Chapter II: Sizing of Transformers

II.0 Introduction to the Design of Electrical Machines

Principles of design; definitions; design approaches.

II.1 Review

Description, operating principle, and study models of transformers.

II.2 Method for Designing a Single-Phase Core Transformer

II.3 Calculation of Parameters

II.4 Performance Analysis

II.0 Introduction to the Design of Electrical Machines

The design problem arises from a specific **need**.

Major considerations in Electrical Machine Design - Electrical Engineering Materials – Space factor – Choice of Specific Electrical and Magnetic loadings - Thermal considerations - Heat flow – Temperature rise - Rating of machines – Standard specifications.

In the field of electrotechnics, this need may be one of the following:

- To drive a mechanical load (elevator, electric traction, etc.);
- To **generate electrical energy**;
- To **supply a device**.

Choice of Structure

Depending on this need, one or more **machine structures** can be selected from among the existing types:

- **Synchronous machines**
- **Asynchronous machines**
- **Direct current (DC) machines**
- **Variable Reluctance Machines (VRM)** (stepper or synchronous VRM)

Type of Excitation:

- Permanent magnets
- Wound field excitation

Type of Supply:

- Current supply
- Voltage supply

Specification Sheet (SS or SD) Definition

The specification sheet establishes:

•**Input:** Type of power supply

•**Output:** Desired power, desired torque/speed characteristic, etc.

•**Operating constraints:** Efficiency, acceptable temperature rise, etc.

•**Economic constraints:** Minimum production cost, minimum losses

In general, during the design process, the goal is to satisfy these constraints while achieving **minimal mass**, and therefore **minimal cost**.

Example of a Specification Sheet

Design a **single-phase transformer** defined by the following specifications:

- U1=220 V   f=50 Hz   S2=100 VA   U2=25 V   Cos(φ)≥0.8

•Tbob≤120 °C (maximum winding temperature)

•Ts≤60 °C   (maximum surface temperature)

Solving the Design Problem

After selecting the **structure** and **materials** to be used, it is necessary to determine all the **geometric dimensions** of the machine.

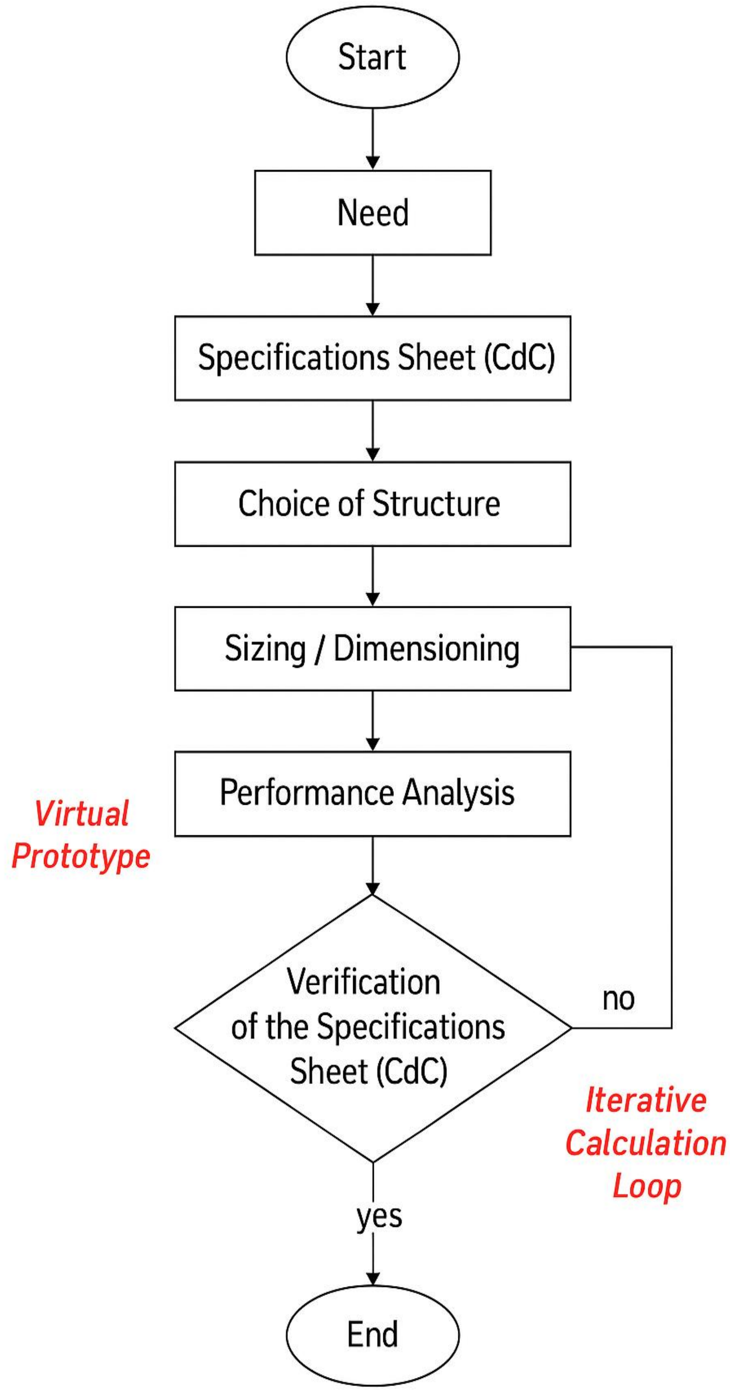
This stage is called **dimensioning** (or sizing).

Performance Analysis

To verify that the machine meets the **requirements of the specification sheet (SS)**, it is necessary to perform a **performance analysis** of the obtained virtual prototype.

The models commonly used for studying electrical machines can also be applied to **evaluate the performance of a given solution**, provided that the **model parameters** (such as phase resistance, synchronous reactance, FEM, etc.) are **expressed as functions of the machine's geometric data and material properties.**

In this course, the **design procedure** will be limited to the **dimensioning (preliminary design) stage**. The resulting machine from the calculations is called a **virtual prototype**.



If the obtained performance **meets the specification sheet requirements**,
the **virtual prototype is accepted**.
Otherwise, the **dimensioning process must be revised** to make corrections.
It should be noted that:
•The **design process is iterative**, and it is rare to find the correct solution on the first attempt.
•The **design problem admits multiple possible solutions**.
The designer's role is to **find the best solution** among all possible ones that satisfy the specification sheet,
according to an **objective criterion**, represented by a **cost function**, which may be:
•The manufacturing cost of the machine;
•Its weight;
•Its overall size (volume or space requirement);
or any other relevant criterion.

a) Description

- Transformers are static electrical machines. Their role is to transform alternating voltage and current into another alternating voltage and current of the same frequency but with values better suited to the transmission or distribution of electrical energy, or to the load to be powered.
- They consist primarily of two coils of copper (Cu) or aluminum (Al) wire threaded onto a ferromagnetic core, made from a stack of thin sheets (0.35 mm thick) to limit the flow of eddy currents.

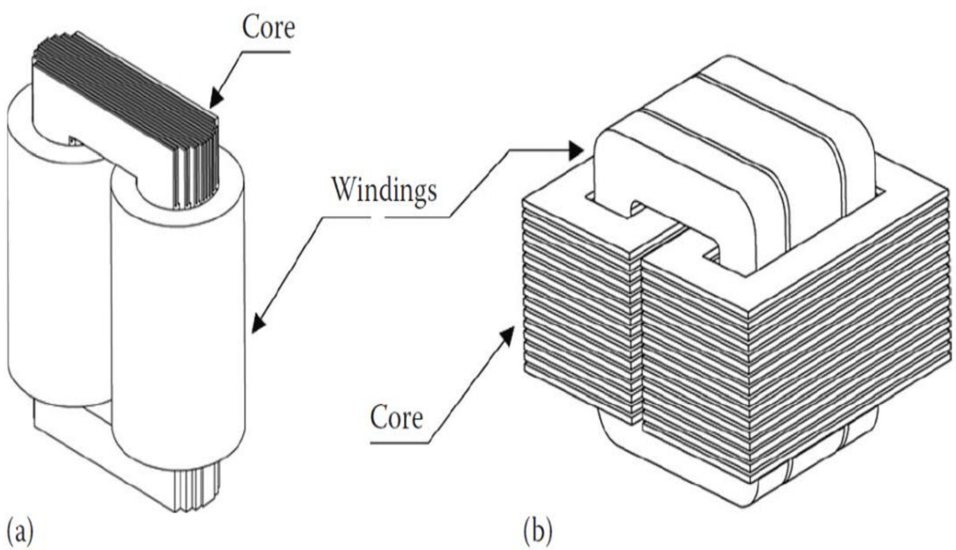


Fig.1 3D single-phase core transformer

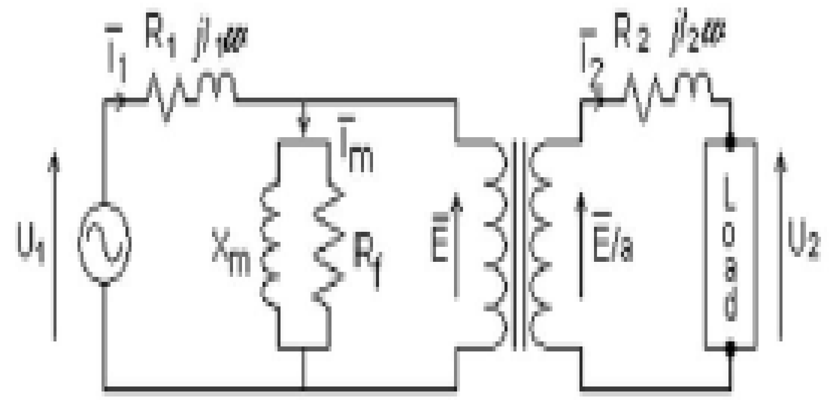


Fig.2 Equivalent electrical circuit type model of a transformer

2- On the basis of Shape of Core

On Shape basis there are three type of transformer classified as follow,

core type, shell type.

2.1 Core type

The structure of core of this transformer is in the form of a rectangular frame made from cold rolled grain oriented sheet steel (C.R.G.O.) with thin silicon steel laminations.

- The primary and secondary windings are uniformly distributed on two adjacent limbs of the core. Both the windings are of cylindrical shape and they are arranged in a concentric manner the Low voltage (LV) winding is place near the core and high voltage (HV) winding is on over it.

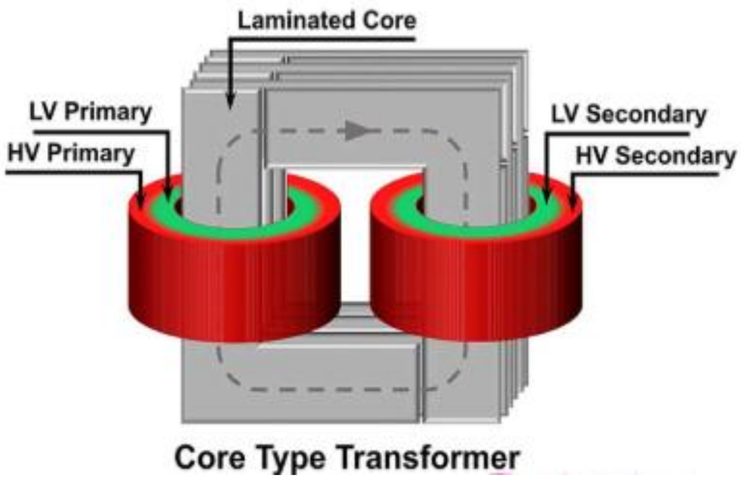
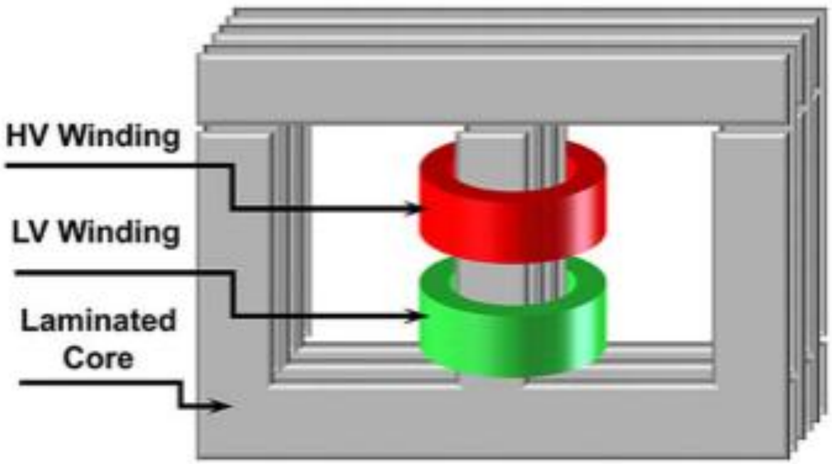


Fig.3 3D single-phase core transformer

2.2. Shell type

The construction of a shell type transformer shown in figure. The primary and secondary windings are placed on the central limb of the core. The high voltage and low voltage windings are arrange in sandwich type. This type of core provides double magnetic circuit and it provides a better mechanical support and protection for the windings. The material used for making core is same as core type transformer.



Shell Type Transformer

Sizing method for a single-phase core-type transformer.

- In this section, we will focus on the sizing of core-type transformers (Fig. 2 and Fig. II.3
- We will consider natural convection cooling. The primary and secondary coils are cylindrical and made of circular copper wires. The core (column) cross-sections are assumed to be square.

a) Materials Used

The table below summarizes the properties of the materials used:

Material	ρ (kg/m³)	ρ [Ω·m] at 75˚C	Iron losses [W/kg] at 1T, 0.35mm
Copper	8.93×10^3	2.16×10^{-8}	-
Iron	7.65×10^3	-	1

The insulation will be made with hardened paper of thickness ($\delta i = 0.25$ mm) ($Ui < 1000V$).

b) Data Required for the Design

To carry out the design, it is necessary to know:

- Data from the Specifications : U_1, U_2, Ac, f, and the type of cooling.
- Design Data

For single-phase transformers (low power < 1 kVA), we will need:

- The current density in the conductors, noted J (A/m^2);

- The maximum magnetic induction in the core, B (T);

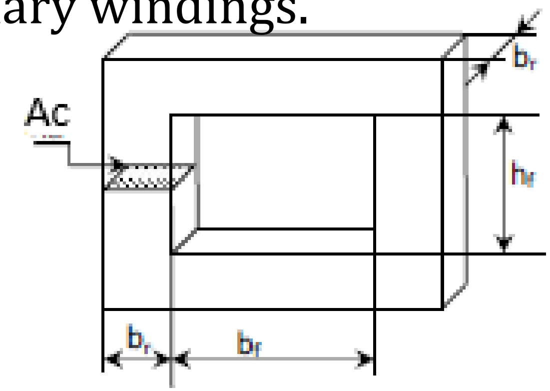
- The window filling factor, Kr.

The following calculations assume that the transformer is ideal and that the current densities in the primary and secondary conductors are identical: $J_1 = J_2 = J$.
For numerical application, we consider the design of a transformer defined by the following specifications:

S_2 = 100 VA ,U_1 = 220 V , U_2 = 12 V , f = 50 Hz	*Natural cooling*
B = 1.5 T and H = 700 A/m (according to the chosen B-H curve).	The window filling factor Kr will be chosen later.

For design data, we take:
To perform the transformer design, it is necessary to find the geometric dimensions of the magnetic circuit: Ac, b_r, b_t, h_r, as well as the number of turns (n_1, n_2) and the cross-sectional areas of the conductors (S_1, S_2) for both primary and secondary windings.

c) Calculation of the Core Section Ac



Under the assumption of an ideal transformer, we can write:$E_1 = U_1$ (no voltage drop) , $E_1 = 4.44\ f\ n_1\ B\ Ac$
Hence:
$n_1 / E_1 = (1 / 4.44\ f\ B\ Ac)$

Core Section and Conductor Calculations

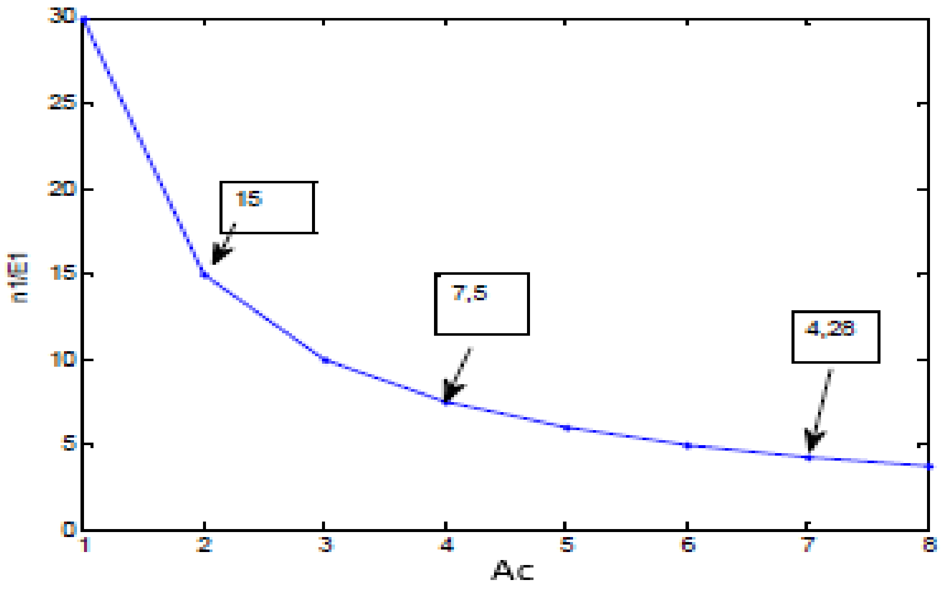
For f = 50 Hz and B = 1.5 T, we have:

$n_1 / E_1 = (30 \times 10^{-4}) / Ac$

Let Ac be the iron cross-section area expressed in [cm^2], we can then write: $n_1 / E_1 = 30 / Ac$

Remarks:

- n_1 / E_1 represents the number of turns per volt. The higher this value, the greater the total number of turns required.
- If we choose a large Ac (i.e., a large amount of iron), the number of turns n_1 is reduced (which saves copper). Conversely, if we choose a small Ac, more turns are needed (saves iron but uses more copper).
  - Generally, Ac and n_1/E_1 are chosen together by finding a compromise between these two parameters.
  - Transformer Type
  - Small Transformers (≤ 1 kVA)
  - Medium Transformers (1–10 kVA)
  - Large Transformers (> 10 kVA)



If we want $n_1 < 1000$ turns, we choose $n_1/E_1 = 4.28$,

which gives $n_1 = 941.6$. Thus, $n_1 = 942$ turns and $Ac = 7\ cm^2$.

Similarly, we can calculate $n_2 = E_2 \times 4.28 = 51.36$ turns. So $n_2 = 52$ turns.

We then deduce the transformation ratio: $a = n_1 / n_2 = 942 / 52 = 18.12$

d) Calculation of Conductor Cross-Sections (Primary and Secondary Windings S_1 and S_2)

- $S_2 = U_2 \times I_2 \rightarrow I_2 = S_2 / U_2 = 100 / 12 = 8.33$ A
- I1=I2/a=8,33/18,12= 0,46 A

Current Density and Conductor Sizing

The current density J [A/m^2] in a conductor is defined by:

$J = I / A\_c \rightarrow A\_c = I / J$

Thus, we obtain:

- $A\_c1 = 0.46 / (2.5 \times 10^6) = 0.184 \text{ mm}^2$
  $A\_c2 = 8.33 / (2.5 \times 10^6) = 3.332 \text{ mm}^2$

Calculation of conductor diameters d_1 and d_2

$A\_c = \pi r^2 \rightarrow r = \sqrt{(A\_c / \pi)} \rightarrow d = 2\sqrt{(A\_c / \pi)}$

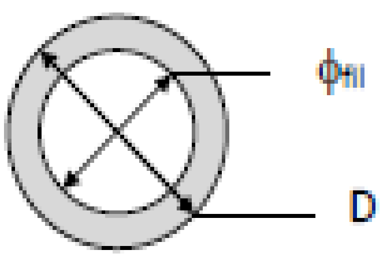
- $d_1 = 2\sqrt{(0.184 / \pi)} = 0.484$ mm
  $d_2 = 2\sqrt{(3.332 / \pi)} = 2.056$ mm

- Choice of standardized conductors

Unfortunately, the calculated sections and diameters are not always available according to standards. Therefore, from standardized values, we select the nearest higher or lower available section.

- According to the given table, we choose:

Winding	φ [mm]	S [mm²]	D (insulated) [mm]	k_{co}
Primary	0.5	0.1963	0.579	0.563
Secondary	2	3.141	2.14	0.656



$k_{co} = k_r$: Filling coefficient, we take $k_r = 0.5$ for both primary and secondary.

φ_{con}: Conductor diameter without insulation

D: Conductor diameter with insulation

e) Calculation of the Window Aw

$Aw = b_t \times h_t$

The total copper surface is calculated as:

$$A\_ct = A\_co_1 + A\_co_2 = n_1 A\_co_1 + n_2 Ao_2$$

$$Act = 942 \times 0.1963 + 52 \times 3.141 = 348.25 \text{ mm}^2$$

The window surface is related to the total copper surface by: $Aw = A_{ct} / k_{co}$

Thus, we get: $Aw = 348.25 / 0.5 = 696.5 \text{ mm}^2 \approx 7 \text{ cm}^2$

For low-power transformers, the ratio h_t / b_t is generally chosen between 2.5 and 3.

If we take $h_t / b_t = 3$, we can solve the system of two equations (h_t and b_t).

$$\begin{cases} hf\_bf = 7 \\ h_f\_b_f = 3h\_{f} \end{cases}$$

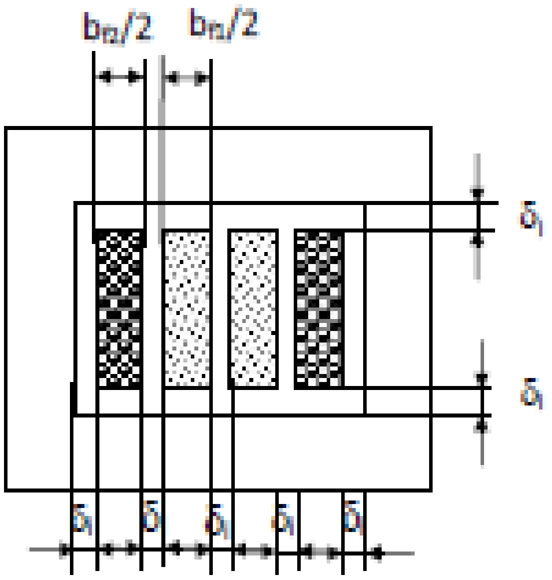
Thus: $b_f = 1.5275$ cm and $h_f = 4.6$ cm

f) Execution of the windings

In this step of the calculations, we verify that the winding is feasible, meaning the wires can be placed without issues in the space provided (window). This will also set the dimensions of the necessary coils for calculating leakage inductances.

If we consider insulation between coil–coil and coil–magnetic circuit, $\delta i = 0.25$ mm, we calculate: $h\_f' = h\_f - 2\delta i \quad = 4.6 - 2 \times 0.025 = 4.55$ mm

$b\_f' = b\_f - 2\delta i = 1.53 - 5i \times 0.025 = 1.405$ cm



The coils will be made with several layers of turns.

We determine the number of conductors per layer:

Primary:

D1 = 0.579 mm = 0.0579 cm → π/D1 = 78.58 → 78 conductors

Number of turns/layer = number of conductors/layer = 1

Total number of turns/layer = 77 turns

942 turns → number of layers = 942/77 = 12.23 layers

12 full layers and the last filled to 1/4

We deduce the final thickness occupied by the primary winding:

$bf1 = (12+¼) \times D1$ = (the last layer is considered complete) $bf1 = 0.7527$ cm

Secondary:

$D2 = 2.14$ mm $= 0.214$ cm $\rightarrow \pi/D2 = 21.26 \rightarrow 21$ conductors

Number of turns/layer = number of conductors/layer – 1

Total number of turns per layer = 20 turns

52 turns → number of layers = 52/20 = 2.6 layers

We consider the last layer filled to 60%

We deduce the final thickness occupied by the secondary winding:

$bf2 = (2+0.6) \times D2$ (the last layer is considered complete) $bf2 = 0.642$ cm

Verification: $bf1 + bf2 \leq d\_bf$?

$0.7527 + 0.642 + 5 \times 0.025 = 1.5197 \leq 1.5275$ (verified)

- The winding can be carried out. Otherwise, the window must be slightly increased.