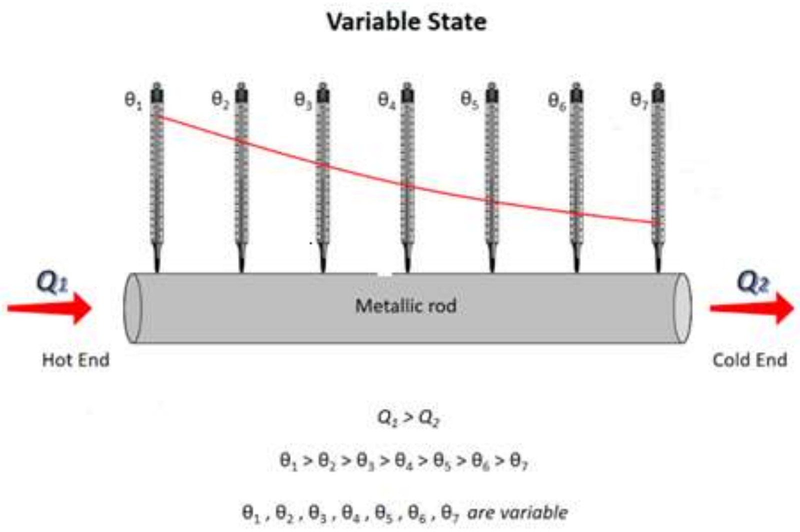
Chapter 3

Heat transfer by conduction in variable regime



Heat transfer by conduction in variable regime

3.1. Introduction

In this chapter, we will present thermal conduction in variable regime, that is to say that the temperature varies, not only in space but also with time.

3.1 Unidirectional conduction in variable regime without change of state:

❖ Main problem

The general equation for heat is given by:

$$\boxed{div(\lambda\,\overrightarrow{grad}\,T) + P = \rho\, C\,\frac{\partial T}{\partial t}} \qquad (3.1)$$

It requires an initial condition at any point T (t=0), and two boundary conditions. In the case where the thermal conductivity λ does not depend on the temperature, we obtain the Fourrier equation:

$$\boxed{\Delta\, T + \frac{P}{\lambda} = \frac{1}{a}\,\frac{\partial T}{\partial t}} \qquad (3.2)$$

- Dimensionless numbers:

The problem of thermal conduction (unidirectional case) is represented by the following system of equations:

$$\boxed{\begin{cases} \dfrac{\partial^2 T}{\partial x^2} + \dfrac{P}{\lambda} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t} & pour \quad 0 \prec x \prec l; \quad t \succ 0 \\ \dfrac{\partial T}{\partial x} = 0 & pour \quad x = 0; \quad t \succ 0 \\ \lambda \dfrac{\partial T}{\partial x} = -h\left(T - T_\infty\right) & pour \quad x = l; \quad t \succ 0 \\ T = T_0 & pour \quad 0 \prec x \prec l; \quad t = 0 \end{cases}} \qquad (3.3)$$

The number of variables in a heat transfer problem can be reduced by the introduction of dimensionless numbers:

$$x^* = \frac{x}{l}; \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}; \quad G = \frac{Pl^2}{\lambda T_0 - T_\infty}$$

System (3.3) becomes:

$$\begin{cases} \dfrac{\partial^2 \theta}{\partial x^{*2}} + G = \dfrac{\partial \theta}{\partial F_0} & pour \quad 0 \prec x^* \prec l; \quad F_0 \succ 0 \\ \dfrac{\partial \theta}{\partial x^*} = 0 & pour \quad x^* = 0; \quad F_0 \succ 0 \\ \lambda \dfrac{\partial \theta}{\partial x^*} = -B_i \theta & pour \quad x^* = l; \quad F_0 \succ 0 \\ \theta = 1 & pour \quad 0 \prec x^* \prec l; \quad F_0 = 0 \end{cases} \quad (3.4)$$

In the variable regime, two dimensionless numbers are important:

- The Biot number: is the ratio between the internal thermal resistance and the external thermal resistance, l is the characteristic languor of the medium (l = r for a sphere).

$$B_i = \frac{hl}{\lambda} \quad (3.5)$$

It calculates the thermal thickness of the domain: a medium is thermally thin if the Biot number is less than 1 (i.e. the external resistance blocks the heat flow). We can also consider that the temperature is uniform depending on the dimension of l.

- Fourier number: is the ratio of flow through l2 to the storage rate in l3, or the ratio of heat passing through to stored heat. The Fourier number describes the penetration of heat under varying conditions.

$$F_0 = \frac{\lambda \dfrac{\Delta T}{l} l^2}{\rho C l^3 \dfrac{\Delta T}{l}} = \frac{\alpha t}{l^2} \quad (3.6)$$

3.1.1 Uniform temperature environment (zero gradient method):

Heat transfer to a uniformly heated medium will be studied in this course, which is initially counterintuitive because a thermal gradient is necessary for heat transfer to occur. This proximity of medium at a constant temperature can however be justified in certain cases which we will specify.

For example, consider the quenching of a metal ball, which consists of immersing a ball initially at temperature Ti in a bath at constant temperature T0. If we assume that the temperature inside the ball is constant, which is more likely given its small size and its high thermal conductivity, we can write the heat balance of the ball between two times t and t +dt:

$$-h \cdot S \cdot (T - T_0) = \rho \cdot V \cdot c \frac{\partial T}{\partial t} \quad \text{soit ;} \quad \frac{dT}{T - T_0} = -\frac{h \cdot S}{\rho \cdot V \cdot c} dt$$

D'où :

$$\frac{T - T_0}{T_i - T_0} = \exp\left(-\frac{h \cdot S}{\rho \cdot V \cdot c} t\right) \tag{3.7}$$

We notice that the grouping $\frac{h \cdot S}{\rho \cdot V \cdot c}$ is homogeneous at one time, we will call it

The constant of system time: $\boxed{\tau = \frac{\rho \cdot V \cdot c}{h \cdot S}}$

This quantity is fundamental insofar as it gives the order of magnitude of time of the physical phenomenon, we have in fact:

$$\frac{T - T_0}{T_i - T_0} = \exp\left(-\frac{t}{\tau}\right) \tag{3.8}$$

The definition of these two numbers (Biot number and Fourier number) allows us to write the expression for the temperature of the ball in the form:

$$\boxed{\frac{T - T_0}{T_i - T_0} = \exp(-B_i \times F_o)} \tag{3.9}$$

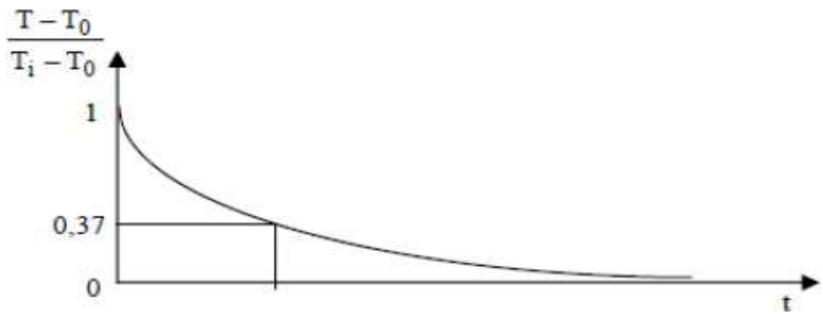


Figure 3. 1. Evolution of the temperature of a medium at uniform temperature

The product of the Biot and Fourier numbers can be used to calculate the evolution of the temperature of the sphere. The Bi < 0.1 criterion is generally called the “thermal accommodation” criterion because a device with Bi < 0.1 can be considered to be at uniform temperature.