Chapter 2

Systems modeling

2.1 Introduction

The model of a system is a mathematical structure that is a translation of reality in order to be able to apply mathematical tools, techniques and theories. Subjected to the same action signals as the real system, this structure reproduces the behavior of an experiment, that is to say that the variables calculated from the model are close to the values measured on the process.
Figure 2.1 illustrates an example of a model, in the case where the model has the same input as the system, in this case we speak of the parallel model.

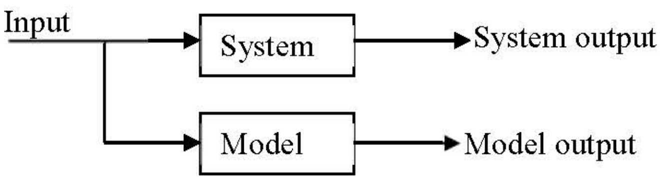
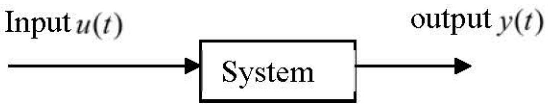


Fig. 2.1: Model configuration.

2.2 Principle

The goal of modeling is to determine the operating or behavior equations of our system. Subsequently, we will limit ourselves to single-variable systems. In the majority of cases, we model a system by differential equations.
So we are looking for a relationship between input and output such that:



2.3 Representation of linear invariant continuous systems

Let us consider a mono-variable stationary linear system described by the following differential equation:

$$a_n \frac{d^n y(t)}{dt^n} + \dots a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_k \frac{d^k u(t)}{dt^k} + \dots b_1 \frac{du(t)}{dt} + b_0 y(t) \qquad (2.1)$$

$a_0, \dots, a_n$ And $b_0, \dots, b_k$ are constants. n is the order of the system.

We have the direct relationship between the input $u(t)$ and the exit $y(t)$ in the form of a linear differential equation with constant coefficients.

In general, for a physical system, we have $k < n$.

In what follows, we will focus on continuous-time linear systems.

2.3.1 Examples of models

A. Modeling of electrical circuits

Charging a capacitor (1st order system)

The input signal considered is the voltage $v_e(t)$ and the output signal is the voltage $v_s(t)$ at the limits of the capacity (C).

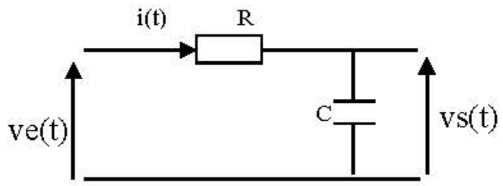


Fig.2.2: RC circuit.

We consider the capacitor discharged at the instant $t = 0$.

Series RLC circuit (2nd order system)

The input signal considered is the voltage $v_e(t)$ and the output signal is the voltage $v_s(t)$ at the limits of the capacity (C).

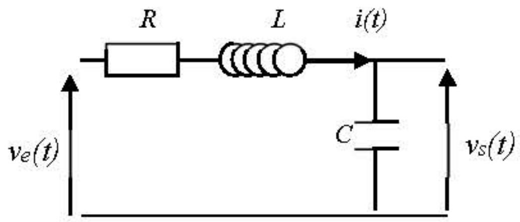


Fig. 2.3: RLC circuit.

B. Modeling of mechanical systems

Mass-spring mechanical system

A mass-spring system is a mechanical system with a degree of freedom. It consists of a mass attached to a spring constrained to move in one direction only. Its movement is due to three forces:

- A restoring force $\vec{F_r}$.
- A damping force f .
- An external force $\vec{F}$.

Consider the following mechanical mass-spring translation system:

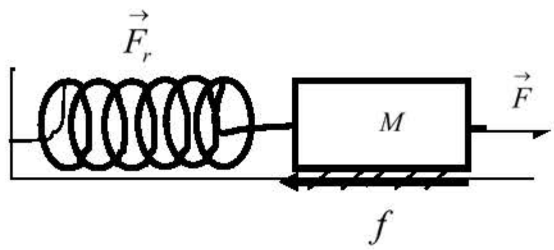
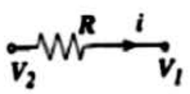
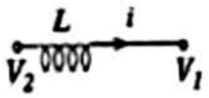
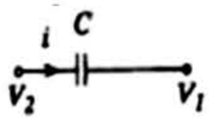
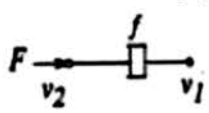
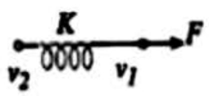
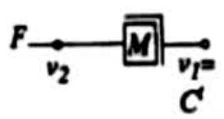
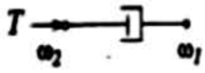
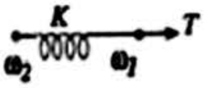
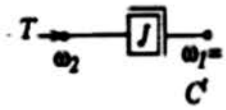
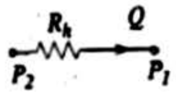
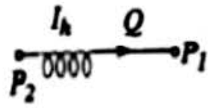
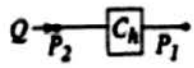
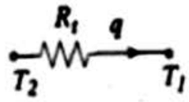
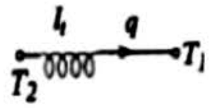
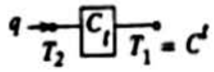


Fig. 2.4: Mass-spring system.

2.4 Analogy of linear differential equations

The analogy presented in Table 2.1 encompasses the basic elements of electrical, mechanical, hydraulic and thermal systems.

E : Energy p : Power	Resistive effect	Inductive effect	Capacitive effect
Electricity (i :intensity V :speed)	Resistance (R)  $i=\frac{1}{R}V_{21};\ p=\frac{1}{R}V_{21}^2$	Inductance (L)  $V_{21}=L\frac{di}{dt};\ E=\frac{1}{2}L.i^2$	Capacity (C)  $i=C\frac{dV_{21}}{dt};\ E=\frac{1}{2}C.V_{21}^2$
Mechanical — Translation (F :force v :speed)	Shock absorber (f)  $F=f.v_{21};\ p=f.v_{21}^2$	Spring (K)  $v_{21}=\frac{1}{K}\frac{dF}{dt};\ E=\frac{1}{2}\frac{F^2}{K}$	Mass (M)  $F=M\frac{dv_2}{dt};\ E=\frac{1}{2}M.v_{21}^2$

	Rotation (T :couple ω :speed)	Shock absorber (f)  $T=f.\omega_{21}; p=f.\omega_{21}^2$	Twist (K)  $\omega_{21}=\frac{1}{K}\frac{dT}{dt}; E=\frac{1}{2}\frac{T^2}{K}$	Inertia (J)  $T=J\frac{d\omega_2}{dt}; E=\frac{1}{2}J.\omega_{21}^2$
Hydraulic (Q:speed P :pressure)		Resistance (R_h)  $Q=\frac{1}{R_h}P_{21}; p=\frac{1}{R_h}P_{21}^2$	Inertia (I_h)  $P_{21}=I_h\frac{dQ}{dt}; E=\frac{1}{2}I_h.Q^2$	Capacity (C_h)  $Q=C_t\frac{dP_{21}}{dt}; E=\frac{1}{2}C_t.P_{21}^2$
Thermal (q : heat flow, V :temperature.)		Resistance (R_t)  $q=\frac{1}{R_t}T_{21}; p=\frac{1}{R_t}T_{21}^2$	Inertia (I_t)  $T_{21}=I_h\frac{dq}{dt}; E=\frac{1}{2}I_t.q^2$	Capacity (C_t)  $q=C_t\frac{dT_2}{dt}; E=\frac{1}{2}C_t.T_{21}^2$

Tab. 2.1: Analogy of linear differential equations.

2.5 Laplace Transform

2.5.1 Definition

To any function f of variable t, such as $f(t)=0$ for $t<0$, we match a function F of complex variable p (with $p=\tau+j\omega$).

τ : is the real part, ω :is the imaginary part.

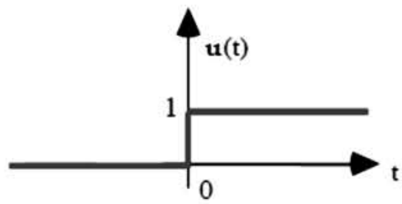
We define the Laplace transform (LT):

$$F(p)=L[f(t)]=\int_0^{+\infty} f(t)\,e^{-pt}\,dt \qquad (2.16)$$

This transformation allows us to move from the time domain to the Laplace domain and allows the resolution in the frequency domain of problems posed in the time domain.

2.5.2 LT of the step function

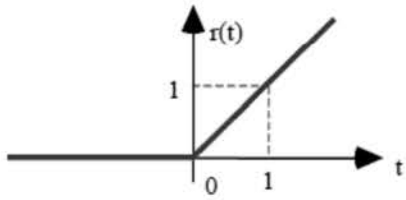
$$u(t)=\begin{cases}0 & if\ t<0\\ 1 & if\ t\geq 0\end{cases}$$



We now calculate $U(p)$ the Laplace transform of $u(t)$.

2.5.3 LT of the Ramp function

$$r(t)=\begin{cases}0 & if\ t<0\\ t & if\ t\geq 0\end{cases}$$



We now calculate $R(p)$ the Laplace transform of $r(t)$.

2.6 Properties and theorems

$f(t)\ (t\geq 0)$	$F(p)=L\{f(t)\}$
$f(t)$	$\int_0^{+\infty} e^{-pt} f(t)dt$
$\lambda_1 f_1(t)+\lambda_2 f_2(t)$	$\lambda_1 F_1(p)+\lambda_2 F_2(p)$
$\frac{df(t)}{dt}$	$pF(p)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$p^n F(p)-\sum_{r=n+1}^{r=2n} p^{2n-r}.\frac{d^{(r-n-1)}f(0)}{dt^{(r-n-1)}}$ With: $f^{(r-n-1)}(0)=\left.\frac{d^{(r-n-1)}f(t)}{dt^{(r-n-1)}}\right\|_{t=0}$ Example: $L\left\{\frac{d^3}{dt^3}f(t)=p^3F(p)-p^2f(0)-pf'(0)-f''(0)\right\}$
$\int_0^t f(t)$	$\frac{1}{p}L\{f(t)\}$
$\int_0^t\int_0^t\ldots\int_0^t f(t).dt^n$ (With conditions zero initials)	$\frac{F(p)}{p^n}$
$f(kt)$	$\frac{1}{k}.F\left(\frac{p}{k}\right)$
$f\left(\frac{t}{k}\right)$	$k.F(k.p)$
$e^{-a.t}f(t)$	$F(p+a)$

$f(t-\tau)$ If $t \geq \tau$	$e^{-p.\tau}F(p)$
$t.f(t)$	$\frac{d}{dp}F(p)$
$f(0^+) = \lim_{p\to\infty}\{pF(p)\}$ *and* $f(\infty) = \lim_{p\to 0}\{pF(p)\}$	

Tab. 2.2: Properties and theorems of the Laplace transform.

2.7 Resolution of differential equations with the symbolic method

2.7.1 First order differential equation

We will consider the solution of the following equation:

$$\begin{cases} a\dfrac{dy(t)}{dt} + by(t) = f(t) \\ C.I: y(0) = A = cte^{\circ} \end{cases} \tag{2.17}$$

We use the Laplace transform:

$$L\left\{a\frac{dy(t)}{dt} + by(t)\right\} = L\{f(t)\}$$
$$a\left(pY(p) - y(0)\right) + bY(p) = F(p)$$
$$Y(p)\left(a.p + b\right) = F(p) + a.y(0) \Rightarrow Y(p) = \frac{F(p) + a.A}{a.p + b}$$

So the solution to the differential equation (2.17) is the inverse Laplace transform of:

$$y(t) = L^{-1}\left\{Y(p) = \frac{F(p) + a.A}{a.p + b}\right\} \tag{2.18}$$

Example 2.1:

RC circuit:

2.7.2 Second-order differential equation

We will consider the solution of the following equation:

$$\begin{cases} a\dfrac{d^2y(t)}{dt^2}+b\dfrac{dy(t)}{dt}+cy(t)=f(t) \\ C.I: y'(0)=A=cte°, y(0)=B \end{cases} \tag{2.19}$$

We use the Laplace transform:

$$L\left\{a\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+cy(t)\right\}=L\{f(t)\}$$

$$a\left(p^2Y(p)-py(0)-y'(0)\right)+b\left(pY(p)-y(0)\right)+cY(p)=F(p)$$

$$Y(p)\left(a.p^2+b.p+c\right)=F(p)+a.B.p+A.a+b.B \Rightarrow Y(p)=\frac{F(p)+B(a.p+b)+a.A}{a.p^2+b.p+c}$$

So the solution to the differential equation (2.19) is the inverse Laplace transform of:

$$y(t)=L^{-1}\{Y(p)\}=L^{-1}\left\{\frac{F(p)+B(a.p+b)+a.A}{a.p^2+b.p+c}\right\} \tag{2.20}$$

Example 2.2:

$$\begin{cases} \dfrac{d^2y(t)}{dt^2}+y(t)=0 \\ C.I: y'(0)=0, y(0)=1 \end{cases}$$

2.8 Transfer function

Let the system be described by the following differential equation:

$$a_n\frac{d^ny(t)}{dt^n}+...a_1\frac{dy(t)}{dt}+a_0y(t)=b_k\frac{d^ku(t)}{dt^k}+...b_1\frac{du(t)}{dt}+b_0y(t) \tag{2.21}$$

The Laplace transform (if the initial conditions are zero):

$$a_np^nY(p)+...a_2p^2Y(p)+a_1pY(p)+a_0Y(p)=b_kp^kU(p)+...b_2p^2U(p)+b_1p^1U(p)+b_0U(p)$$

$$(a_np^n+...a_2p^2+a_1p+a_0)Y(p)=(b_kp^k+...b_2p^2+b_1p^1+b_0)U(p)$$

The transfer equation is defined by the quotient of the output and input quantities:

$$H(p)=\frac{Y(p)}{U(p)}=\frac{b_kp^k+...b_2p^2+b_1p+b_0}{a_np^n+...a_2p^2+a_1p+a_0}=\frac{N(p)}{D(p)} \tag{2.22}$$

It is also the ratio of the Laplace transform of the output to the Laplace transform of the input when all initial conditions are zero. In this case, we have:

$$Y(p) = H(p).U(p) \tag{2.23}$$

<u>Noticed :</u> The transfer function characterizes the dynamics of the system. In a real physical system the degree n of $D(p)$, is greater than the degree k of $N(p)$.

2.9 Characteristic equation

The denominator equation of the transfer function of a system is called the characteristic equation:

$$D(p) = a_n p^n + \ldots a_2 p^2 + a_1 p + a_0 \tag{2.24}$$

- Solutions of the characteristic equation $D(p) = 0$ are called the roots or poles of the system.
- The solutions of the equation $N(p) = 0$ are called the zeros of the system.

$$\begin{aligned} D(p) = 0 &\Leftrightarrow a_n p^n + \ldots a_2 p^2 + a_1 p + a_0 = 0 \\ &\Leftrightarrow (p - p_n)(p - p_{n-1})\ldots(p - p_0) \\ &\Leftrightarrow \prod_{i=0}^{n}(p - p_i) = 0 \end{aligned} \tag{2.25}$$

$$\begin{aligned} N(p) = 0 &\Leftrightarrow b_k p^k + \ldots b_2 p^2 + b_1 p + b_0 = 0 \\ &\Leftrightarrow (p - z_k)(p - z_{k-1})\ldots(p - z_0) \\ &\Leftrightarrow \prod_{j=0}^{k}(p - z_j) = 0 \end{aligned} \tag{2.26}$$

So:

$$H(p) = \frac{Y(p)}{U(p)} = \frac{\prod_{j=0}^{k}(p - z_k)}{\prod_{i=0}^{n}(p - p_i)} \tag{2.27}$$

$$Y(p) = \frac{\prod_{j=0}^{k}(p - z_k)}{\prod_{i=0}^{n}(p - p_i)}.U(p) \tag{2.28}$$

Noticed :

- The poles (p_i) can be real or complex.
- If the poles (p_i) are real different: $y(t)=\sum_{i=0}^{n} C_i e^{p_i t} \Rightarrow \lim y(t)=\lim \sum_{i=0}^{n} C_i e^{p_i t}$

  If the $p_i < 0 \Rightarrow y(t) \to 0$ if $t \to +\infty$: stable system.

  If the $p_i > 0 \Rightarrow y(t) \to +\infty$ if $t \to +\infty$: unstable system.

Theorem:

A system is said to be stable if the real parts of its poles have negative real parts.

Example 2.3:

- Calculate the transfer function of the circuit (Fig.2.5)?
- (AN: C=0.1µ F and R=100 KΩ).

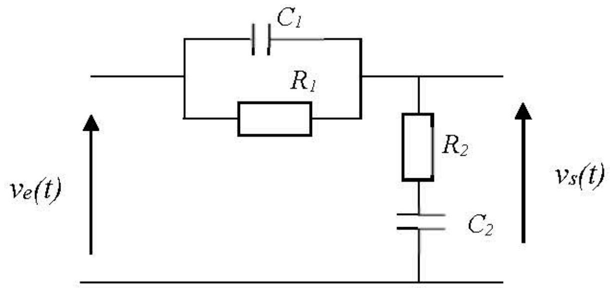


Fig. 2.5: RC circuit.

2.10 Elementary table of Laplace transforms

The following table shows the Laplace transforms of some elementary functions:

The function	Laplace transform
$\delta(t)$	1
$u(t)$	$\frac{1}{p}$
$t.u(t)$	$\frac{1}{p^2}$
$t^n.u(t)$	$\frac{n!}{p^{n+1}}$

$e^{-a.t}.u(t)$	$\frac{1}{p+a}$
$t.e^{-a.t}.u(t)$	$\frac{n!}{(p+a)^{n+1}}$
$\sin(wt).u(t)$	$\frac{w}{p^2+w^2}$
$\cos(wt).u(t)$	$\frac{p}{p^2+w^2}$
$e^{-a.t}\sin(wt).u(t)$	$\frac{w}{(p+a)^2+w^2}$
$e^{-a.t}\cos(wt).u(t)$	$\frac{p+a}{(p+a)^2+w^2}$

Tab. 2.2: Laplace transforms of some elementary functions.

2.11 Inverse Laplace Transform

Sometimes it is necessary to invert the Laplace transform to obtain the time domain solution t. The transformation from the domain of p in the field of t, is called the inversion of the Laplace transform.

Either $F(p)$ the Laplace transform of $f(t), t > 0$. The integral:

$$f(t) = L^{-1}[F(p)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(p)e^{pt}dp \qquad (2.36)$$

It is called: the inverse transform of $F(p)$. It is a real value. The symbol L^{-1} reads "the inverse Laplace transform operator".

Another method to determine $f(t)$ from $F(p)$ is to decompose the function $F(p)$ fractions (elements), using the transform tables, we determine $f(t)$. We will explain this by taking three categories. We seek to find $f(t)$.

2.11.1 If $F(p)$ contains only distinct poles

$$F(p)=\frac{N(p)}{D(p)}=\frac{\prod_{j=0}^{k}(p-z_k)}{\prod_{i=0}^{n}(p-p_i)} \tag{2.37}$$

$F(p)$ Can then be decomposed into a sum of partial fractions:

$$F(p)=\frac{N(p)}{D(p)}=\frac{a_1}{p+p_1}+\frac{a_1}{p+p_1}+...+\frac{a_n}{p+p_n} \tag{2.38}$$

With:

$a_i=\left[\frac{N(p)}{D(p)}(p+p_i)\right]_{p=p_i}$; a_i : constant called "residue at pole p = pi".

Example 2.4:

Find the Inverse Transform of $F(p)=\dfrac{p+3}{(p+1)(p+2)}$

<u>Noticed:</u>

In the event that the degree of $N(p)>$ degree of $D(p)$ In $F(p)=\dfrac{N(p)}{D(p)}$, we must then divide the numerator by the denominator, then apply the partial fraction method.

Example 2.5:

Either $F(p)=\dfrac{p^3+5p^2+9p+7}{(P+1)(p+2)}$

2.11.2 If $F(p)$ contains complex conjugate poles

Let p_1 And p_2 the 2 complex poles conjugate, then:

$$F(p)=\frac{N(p)}{D(p)}=\frac{\alpha_1 p+\alpha_2}{(p+p_1)(p+p_2)}+\frac{a_3}{p+p_3}+...+\frac{a_n}{p+p_n} \tag{2.39}$$

With:

α_1 And α_2 residues at the poles p_1 And p_2 :

$$\left(\alpha_1 p+\alpha_2\right)_{p=-p_1}=\left[\frac{N(p)}{D(p)}\left(p+p_1\right)\left(p+p_2\right)\right]_{p=-p_1\ ou\ p=-p_2}$$

Example 2.6:

Find the Inverse Transform of $F(p)=\dfrac{p+1}{p\left(p^2+p+1\right)}$

2.11.3 If $F(p)$ contains multiple poles

Either p_1 the multiple pole of $F(p)$, r being the multiplicity index of this pole.

$$F(p)=\frac{N(p)}{D(p)} \qquad D(p)=\left(p+p_1\right)^r\left(p+p_{r+1}\right)\left(p+p_{r+2}\right)\ldots\left(p+p_n\right) \tag{2.40}$$

SO $F(p)$ is written:

$$F(p)=\frac{b_r}{\left(p+p_1\right)^r}+\frac{b_{r-1}}{\left(p+p_1\right)^{r-1}}+\ldots+\frac{b_1}{\left(p+p_1\right)}+\frac{b_r}{\left(p+p_1\right)^r}+\frac{a_{r+1}}{\left(p+p_{r+1}\right)}+\frac{a_{r+2}}{\left(p+p_{r+2}\right)}+\ldots+\frac{a_n}{\left(p+p_n\right)}$$

With : $a_k=\left[\dfrac{N(p)}{D(p)}(p+p_k)\right]_{p=-p_k} \quad (k=r+1, r+2, \ldots, n)$

And:

$$b_r=\frac{1}{0!}\left[\frac{N(p)}{D(p)}(p+p_1)^r\right]_{p=-p_1}$$

$$b_{r-1}=\frac{1}{1!}\left[\frac{d}{dp}\left\{\frac{N(p)}{D(p)}(p+p_1)^r\right\}\right]_{p=-p_1}$$

$$b_{r-j}=\frac{1}{\mathrm{j}!}\left[\frac{d^j}{dp^j}\left\{\frac{N(p)}{D(p)}(p+p_1)^r\right\}\right]_{p=-p_1}$$

$$b_1=\frac{1}{(r-1)!}\left[\frac{d^{r-1}}{dp^{r-1}}\left\{\frac{N(p)}{D(p)}(p+p_1)^r\right\}\right]_{p=-p_1}$$

Noticed : $L^{-1}\left[\dfrac{1}{(p+p_1)^n}\right]=\dfrac{t^{n-1}}{(n-1)!}e^{-p_1 t}$

Example 2.7:

Either the transfer function:

$$F(p)=\frac{p^2+2p+3}{(p+1)^3}$$

2.12 State representation

a state representation allows to model a dynamic system using state variables. This representation, which can be linear or not, continuous or discrete, makes it possible to determine the state of the system at any future instant if we know the state at the initial instant and the behavior of the exogenous variables which influence the system.

The state representation of these systems, when they are in time continuous, is written as follows:

$$\begin{cases}\dot{x}(t)=\mathrm{A}\,x(t)+\mathrm{B}u(t)\\ \mathrm{y(t)}=\mathrm{C}\,x(t)+Du(t)\end{cases}\tag{1.41}$$

With:

$x(t)\in\mathbb{R}^n$: column which represents the n state variables.

$u(t)\in\mathbb{R}^m$: column which represents the m orders.

$y(t)\in\mathbb{R}^p$: column which represents the p outings.

$A\in\mathbb{R}^{n\times n}$: State matrix.

$B\in\mathbb{R}^{n\times m}$: Command matrix.

$C\in\mathbb{R}^{p\times n}$: Observation matrix.

$D\in\mathbb{R}^{p\times m}$: Direct Action Matrix

Example 2.8:

- Establish the state representation of the circuit below?

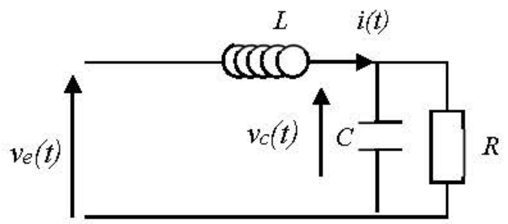


Fig. 2.6: RLC circuit.

2.13 Block diagram of a linear system

Consider the following system:

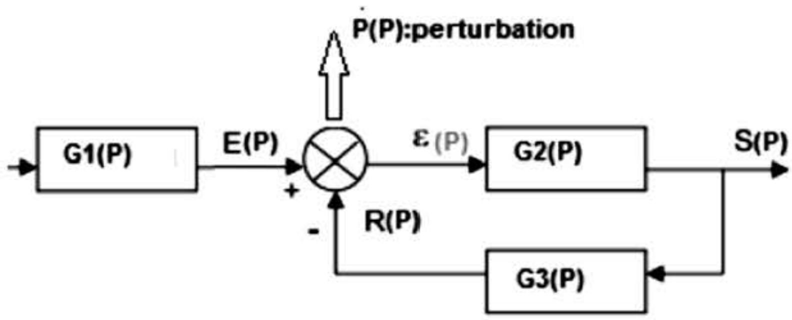


Fig. 2.7: Block diagram of a linear system.

Direct chain of $e(t)$ towards $s(t)$ and the return chain of $s(t)$ towards $r(t)$ and their Laplace transforms.

$G_1(p)$: used to treat $e(t)$ so that $e(t)$ And $r(t)$ be of the same nature.

$G_2(p)$: main control system.

$G_3(p)$: return system.

$\varepsilon(p)$: Error signal.

$E(p)$ And $S(p)$: Laplace transforms of input and output signals.

$P(p)$: perturbation or disturbances or parasites.

Transfer function or total transmittance: $F(p) = \dfrac{S(p)}{E(p)}$

Transfer function or parasitic transmittance: $K(p) = \dfrac{S(p)}{P(p)}$

2.14 Transformation of block diagrams

2.14.1 Serial Block Association

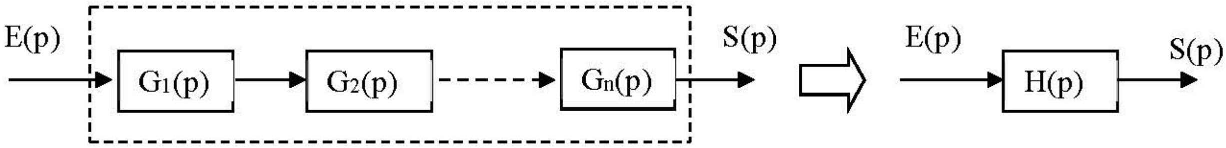


Fig. 2.8: Series connection (or cascade) of transfer functions.

The transfer function of the set is equal to the product of the transfer functions of each element :

$$H(p)=\frac{S(p)}{E(p)}=G_1(p).G_2(p)....G_n(p) \tag{2.45}$$

2.14.2 Association of blocks in parallel

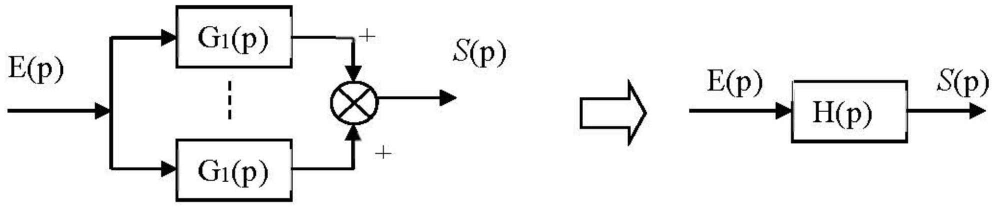


Fig. 2.9: Parallel connection of transfer functions.

The equivalent transfer function $H(p)$ has the expression:

$$H(p)=\frac{S(p)}{E(p)}=G_1(p)+G_2(p)+...+G_n(p) \tag{2.46}$$

2.15 Open Loop Transfer Function

The Open Loop Transfer Function (also called OLTF) is the transfer function that relates the Laplace transforms of the output of the feedback chain $S'(p)$ to the error $\varepsilon(p)$. It corresponds to the opening of the loop (Fig. 2.10):

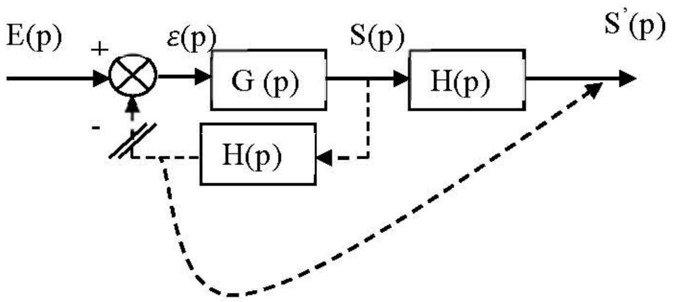


Fig. 2.10: Block diagram of an Open Loop servo system.

The Open Loop Transfer Function $T(p)$ has the expression: $T(p) = \dfrac{S'(p)}{\varepsilon(p)}$

We have:

$$S(p) = G(p).\varepsilon(p)$$
$$\varepsilon(p) = E(p) - S'(p)$$

And : $S'(p) = H(p).S(p)$

$$T(p) = \frac{S'(p)}{\varepsilon(p)}, \varepsilon(p) = \frac{S(p)}{G(p)}$$
$$T(p) = \frac{H(p).S(p)}{\dfrac{S(p)}{G(p)}} = H(p).G(p)$$

So the Open Loop transfer function:

$$T(p) = G(p).H(p) \tag{2.47}$$

2.16 Closed loop transfer function

Let us consider a controlled system, the most general, represented by the diagram in Fig. 2.11:

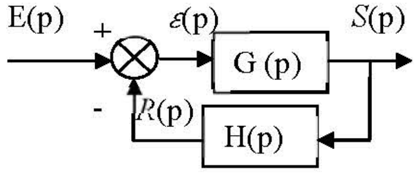


Fig. 2.11: Block diagram of a closed loop servo system.

Either $G(p)$ And $H(p)$, respectively, the forward and return chain transfer functions.

Let us find the transfer function of the complete system: $F(p)=\dfrac{S(p)}{E(p)}$

We have the following relationships:

$$S(p)=G(p).\varepsilon(p)\,,\quad R(p)=H(p).S(p)\,,\quad \varepsilon(p)=E(p)-R(p)$$

$$S(p)=G(p).\left[E(p)-R(p)\right]=G(p).\left[E(p)-H(p).S(p)\right]$$

From where:

$$S(p)=\frac{G(p)}{1+G(p).H(p)}E(p)$$

The transfer function of a closed loop system (FTBF) is therefore:

$$F(p)=\frac{S(p)}{E(p)}=\frac{G(p)}{1+T(p)} \tag{2.48}$$

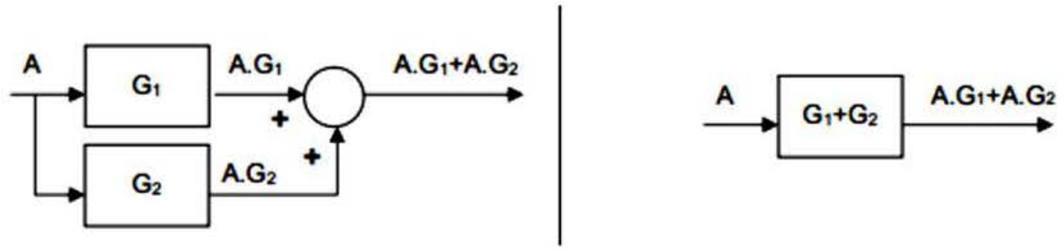
With : $T(p)=G(p).H(p)$

2.17 Simplifications of functional diagrams

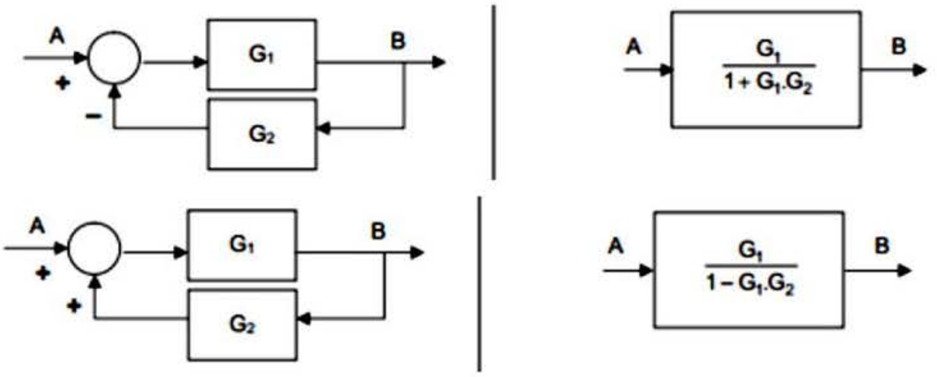
To simplify a functional diagram, it is necessary to reduce the internal loops and for this, it is necessary to move the connection points, the comparators or to interchange them to have in the end a single reduced mesh to determine the transfer function of the simplified system and to move on to its study namely stability, precision etc.

2.17.1 Cascading or series elements

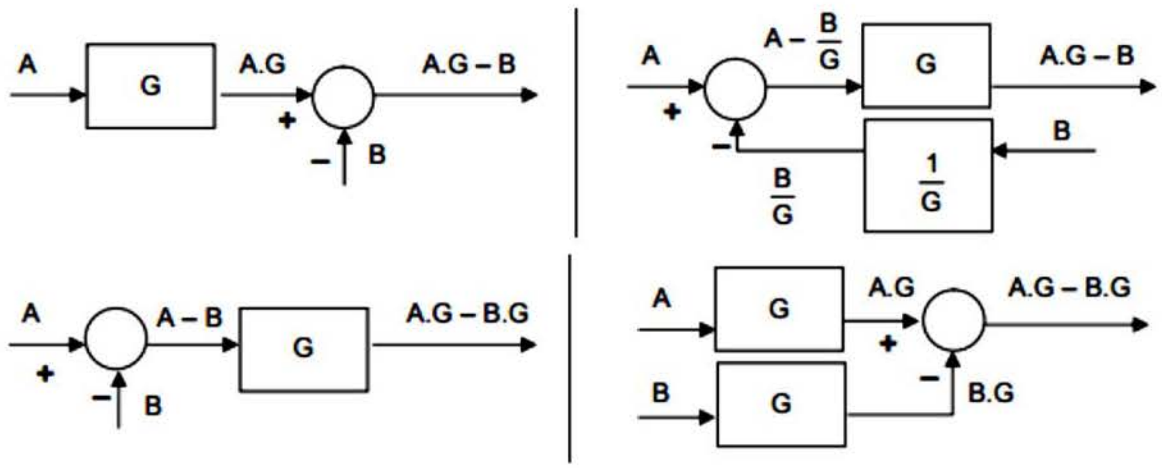
2.17.2 Parallel Elements



2.17.3 Eliminating a feedback loop

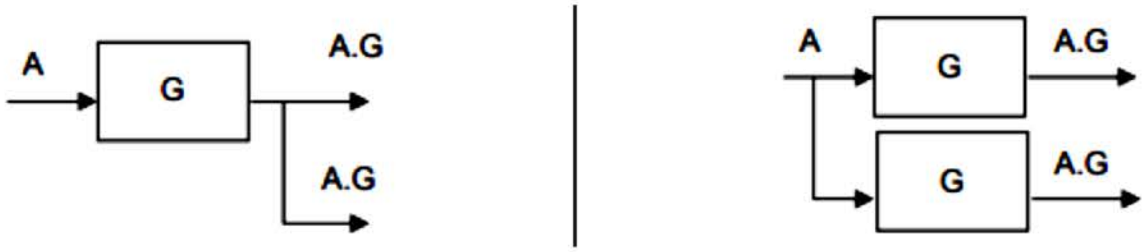


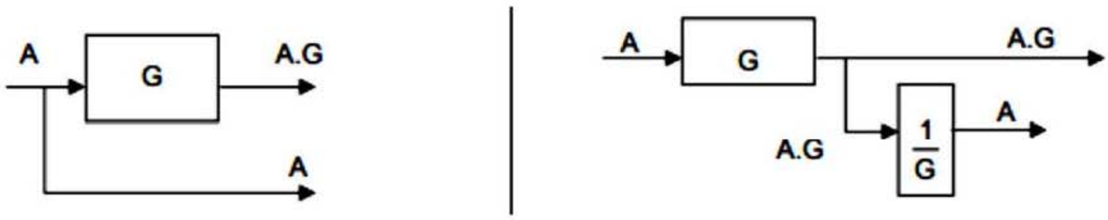
2.17.4 Moving a comparator either upstream or downstream of another element



2.17.5 Moving one comparator relative to another

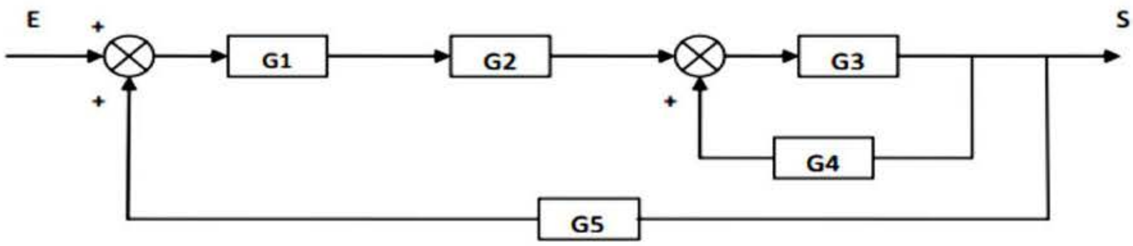
2.17.6 Moving a deviation point either upstream or downstream of another element



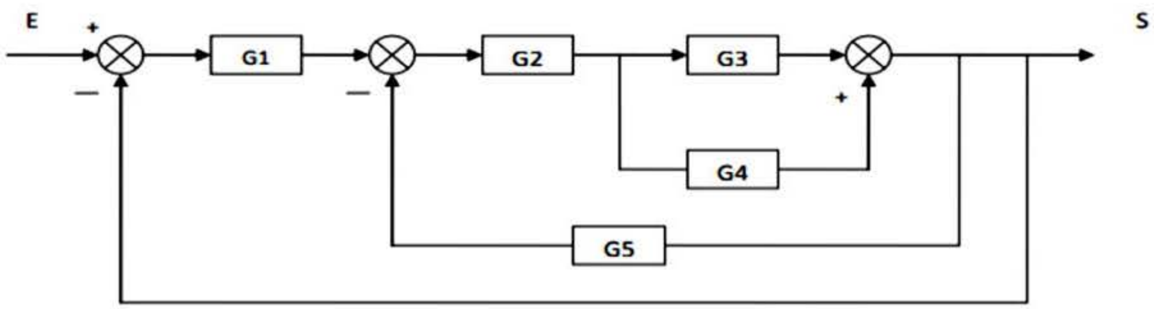


Example 2.9:

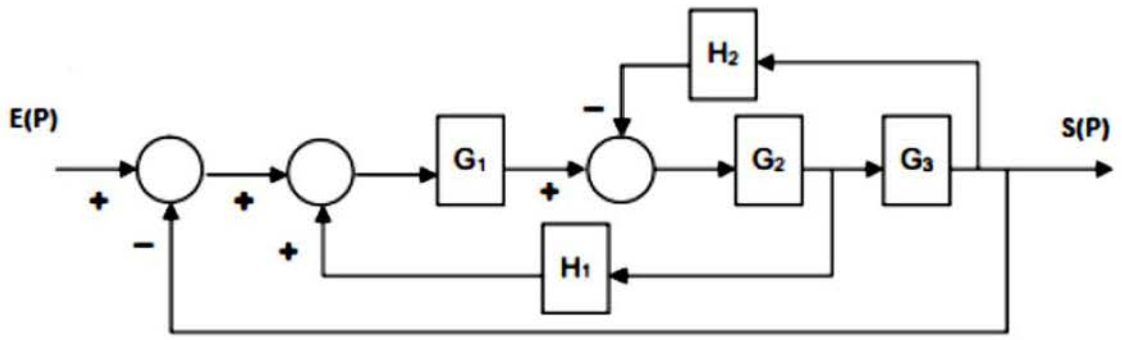
Simplify the following diagrams and give their transfer functions. $F(p) = \dfrac{S(p)}{E(p)}$



Example 2.10:



Example 2.11:



Example 2.12: