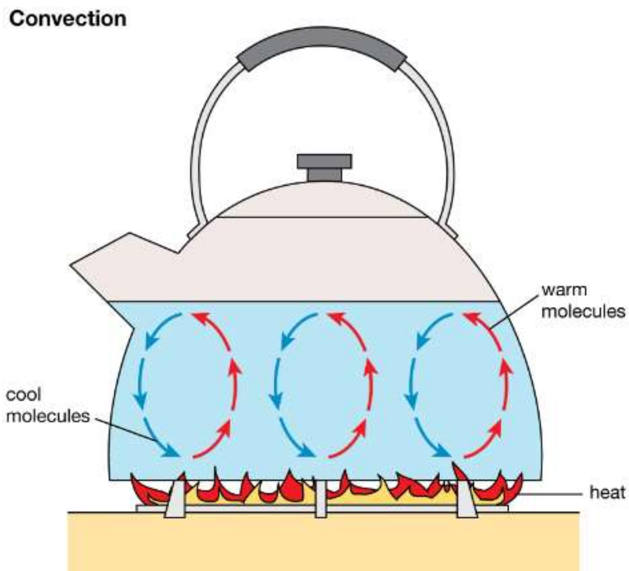
Chapter 4

Heat transfer by convection



Heat transfer by convection

4.1. Introduction

When heat transfer is accompanied by mass transfer, it is called convective transfer. This mode of heat exchange occurs in non-isothermal fluid environments or when a fluid circulates around a solid. Convection is a mode of transport of energy by the combined action of conduction, the accumulation of energy and the movement of the fluid medium. The study of heat transfer by convection essentially makes it possible to determine the heat exchanges occurring between a fluid and a wall. The quantity of heat exchanged per unit of time depends on several parameters:
the temperature difference between the wall and the fluid;

- fluid speed;
- the heat capacity of the fluid;
- the exchange surface;
- the surface condition of the solid;
- its size
- …etc.

4.1. Natural convection-forced convection

Heat transmission by convection is designated, depending on the mode of fluid flow, by natural convection (also called free) and forced convection:

- When currents occur within the fluid simply due to temperature differences, we say that the convection is natural or free.
- On the other hand, if the movement of the fluid is caused by an external action, such as a pump or a fan, the process is called forced convection.

4.2. Newton's law

Newton's law gives the expression for the quantity of heat δQ exchanged between the surface of a solid at temperature Ts and the fluid at temperature Tf (often denoted T∞).

The study of heat transfer by convection makes it possible to determine the heat exchanges occurring between a fluid and a wall.

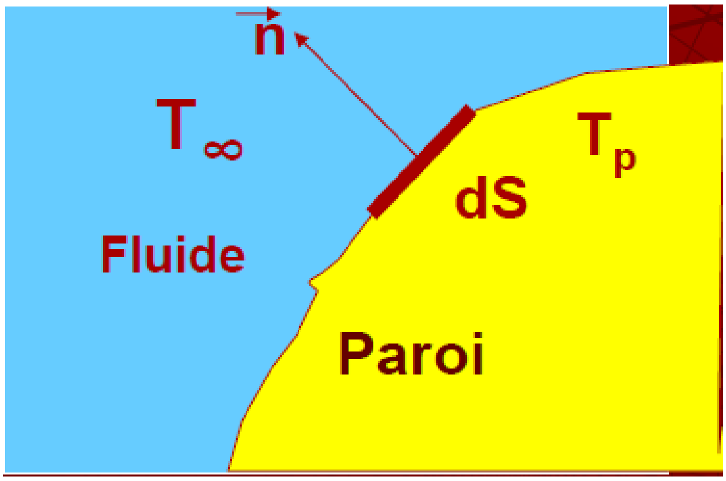


Figure 4. 1. Newton law

The quantity of heat δQ which passes through dS during the time interval dt can be written:

$$\boxed{\delta Q = h\, dS\, (T_p - T_\infty)\, dt}$$

- h is the convection exchange coefficient, it is expressed in W/(m2.K).
- δQ is expressed in Joules.
- δQ/dt in Watts.

Whatever the type of convection (natural or forced) and whatever the flow regime of the fluid (laminar or turbulent), the transmitted heat flow is given by the relationship known as Newton's law (empirical relationship):

$$\boxed{\phi = h\, dS\, (T_p - T_\infty)}$$

- ϕ is the transmitted power (W)

- h is the exchange coefficient (in $W/(m^2.K)$)
- Tp -T∞ is the temperature difference between the body and the fluid (in K).
- dS is the exchange surface (in m^2)

4.2 Convective exchange coefficient h

The major problem to resolve before any calculation of the heat flow consists of determining the convective exchange coefficient h which depends on numerous parameters:

- Type of convection (natural or forced)
- fluid characteristics,
- nature of the flow,
- temperature,
- the shape of the exchange surface,
- ...etc.

4.3. Natural convection

4.3.1 Principle of natural convection

The hot particle starts moving and directly ensures the transfer of heat towards the colder medium: The regime becomes convective.

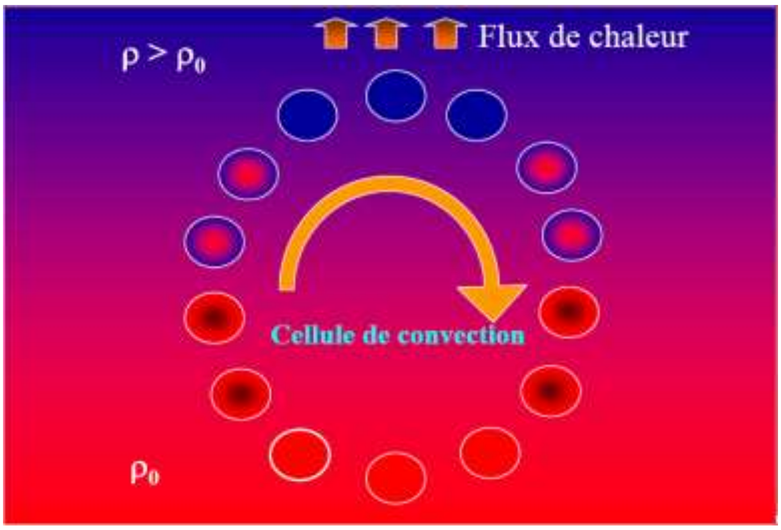


Figure 4. 2. Heat transfer by convection

4.4. Dynamic study

Consider a horizontal surface (S) at a temperature Ts in contact with a still fluid of density ρ at a temperature Tf.

A particle (P) of the fluid of volume V in contact with the surface (S) has a temperature close to Ts.

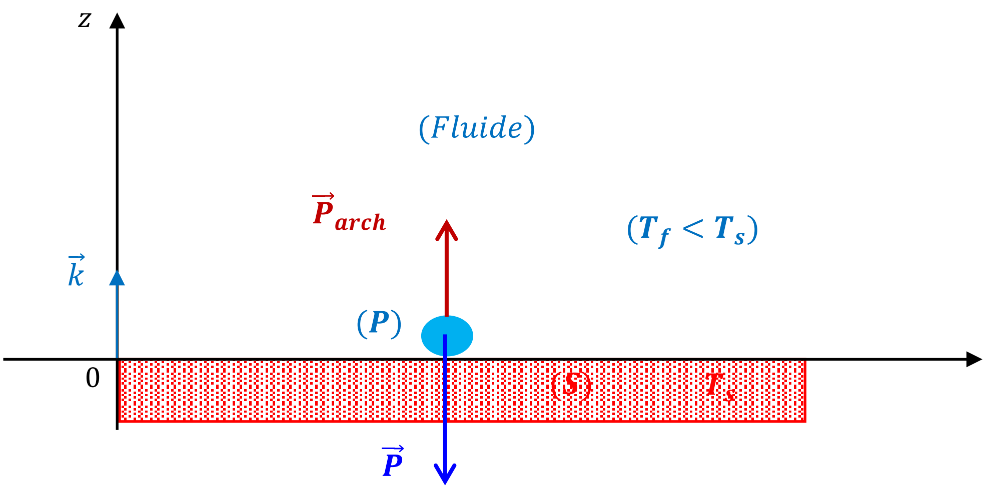


Figure 4. 3. Dynamic study

- ➢ Balance of forces acting on the particle (P):
- Archimedes' thrust: $\vec{P}arch = \rho(Tf).V.g\,\vec{k}$
- The weight: $\vec{P} = -mg\vec{k} = -\rho(Ts).V.g\,\vec{k}$

As Ts>Tf, we of course have $\rho(Tf) > \rho(Ts)$ and consequently ‖$\vec{P}$arch‖ >‖$\vec{P}$‖, which allows us to conclude that the densest parts are located below the least dense. Movements in the fluid will then be favored: this is the phenomenon of natural convection.

The fundamental principle of dynamics is written in this case:

$$\sum \vec{F}ext = m\vec{\gamma}$$
$$\Rightarrow \vec{P}arch + \vec{P} = \rho(Ts).V\,\vec{\gamma}$$
$$\Rightarrow \rho(Tf).V.g - \rho(Ts).V.g = \rho(Ts).V\,(d^2 z)/(dt^2)$$

The equation of movement of the particle (P) in the immediate vicinity of (S) is therefore written:

$$\Longrightarrow (d^2 z)/(dt^2) = (\rho(Tf) - \rho(Ts))/(\rho(Ts)).g$$

4.5. Study of the convection phenomenon

4.5.1 Boundary layers

The study of flows near the walls is necessary for the determination of heat exchanges by convection between a solid and the fluid which surrounds it. Consider a fluid flowing along a surface S:

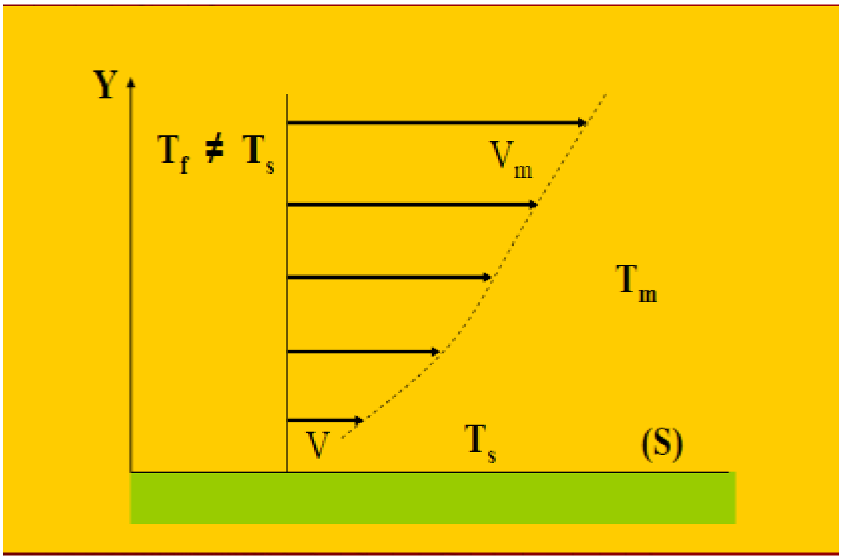


Figure 4. 4. Boundary lyers

- Far from the surface, the fluid has an average speed Vm and an average temperature Tm.
- In the immediate vicinity of the surface, the temperature of the fluid is very close to that of the surface. The speed of the fluid is almost zero.

The speed and temperature diagrams, in the direction perpendicular to the surface (the Y direction), define a layer of fluid called the “boundary layer”whose temperature and speed have the appearance of the following curves:

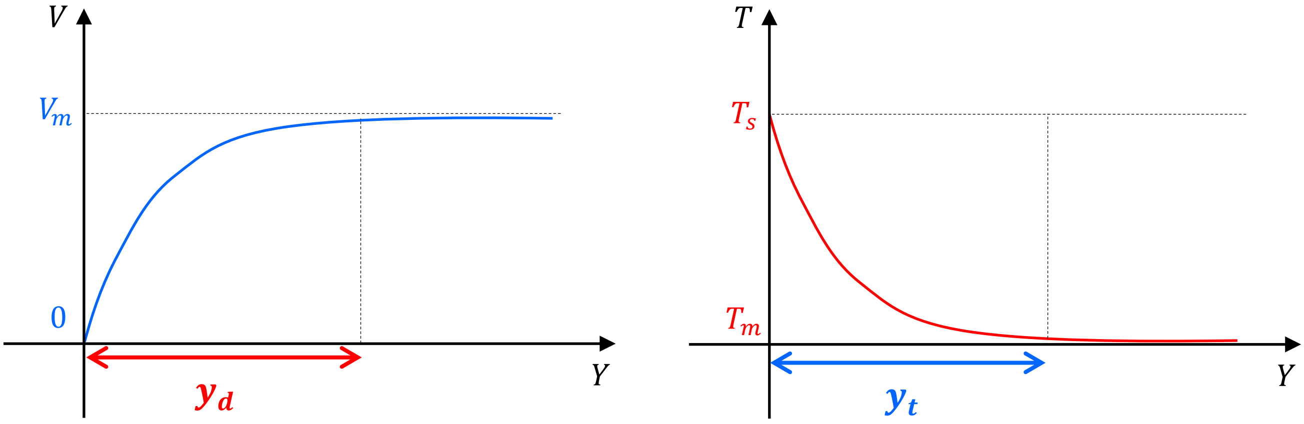


Figure 4. 5. The speed and temperature diagrams

We thus define two types of boundary layers:

- dynamic boundary layer that we note *yd*
- thermal boundary layer that we note *yt*

It should be noted that these two boundary layers generally have different thicknesses.

The velocity and temperature profiles developed within a flowing fluid can be represented like this:

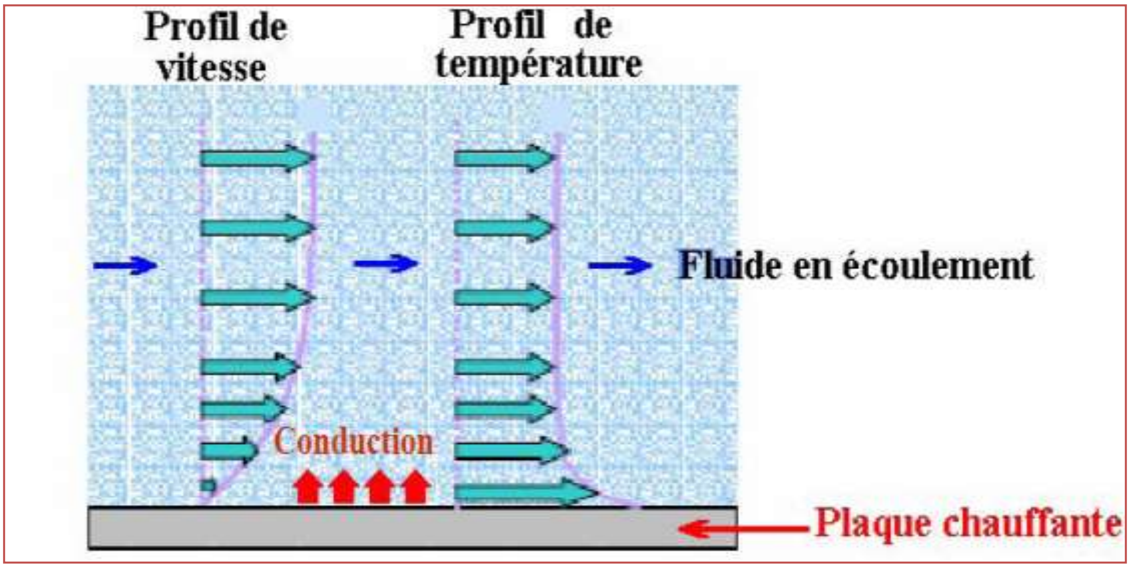


Figure 4. 6. The velocity and temperature profiles

Note that in the vicinity of the plate, dynamic and thermal boundary layers develop in which variations in speed and temperature are observed.

Heat transfer from the plate to the fluid results from two mechanisms:

- In the immediate vicinity of the surface, the transfer takes place by conduction.
- Far from the surface the transfer also results from the movement of the fluid.

In the boundary layer, if we admit that the heat transfer takes place essentially by conduction, therefore without transfer of matter in the y direction, we can write:

- The amount of heat through the surface (S):

$$\boxed{\delta Q = -\lambda_s . \frac{\partial T_s}{\partial y} . dS . dt} \quad (1)$$

Ts is the surface temperature of the solid.

- The amount of heat through the boundary layer:

$$\boxed{\delta Q = -\lambda_f . \frac{\partial T_f}{\partial y} . dS . dt} \quad (2)$$

Tf is the average temperature of the fluid quite far from the wall of the solid.

However, the variations of Tf and Ts as a function of y are generally not known to be able to deduce δQ from equalities (1) and (2). Newton's law allows this difficulty to be avoided by using only the temperature difference (Ts- Tf):

$$\boxed{\delta Q = h.(T_s - T_f).dS.}$$

4.5.2 Nature of the convective exchange coefficient h

The convective exchange coefficient, h, depends on several parameters and the heat exchange is all the more active (i.e. h greater) when:

- The fluid flow speed v is greater.
- Its specific heat c_p is greater.
- Its thermal conductivity λ (or its thermal diffusivity) is higher.
- Its kinematic viscosity ν=μ/ρ is lower.

h can also depend on the dimensions of the wall, d, its nature and its shape.We can then write:

$$\boxed{\boldsymbol{h = h(\mathrm{v}, c_p, \lambda, \mu, d)}}$$

The large number of factors influencing heat transfer by convection explains the difficulty of any theoretical or even experimental study, especially if the coefficients which characterize the material vary with pressure and temperature. Hence the idea of thinking of an effective method allowing the determination of the expression of the coefficient h. The method using what is called "dimensional analysis" seems to be the easiest to implement for determining the expression of the convection coefficient h.

4.5.3 Determination of the coefficient h by the dimensional analysis method

Suppose that h is a function of the variables $\boldsymbol{v, c_p, \lambda, \mu}$ and $\boldsymbol{d}$

$$\boxed{\boldsymbol{h = h(v, c_p, \lambda, \mu, d)}}$$

h is also an implicit function of the temperature T since cp and λ depend on it.

Let's use the equations for the dimensions of each term:

$* \ [\delta Q] = [energy] = [force][displacement] = M.L^2.t^{(-2)}$

$* \ [\boldsymbol{\lambda}] = \dfrac{[\delta Q]}{L.T.t} = \dfrac{M.L^2.t^{-2}}{L.T.t} = M.L.T^{-1}.t^{-3}$

$$* \quad [c_p] = \frac{[\delta Q]}{M.T} = \frac{M.L^2.t^{-2}}{M.T} = L^2.t^{-2}.T^{-1}$$

$$* \quad [\mu] = M.L^{-1}.t^{-1}$$

$$* \quad [v] = L.t^{-1}$$

$$* \quad [d] = L$$

The equation with dimensions of h is obtained from Newton's law:

$$\boxed{[h] = \frac{[\delta Q]}{[(T_s - T_f)].[dS].[dt]} = \frac{M.L^2.t^{-2}}{T.L^2.t} = M.T^{-1}.t^{-3}}$$

By writing [h] in the form:

$$\boxed{[h] = [c_p]^a.[\lambda]^b.[\mu]^c.[d]^d.[v]^e}$$

or

$$\boxed{[h] = (L^2.t^{-2}.T^{-1})^a.(M.L.T^{-1}.t^{-3})^b.(M.L^{-1}.t^{-1})^c.(L)^d.(L.t^{-1})^e = M.T^{-1}.t^{-3}}$$

Note that the fundamental quantities involved in the calculation of h are:

The mass M, the time t, the length L and the temperature T.

Identifying the exponents in the equation with dimensions of h provides a linear system of equations for calculating a, b, c, d and e:

- the exponent of M: b+c=1
- the exponent of T: a+b=1
- the exponent of L: 2a+b-c+d+e=0
- the exponent of t: 2a+3b+c+e=3

4.5.4 Dimensionnels numbers

For convection problems, solving the dimensional equations reveals dimensionless numbers that are very useful in the study of fluid mechanics and in particular in convective phenomena. These numbers are in particular:

- The Reynolds number
- The Nusselt number
- The Prandtl number

a) The Reynolds number

The flow regime of a fluid can be laminar or turbulent. The transition from one regime to another is characterized by the Reynolds number:

$$\boxed{Re = \frac{v.d}{\nu}}$$

In the case of a tube, d represents the internal diameter. Experience shows that for Re less than a critical value Re_{cr}=2200, the flow in a pipe is always laminar:

- If $Re<Re_{cr}$: the regime is said to be laminar
- If $Re>Re_{cr}$: the regime is said to be turbulent

b) The Nusselt number

The Nusselt number characterizes the importance of convection compared to conduction. It is the ratio of the quantity of heat exchanged by convection h.S.ΔT to a quantity of heat exchanged by conduction λ.S.ΔT/d:

$$\boxed{Nu = \frac{h.S.\Delta T}{\lambda.S.\frac{\Delta T}{d}} \quad \Rightarrow \quad Nu = \frac{h.d}{\lambda}}$$

Nu is a direct function of h, its knowledge makes it possible to determine the value of h.c.

c) The Prandtl number

It characterizes the speed distribution in relation to the temperature distribution (characterizes the thermal properties of the fluid):

$$\boxed{Pr = \frac{\mu.c_p}{\lambda}}$$

d) The Grashof number:

The viscosity force of the fluid is characterized by the Grashof number:

$$Gr = \frac{g \cdot d^3 \cdot \beta_p \cdot \Delta T}{\nu^2}$$

d: characteristic dimension of the wall [m].

g: acceleration of gravity [m.s^{-2}].

$\Delta T = T_p - T_f$: Characteristic temperature difference [°C].

β_p : Volume expansion factor of the fluid [°C^{-1}].

pour un gaz parfait : $\beta = \frac{1}{T}$

e) The Rayleigh number:

It is identical to the Reynolds number in natural convection.

$$\boxed{Ra = \Pr \cdot Gr}$$

4.5.5 Thermal resistance

Consider a fluid of temperature T_1 which circulates in the vicinity of a wall of temperature T2. The exchanged heat flow is written:

$$\phi = hS(T_2 - T_1)$$

$$(T_2 - T_1) = \frac{1}{hS}\phi = R_{th}\phi$$

The analogy with Ohm's law makes it possible to define the thermal resistance R_{th} of convection

$$\boxed{R_{th} = \frac{1}{hS}}$$

4.6. Forced convection:

In this section of the chapter we will see how to transfer heat by convection without phase change. The most common experimental correlations in forced convection are of the following type: $\boxed{Nu = f(\mathrm{Re}, \mathrm{Pr})}$

For a given fluid temperature, the Prandtl number, which appears in

the expression of the Nusselt number and characterizes the flowing fluid, must be determined. Alternatively, if there is heat exchange, the surface temperature and the fluid temperature would be different; the fluid properties would simply be taken for the average temperature T_m.

$$\boxed{T_m = \frac{T_p + T_f}{2}}$$

4.6.1. Heat exchange along a flat plate (External forced convection Rec = 5*105):

- Laminar speed: $\mathrm{Re} \le 5.10^5$
  $$Nu_L = 0.66(\mathrm{Re}_L)^{1/2}(\mathrm{Pr})^{1/3}$$
- Turbulent regime: $\mathrm{Re} \succ 5.10^5$
  $$Nu_L = 0.036(\mathrm{Re}_L)^{4/5}(\mathrm{Pr})^{1/3}$$

4.6.2. Flow inside smooth cylindrical tubes (Internal forced convection Rec =2300):

Laminar speed: $\mathrm{Re} \le 2300$

Valid for : $Nu_L = \mathrm{Re}.\mathrm{Pr} \times \dfrac{D}{L} \prec 100$

Turbulent regime $\mathrm{Re} \succ 2300$

$$\boxed{Nu_L = 0.023(\mathrm{Re})^{0.8}(\mathrm{Pr})^n}$$

Valid for

$$\begin{array}{lll} n = 0.3 & si & T_f \succ T_p \\ n = 0.4 & si & T_f \prec T_p \end{array}$$

4.6.3. Flow in annular spaces (Internal forced convection)

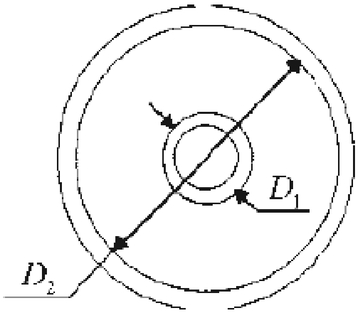


Figure 4. 7. Flow in the annular space formed by the two pipes

$$Nu_{Dh} = 0.023\left(\mathrm{Re}_{Dh}\right)^{0.8}\left(\mathrm{Pr}\right)^{n}$$

Avec ; $\mathrm{Re}_{Dh} = \dfrac{U_m.D_h}{\upsilon}$ et $Nu_{Lh} = \dfrac{h.D_h}{\lambda}$

D_h : Diamètre hydraulique. Dans ce cas: $D_h = D_1 - D_2$

n = 0,4 pour chauffage

n = 0,3 pour refroidissement

4.7. Natural convection:

a) Practical method for calculating heat flow in natural convection:

The use of dimensional analysis reveals that the relationship between the flow of heat transported by convection and the factors on which it depends is found in the form of a relationship between three dimensionless numbers: $Nu = f\left(Gr, \mathrm{Pr}\right)$

The following steps are used to calculate a heat flux transmitted by natural convection:

- The determination of the following dimensionless numbers: Gr and Pr.
- According to the result of Gr, we choose an experimental correlation corresponding to the configuration studied.
- By applying this correlation for the calculation of the number Nu.
- Calculation of $h = \dfrac{\lambda \cdot Nu}{L}$

- Using Newton's law to calculate the heat exchanged flow:

$$\boxed{\Phi = h \cdot S\left(T_p - T_\infty\right)}$$

Noticed :

The following should be taken into consideration when calculating h:

- The nature of the fluid (liquid or gas).
- The temperature of the fluid.
- Know the type of convection (natural or forced).
- Know the type of flow regime (laminar or turbulent).
- Know the type of contact between the fluid and surface (flat surface, or circulates between two flat surfaces, or circulates in a tube, etc.).

b) Different correlations for calculating h

We noticed that the relationships describing a natural convection problem can take the following form: $\boxed{Nu = f\left(Gr, \Pr\right)}$

The relationship between these three dimensionless numbers cannot be established theoretically, but must be determined experimentally. Many scientific discoveries have been compiled in the literature. They are known as "experimental correlations." In this section, we will review the most common experimental correlations in natural convection.

The most common experimental correlations in natural convection are of the following type: $\boxed{Nu = C \cdot Ra^n \qquad \text{avec ;} \quad Ra = \Pr \cdot Gr}$

The value of the coefficient C depends on the nature of the regime and the fluids. It is determined by calculating Ra, depending on the value found we choose the suitable values of c and n which are given in the following table:

Geometry and Orientation	Characteristic Dimension	C in Laminar Convection (n = 1/4)	C in Turbulent Convection (n = 1/3)
Vertical Plate	Height	0.59 $Ra \leq 10^9$	0.10 $Ra > 10^9$
Horizontal Plate (Heating Upward)	Width	0.54 $10^4 \leq Ra \leq 10^7$	0.15 $10^7 < Ra \leq 10^{11}$
Horizontal Plate (Heating Downward)	Width	0.27 $10^5 \leq Ra \leq 10^{10}$	0.54 $10^{10} < Ra \leq 10^{13}$
Horizontal Cylinder	Outer Diameter	C = 1.02 and n = 0.148 $10^{-2} \leq Ra \leq 10^2$ 0.54 $5\times10^2 \leq Ra \leq 2\times10^7$	0.135 $2\times10^7 < Ra \leq 10^{13}$