The per-unit (Pu) system

III.1. Introduction

Power system quantities such as current, voltage, impudence and power are often expressed in pu values. For examples: base voltage if 220Kv is specified. Then the voltage is 210 Kv, the value 210/220=0.954 Pu is per unit value. The advantage of the per unit value is that by properly specifying base quantities, the equivalent impedance of transformer can be simplified. ($(Z_p = Z_s)$.

Another advantage of the per-unit system is that the comparison of the characteristics of the various electrical apparatus of different types and ratings is facilitated by expressing the impedance in per-unit based on their rating.

The different voltage levels disappear and power networks (generators, transformers, lines) reduces to a system of simple impedance.

$$Per-unit\ quantity = \frac{Actual\ quantity}{base\ value\ of\ quantity}$$

Let us define,

$$S_{pu} = \frac{S_{act}}{S_b}, V_{pu} = \frac{V_{act}}{V_b}, I_{pu} = \frac{I_{act}}{I_b}\ and\ Z_{pu} = \frac{Z_{act}}{Z_b}$$

$S_b, V_b, and\ Z_b\ are\ always\ real\ numbers$

Usually, the $S_b(MVA)\ and\ line\ to\ line\ (V_b(Kv))$ are selected

$$S = \sqrt{3}VI \Rightarrow$$

$$I_b = \frac{S(MVA)}{\sqrt{3}V_b(K_v)}$$

$$Z_b = \frac{V_b^2}{S_b}$$

Here $S_{pu} = V_{pu}.I_{pu}^*$ $(I^*: complexe\ conjugate\ of\ per-unit\ current\ I_{pu})$.

$$Z_{pu} = |V\_pu\ |^2/S_{pu}^*$$

Furthermore, the impedance of generators, transformers and motors supplied by the manufacturer are generally given in P.U values on their own rating. For power system analysis all impedances must expressed in per-unit values on common base.

When base quantities are changed from $S_{b,old}$ to S_b^{new} and from V_b^{old} to V_b^{new} , the new per-unit impedance com be given by:

$$Z_{Pu}^{new} = Z_{Pu}^{old} \cdot \frac{S_b^{new}}{S_b^{old}} \cdot \left(\frac{V_b^{old}}{V_b^{new}}\right)^2$$

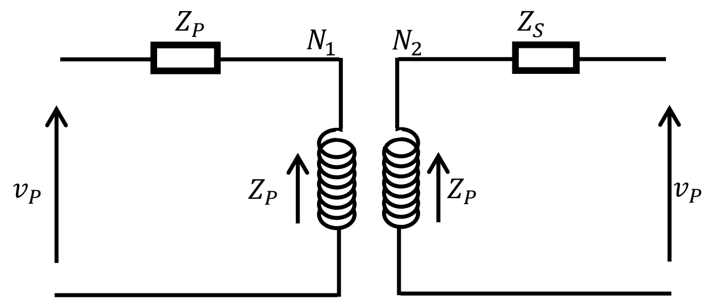
III.2. Per-unit of transformer

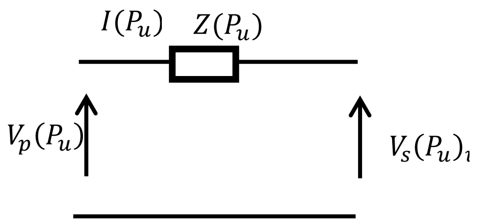
The per-unit of transformer of a transformer is the same whether computed from primary or secondary side

$$Z_1(P_u) = Z_2(P.U) = Z_{pu}$$

$$Z_1(P_u) = Z_p(P.U) + Z_s(P_u)$$

$$Z_1(P_u) = Z_s(P.U) + Z_p(P_u)$$





Per-unit equivalent circuit of single-phase transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \left(\frac{1}{\alpha}\right) \leftarrow \text{(transformer ratio)}$$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{1}{\alpha}$$

III.3. Example 01

A single phase two winding transformer is rated 25KVA, 1100/440V, 50Hz. The equivalent leakage impedance of the transformer referred to the low voltage side is $0.06\ \angle 78°\Omega$.

Using transformer rating as base values, determine the pre-unit leakage impedance referred to low voltage winding and referred to heigh voltage winding.

Solution

$$S_b = 25Kva, V_{p,b} = 1.1KV\ , V_{s,b} = 0.44KV$$

Base impedance on the low side is

$$Z_{s,b} = \frac{V_{s,b}^2}{S_b} = \frac{(0.44)^2}{0.025} = 7.744\Omega$$

Per-unit leakage impedance referred to the low voltage side is

$$Z_{s,pu} = \frac{Z_{s,eq}}{Z_{sb}} = \frac{0.06\angle 78°}{7.444} = 7.74.10^{-3}\angle 78° pu$$

If $Z_{p.eq}$ referred to primary winding (HVSide)

$$Z_{P.eq} = \alpha^2.Z_{s.eq} = \left(\frac{N_1}{N_2}\right)^2.Z_{s.eq} = \left(\frac{1.1}{0.44}\right)^2.0.06\angle 78°$$

$$Z_{P.eq} = 0.375\angle 78°\Omega$$

Hence, base impedance in heigh voltage is 1.1Kv

$$Z_{p.b} = \frac{V_b^2}{S_b} = \frac{(1.1)^2}{0.025} = 48.4\Omega$$

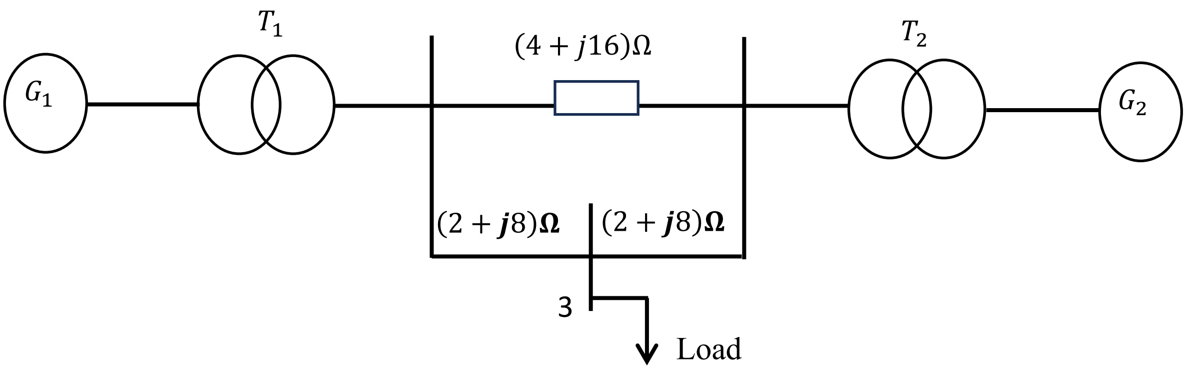
$$Z_{p.pu} = \frac{Z_{peq}}{Z_{pb}} = \frac{0.375\angle 78°}{48.4} = 7.47.10^{-3}\angle 78° pu$$

Therefore, per-unit leading impedance remains unchanged and this has been achieved by specifying

$$\frac{V_{PB}}{V_{SB}} = \frac{1.1}{0.44} = 2.5$$

Example 02

Draw the per-unit impedance diagram of the system shown figure below, assumed base values are 100 MVA and 100KV.



G_1: $50\ MVA, 12,2KV$, $xg_1 = 0.10\ P_u$.

G_2: $20\ MVA, 13.8Kv$, $xg_2 = 0.10\ P_u$.

T1:80MVA, 12,2/132KV, $x_{t1} = 0.10P_u$

T2:40MVA, 13,8/132KV, $x_{t2} = 0.10P_u$

Load: 50MVA, 0.80 PF lagging operating at 124 KV

Solution:

Base KV is 100 KV in transmission line

$V_b(G_1) = V_b \frac{V_1}{V_2} = 100\frac{12.2}{132} = 9.24Kv,\ V_{pu} = \frac{12.2}{9.4}\ \left(\frac{V_1}{V_2} = \frac{N_1}{N_2}\right)$

$V_b(G_2) = V_b\frac{V_1}{V_2} = 100\frac{13.8}{132} = 10.45KV$

Now for:

$x_{g1}^{(new)} = x_{g1}^{old}.\left(\frac{S_b^{new}}{S_b^{old}}\right)\left(\frac{v_b^{old}}{V_b^{new}}\right)^2 = 0.10\left(\frac{100}{50}\right)\left(\frac{12.2}{9.24}\right)^2$

$x_{g1}^{new} = 0.3486P_u$

$x_{g2}^{(new)} = x_{g1}^{old}\left(\frac{S_b^{new}}{S_b^{old}}\right)\left(\frac{V_b^{old}}{V_b^{new}}\right)^2 = 0.10\left(\frac{100}{20}\right)\left(\frac{13.8}{10.45}\right)^2$

$x_g^{new} = 0.8719\ P_u$

For $T_1, x_{T1}^{(new)} = 0.10\frac{100}{80}.\left(\frac{132}{100}\right)^2 = 0.2179pu$

For $T_2, x_{T2}^{new} = 0.10\frac{100}{40}.\left(\frac{132}{100}\right)^2 = 0.4359$pu

$\rightarrow\ base\ impedance\ of\ transmission\ line$:

$Z_b(Line) = \left(\frac{V_b^2}{S_b}\right) = \frac{(100)^2}{100} = 100\Omega$

$Z_{pu}^{(12)} = \frac{Z_{12}}{Z_{b(line)}} = \frac{4+j16}{100} = (0.04+j0.16)pu$

$Z_{pu}(13) = Z_{pu}(23) = \frac{2+j8}{100} = (00.2+j0.08)pu$

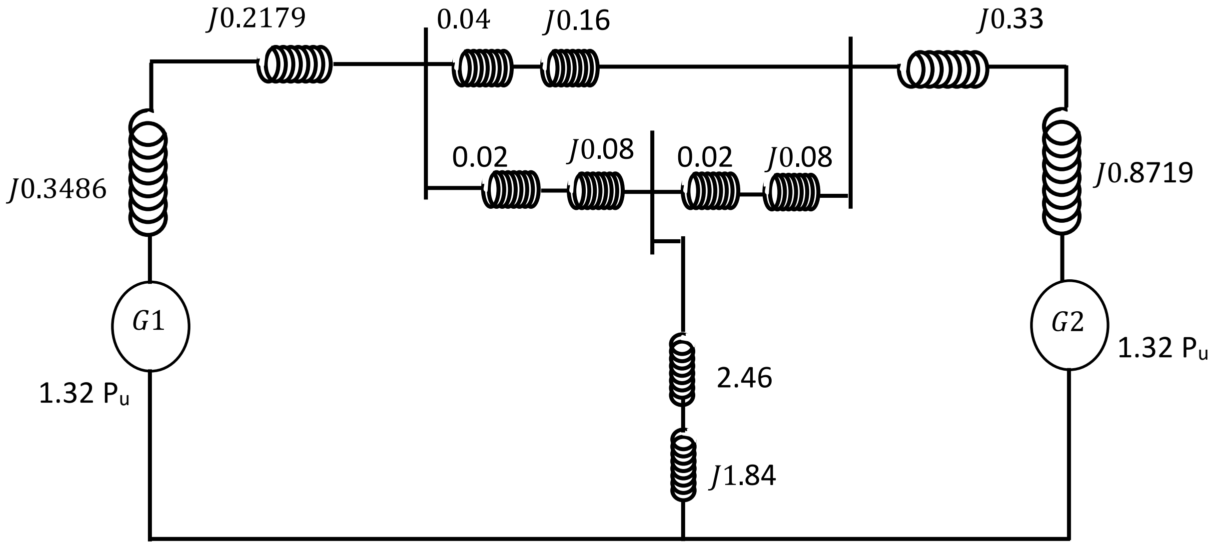
The load is specified as:

$S_b = 50(0.8+j0.6) = 40+j30MVA\ \leftarrow\ S_{complex} = S(\cos\varphi + J\sin\varphi)$

$Z_{load} = \frac{V^2}{S} = \frac{(124)^2}{(40+30)} = 307.52\angle-36.87°\Omega$

$Z_{load}(P_u) = \frac{Z_{act}}{Z_{b(line)}} = \frac{307.52\angle-36.78°}{100} = (2.46+j1.845)$pu

$R_{series} = 2.46P_u, X_{series} = 1.845P_u$



J0.2179
0.04
J0.16
J0.33
0.02
J0.08
0.02
J0.08
J0.3486
J0.8719
G1
G2
1.32 Pu
1.32 Pu
2.46
J1.84