	1.5
Settling Time ($\sim 5\%$)			

*Table 2: $\tau=1$

k Value	1	3	10
Steady-State Error			

b) Closed-Loop Study

Let us now consider the previous case controlled in closed-loop with unit feedback. Repeat the same steps mentioned in (a) by completing the tables below for:

Table 3: $k=1$

τ Value	0.5	1.0	1.5
Time Constant T'			
Settling Time ($\sim 5\%$)			

Table 4: $\tau=1$

k Value	1	3	10
Gain K' (CL)			
Steady-State Error E'(∞)			

Work Required:

- Interpret the step responses obtained in open-loop and closed-loop. Deduce the advantage of closed-loop control.
- Discuss the benefit of proportional action in both steady-state and dynamic performance.

2. SECOND ORDER SYSTEM

a) Open-Loop Study

Implement the corresponding simulation diagram using Simulink.

- **1st Case:** Set ω_0 to 1 [rad/s] with $k=1$

- Plot the system's step response for: $\xi = 0; 0.1; 0.5; 0.7; 1; 1.5; 2$.
- Record the maximum value of each response and complete the table below:

Table 5: $\omega=1$ rad/s, $k=1$

ξ Value	0	0.1	0.5	0.7	1.0	1.5	2.0
y_max							
D_1(%)							

• **Second Case:** Taking $\omega = 1$ [rad/s], $\xi = 0.25$, and $k = 1$

- Determine the characteristic times of the system: t_m , t_p , and t_r
- Determine graphically the value of ω_8 (pseudo-oscillation frequency)

• **Third Case:** With $k = 1$ and $\xi = 0.5$

Plot the step response for: $\omega = 1, 3$, and 10 [rad/s]

What do you observe?

b) Closed-Loop Study:

- Implement the simulation diagram in Simulink for the system in closed-loop with unit feedback and a gain k in the forward path. Take $\omega = 1$ [rad/s], $\xi = 0.3$
- Excite the system with a unit step and observe the output while varying the gain k : $k = 1, 5, 10, 100$
- For each response, record the overshoot ($D_1\%$), peak time (t_p), and the corresponding steady-state error $E(tf)$.

c) Case of an Unstable Second-Order System

Plot the step response for the following two cases:

- $\xi = -0.1$; $\omega_0 = 1$ rad/sec
- $\xi = -0.1$; $\omega_0 = 10$ rad/sec

What can be concluded in this case?

WORK REQUIRED

1. Compare open-loop vs. closed-loop responses
2. Discuss proportional action effects
3. Analyze system performance in both regimes
4. Draw conclusions about control system behavior