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UNIVERSITY MOHAMED BOUDIAF OF M'SILA
FACULTY OF TECHNOLOGY
DEPARTMENT OF ELECTRICAL ENGINEERING

Course Material for the Subject:

Solar and Wind Resources

Intended for 2nd Year Bachelor's Students
Specialty: Renewable Energies and Environment

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Foreword

This manuscript is intended for 2^{nd} Year Bachelor's (LMD) students, specializing in Renewable Energies and Environment. It complies with the official program.

The aim of this course is to familiarize students with meteorological data (solar irradiation and its various components, wind speed, relative humidity, temperature…) and the assessment of renewable energy production for a site (solar resource, wind resource).

I hope this document will be appreciated by my colleagues and will help the students. I would be very pleased to receive their comments and suggestions with gratitude.

Chapitre II. The solar resource

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II.1 Introduction

Solar energy is the radiant light and heat from the sun that has been harnessed by humans since ancient times using a range of ever-evolving technologies. Solar radiation along with secondary solar resources account for most of the available renewable energy on earth. All other renewable energies other than geothermal derive their energy from energy received from the sun.

To design and analyze solar systems, we need to know how much sunlight is available. A fairly straightforward, though complicated-looking, set of equations can be used to predict where the sun is in the sky at any time of day for any location on earth, as well as the solar intensity (or *insolation: in*cident *sol*ar Radi*ation*) on a clear day. To determine average daily insolation under the combination of clear and cloudy conditions that exist at any site we need to start with long-term measurements of sunlight hitting a horizontal surface. Another set of equations can then be used to estimate the insolation on collector surfaces that are not flat on the ground.

II.2 The solar spectrum

The sun is the source of 'Solar energy, it is a gigantic, glowing sphere of hot gas with 1.4 million kilometer diameter. The resulting loss of mass is converted into about 3.8×10^{20} MW of electromagnetic energy that radiates outward from the surface into space.

Terms used in Solar Energy: Irradiance, Irradiation & Insolation:

- **Irradiance**: is the rate at which radiant energy is incident on a surface per unit area (W/m^2) and is represented by the symbol G.
- **Irradiation**: is the incident energy per unit area (J/m^2) on a surface -determined by integration of irradiance over a specified time, usually an hour or a day.
- **Insolation**: is a term used to indicate 'Solar Energy Irradiation'. (An abbreviation for '**In**cident **So**lar Radi**ation**')

Objects emit radiant energy based on their temperature. To quantify this emission, they are compared to an idealized **blackbody**, which is a **perfect emitter and absorber**. A blackbody radiates more energy per unit area than any real object at the same temperature and absorbs all incoming radiation without reflection or transmission. The wavelengths emitted by a blackbody depend on its temperature as described by *Planck's law*:

$$E_\lambda = \frac{3.74 \times 10^8}{\lambda^5 \left[\exp\left(\frac{14{,}400}{\lambda T}\right) - 1\right]} \tag{II.1}$$

Where E_λ is the emissive power per unit area of a blackbody (W/m^2 μm), T is the absolute temperature of the body (K), and λ is the wavelength (μm).

Modeling the earth itself as a 288 K (15◦C) blackbody results in the emission spectrum plotted in Fig. 2.1.

The area under Planck's curve between any two wavelengths is the power emitted between those wavelengths, so the total area under the curve is the total radiant power emitted. That total is conveniently expressed by the *Stefan–Boltzmann law of radiation*:

$$E = A\sigma T^4 \tag{II.2}$$

Where E is the total blackbody emission rate (W), σ is the Stefan–Boltzmann constant = 5.67×10^{-8} W/m^2-K^4, T is the absolute temperature of the blackbody (K), and is the surface area of the blackbody (m^2).

Another convenient feature of the blackbody radiation curve is given by *Wien's displacement rule*, which tells us the wavelength at which the spectrum reaches its maximum point:

$$\lambda_{max}(\mu m) = \frac{2898}{T\,(K)} \tag{II.3}$$

Where the wavelength is in microns (μm) and the temperature is in kelvins.

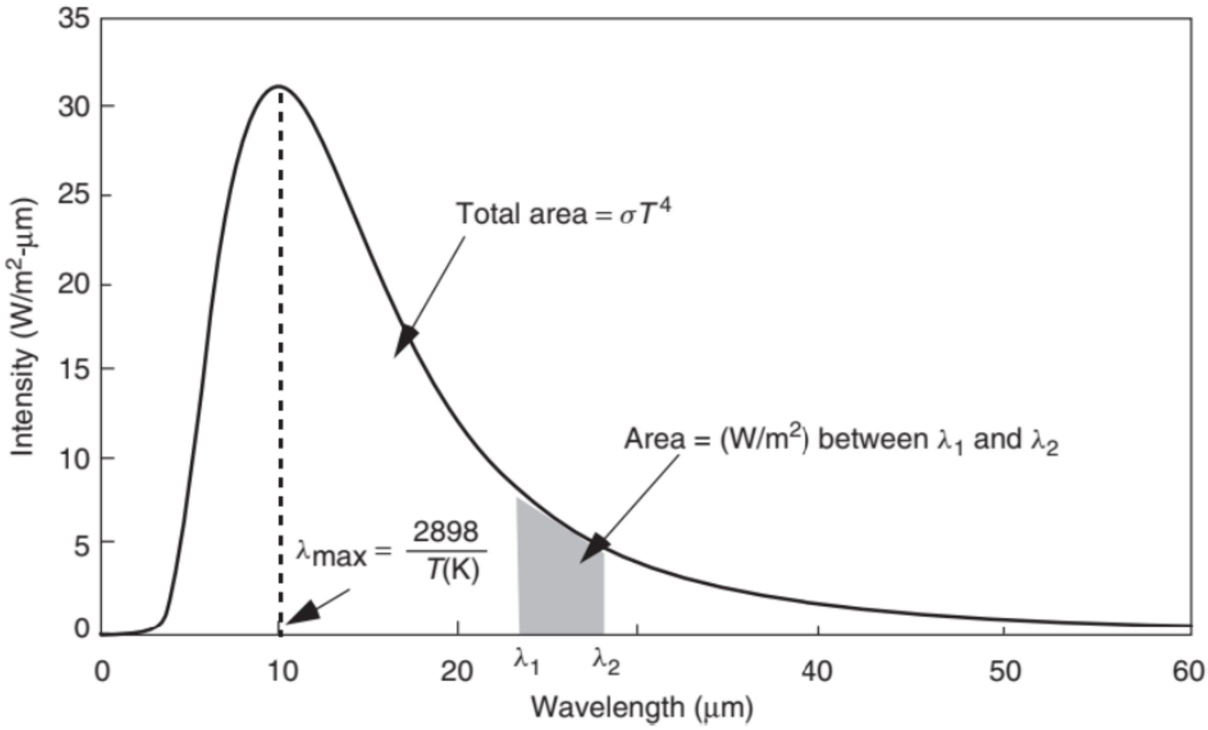

Figure II.1– The spectral emissive power of a 288 K blackbody.

II.2.1 Extra Terrestrial &Terrestrial Solar Radiation

While the solar radiation incident on the Earth's atmosphere which is known as ***Extraterrestrial Solar Radiation*** is relatively constant, the radiation at the Earth's surface which is known as ***Terrestrial Solar Radiation*** varies widely due to:

- Atmospheric effects, including absorption and scattering.
- Local variations in the atmosphere, such as water vapour, clouds, and pollution.
- Latitude of the location and
- Season of the year and the time of day.

The above effects have several impacts on the solar radiation received at the Earth's surface. These changes include:

- Variations in the overall power received, the spectral content of the energy and the angle from which light is incident on a surface.
- In addition, a key change is that the variability of the solar radiation at a particular location increases dramatically. The variability is due to both local effects

such as clouds and seasonal variations, as well as other effects such as the length of the day at particular latitude.

- Desert regions tend to have lower variations due to local atmospheric phenomena such as clouds. Equatorial regions have low variability between seasons.

- As solar radiation makes its way toward the earth's surface, some of it is absorbed by various constituents in the atmosphere, giving the terrestrial spectrum an irregular, bumpy shape.

- The terrestrial spectrum also depends on how much atmosphere the radiation has to pass through to reach the surface. This is explained by a term called *Air Mass Ratio*.

II.2.2 Characteristics of Solar Radiation & Radiation Spectrum

The characteristics of Solar Radiation are best explained with the help of the solar spectrum plots which give data on intensity as spectral content. These characteristics are normally shown at ***Extra Terrestrial*** (above the atmosphere) level and at ***Terrestrial level*** (sea level).

The interior of the sun is estimated as **15 million K**, but its surface radiation follows **Planck's law** for a **5800 K blackbody**. The **solar insolation** outside Earth's atmosphere is **1.37 kW/m²**, with **7% UV, 47% visible, and 46% IR** radiation. As solar radiation passes through the atmosphere, some of it is absorbed, giving the terrestrial spectrum it irregular and bumpy shape.

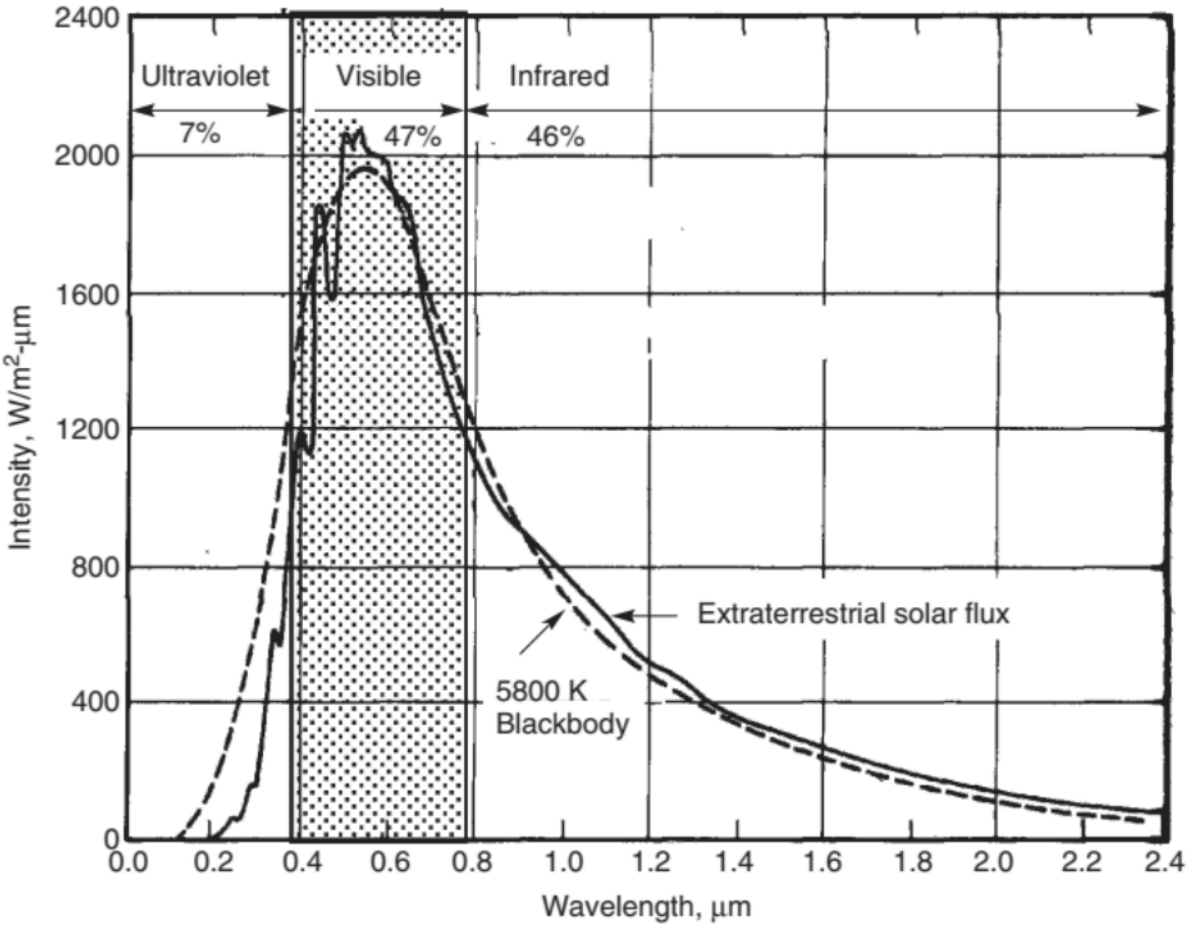

Figure II.2– The extraterrestrial solar spectrum compared with a 5800 K blackbody.

The terrestrial spectrum also depends on how much atmosphere the radiation has to pass through to reach the surface. The *air mass ratio* can be expressed as:

$$m = \frac{h_2}{h_1} = \frac{1}{\sin\beta} \tag{II.4}$$

Where h_1 = path length through the atmosphere with the sun directly overhead, h_2 = path length through the atmosphere to reach a spot on the surface, and β = the altitude angle of the sun (see Fig. 2.3).

Thus, an air mass ratio of 1 (designated “AM1”) means that the sun is directly overhead. By convention, AM0 means no atmosphere; that is, it is the extraterrestrial solar spectrum. Often, an air mass ratio of 1.5 is assumed for an average solar spectrum at the earth’s surface.

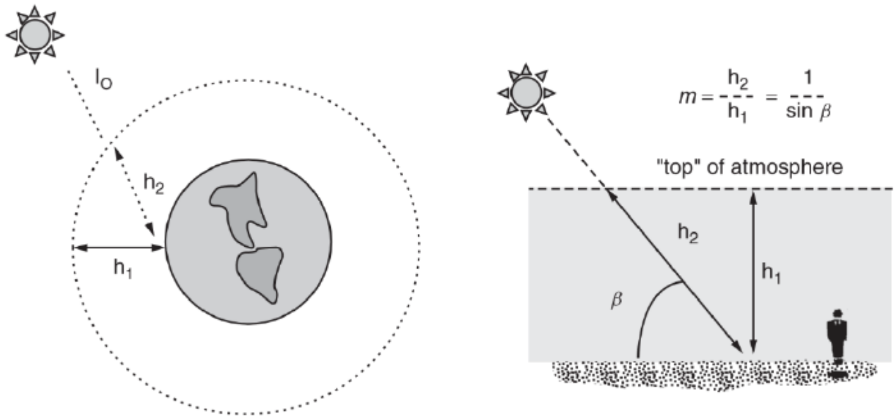

Figure II.3– The air mass ratio m is a measure of the amount of atmosphere the sun's rays must pass through to reach the earth's surface. For the sun directly overhead, m = 1.

As sunlight passes through more atmosphere, less energy arrives at the earth's surface and the spectrum shifts some toward longer wavelengths. The impact of the atmosphere on incoming solar radiation for various air mass ratios is shown in Fig. 2.4.

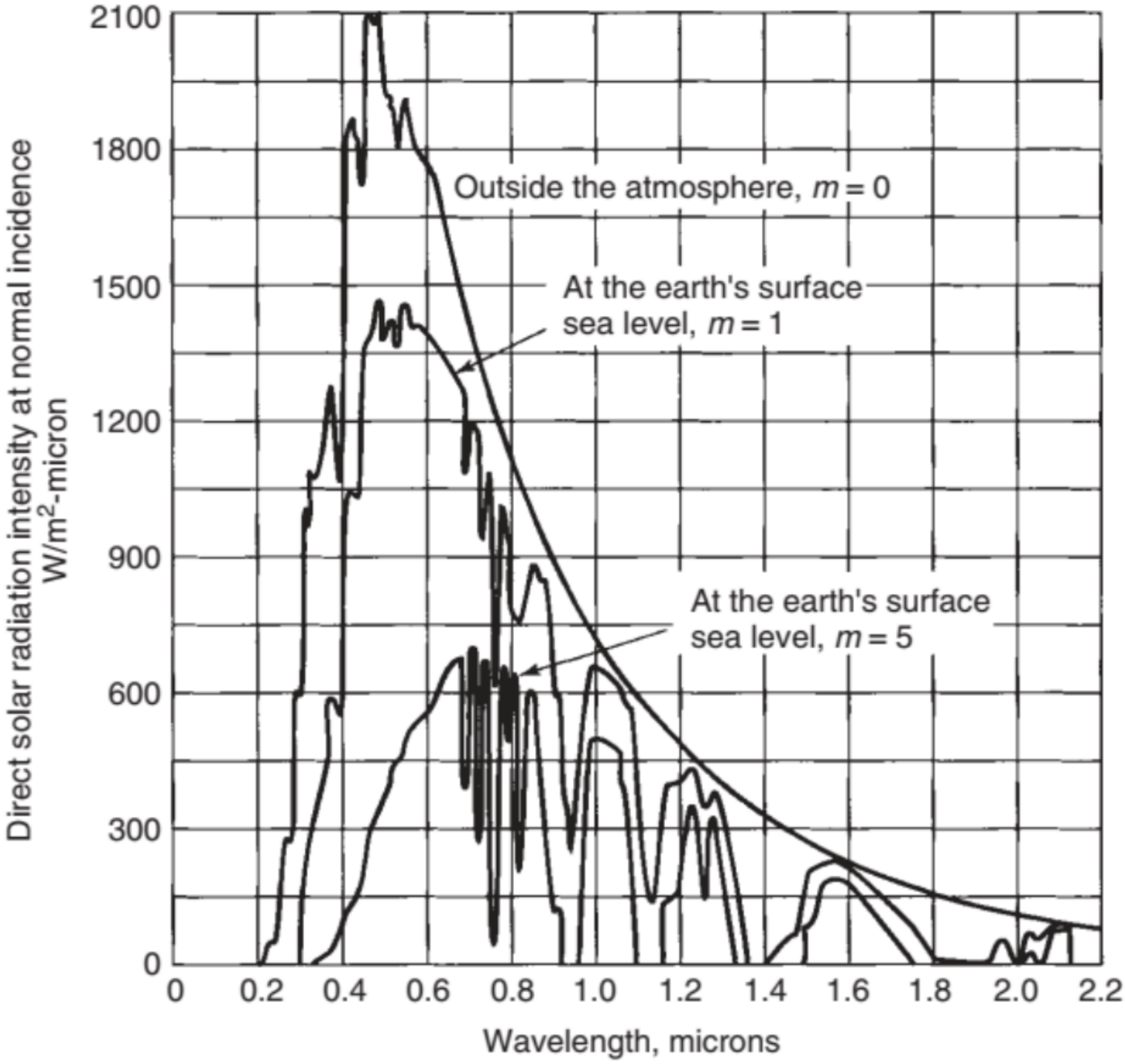

Figure II.4– Solar spectrum for extraterrestrial (m = 0), for sun directly overhead (m = 1), and at the surface with the sun low in the sky, m = 5.

II.3 The earth's orbit

The earth revolves around the sun in an elliptical orbit, making one revolution every 365.25 days. The eccentricity of the ellipse is small and the orbit is, in fact, quite nearly circular. The point at which the earth is nearest the sun, the perihelion, occurs on January 2, at which point it is a little over 147 million kilometers away. At the other extreme, the aphelion, which occurs on July 3, the earth is about 152 million kilometers from the sun. This variation in distance is described by the following relationship:

$$d = 1.5 \times 10^{8} \left\{ 1 + 0.017 \sin \left[\frac{360(n-93)}{365} \right] \right\} [\mathrm{km}] \qquad \text{(II.5)}$$

Where n is the *day number*, with January 1 as day 1 and December 31 being day number 365. Table 2.1 provides a convenient list of day numbers for the first day of each month. *It should be noted that (2.5) and all other equations developed in this chapter involving trigonometric functions use angles measured in degrees, not radians*.

Each day, as the earth rotates about its own axis, it also moves along the ellipse. If the earth were to spin only 360° in a day, then after 6 months time our clocks would be off by 12 hours; that is, at noon on day 1 it would be the middle of the day, but 6 months later noon would occur in the middle of the night. To keep synchronized, the earth needs to rotate one extra turn each year, which means that in a 24-hour day the earth actually rotates 360.99°, which is a little surprising to most of us.

As shown in Fig. 2.5, the plane swept out by the earth in its orbit is called the ecliptic plane. The earth's spin axis is currently tilted 23.45° with respect to the ecliptic plane and that tilt is, of course, what causes our seasons. On March 21 and September 21, a line from the center of the sun to the center of the earth passes through the equator and everywhere on earth we have 12 hours of daytime and 12 hours of night, hence the term *equinox* (equal day and night). On December 21, the winter *solstice* in the Northern Hemisphere, the inclination of the North Pole reaches its highest angle away from the sun (23.45°), while on June 21 the opposite occurs. By the way, for convenience we are using the twenty-first day of the month for the solstices and equinoxes even though the actual days vary slightly from year to year.

Tableau II.1 – Day Numbers for the First Day of Each Month.

January	$n = 1$	July	$n = 182$
February	$n = 32$	August	$n = 213$
March	$n = 60$	September	$n = 244$
April	$n = 91$	October	$n = 274$
May	$n = 121$	November	$n = 305$
June	$n = 152$	December	$n = 335$

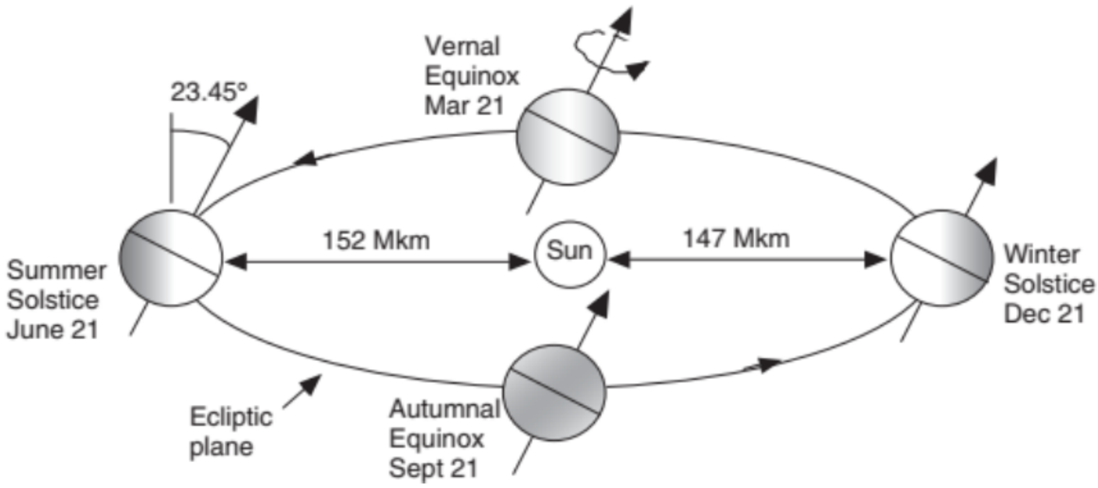

Figure II.5– The tilt of the earth's spin axis with respect to the ecliptic plane is what causes our seasons. "Winter" and "summer" are designations for the solstices in the Northern Hemisphere.

For solar energy applications, the characteristics of the earth's orbit are considered to be unchanging, but over longer periods of time, measured in thousands of years, orbital variations are extremely important as they significantly affect climate. The shape of the orbit oscillates from elliptical to more nearly circular with a period of 100,000 years (*eccentricity*). The earth's tilt angle with respect to the ecliptic plane fluctuates from 21.5° to 24.5° with a period of 41,000 years (*obliquity*). Finally, there is a 23,000-year period associated with the *precession* of the earth's spin axis. This precession determines, for example, where in the earth's orbit a given hemisphere's summer occurs. Changes in the orbit affect the amount of sunlight striking the earth as well as the distribution of sunlight both geographically and seasonally. Those variations are thought to be influential in the timing of the coming and going of ice ages and interglacial periods. In fact, careful analysis of the historical record of global temperatures does show a primary cycle between glacial episodes of about 100,000 years, mixed with secondary oscillations with periods of 23,000 years and 41,000 years that match these orbital changes. This connection between orbital variations and climate were first proposed in the 1930s by an astronomer named Milutin Milankovitch, and the orbital cycles are now referred to as *Milankovitch oscillations*. Sorting out the impact of human activities on climate from those caused by natural variations such as the Milankovitch oscillations is a critical part of the current climate change discussion.

II.4 Altitude angle of the sun at solar noon

We all know that the sun rises in the east and sets in the west and reaches its highest point sometime in the middle of the day. In many situations, it is quite useful to be able to predict exactly where in the sky the sun will be at any time, at any location on any day of the year. Knowing that information we can, for example, design an overhang to allow the sun to come through a window to help heat a house in the winter while blocking the sun in the summer. In the context of photovoltaics, we can, for example, use knowledge of solar angles to help pick the best tilt angle for our modules to expose them to the greatest insolation.

While Fig. 2.5 correctly shows the earth revolving around the sun, it is a difficult diagram to use when trying to determine various solar angles as seen from the surface of the earth. An alternative (and ancient!) perspective is shown in Fig. 2.6, in which the earth is fixed, spinning around its north–south axis; the sun sits somewhere out in space, slowly moving up and down as the seasons progress. On June 21 (the summer solstice) the sun reaches its highest point, and a ray drawn at that time from the center of the sun to the center of the earth makes an angle of 23.45° with the earth's equator. On that day, the sun is directly over the Tropic of Cancer at latitude 23.45°. At the two equinoxes, the sun is directly over the equator. On December 21 the sun is 23.45° below the equator, which defines the latitude known as the Tropic of Capricorn.

As shown in Fig. 2.6, the angle formed between the plane of the equator and a line drawn from the center of the sun to the center of the earth is called the solar declination, δ. It varies between the extremes of ± 23.45°, and a simple sinusoidal relationship that assumes a 365-day year and which puts the spring equinox on day n = 81 provides a very good approximation. Exact values of declination, which vary slightly from year to year, can be found in the annual publication *The American Ephemeris and Nautical Almanac*.

$$\delta = 23.45 \sin\left[\frac{360}{365}(n-81)\right] \tag{II.6}$$

Computed values of solar declination on the twenty-first day of each month are given in Table 2.2.

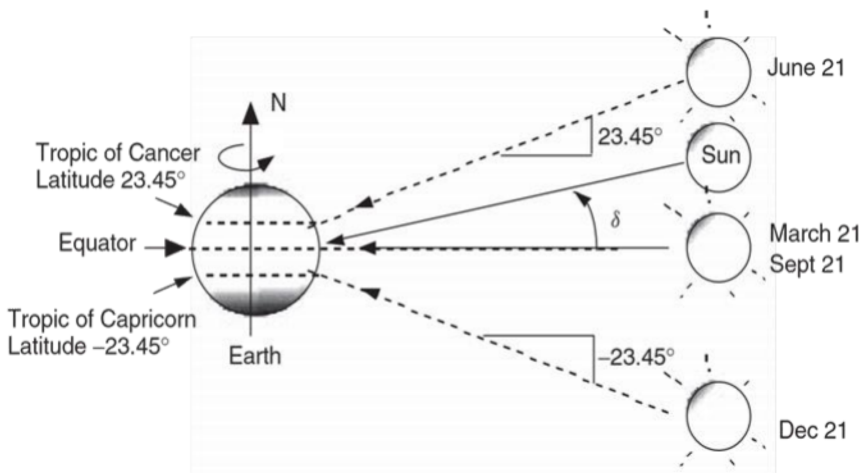

Figure II.6– An alternative view with a fixed earth and a sun that moves up and down. The angle between the sun and the equator is called the solar declination δ.

Tableau II.2 – Solar Declination δ for the 21st Day of Each Month (degrees).

Month:	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec
δ:	−20.1	−11.2	0.0	11.6	20.1	23.4	20.4	11.8	0.0	−11.8	−20.4	−23.4

While Fig. 2.6 does not capture the subtleties associated with the earth's orbit, it is entirely adequate for visualizing various latitudes and solar angles. For example, it is easy to understand the seasonal variation of daylight hours. As suggested in Fig. 2.7, during the summer solstice all of the earth's surface above latitude 66.55° (90° − 23.45°) basks in 24 hours of daylight, while in the Southern Hemisphere below latitude 66.55° it is continuously dark. Those latitudes, of course, correspond to the Arctic and Antarctic Circles.

It is also easy to use Fig. 2.6 to gain some intuition into what might be a good tilt angle for a solar collector. Figure 2.8 shows a south-facing collector on the earth's surface that is tipped up at an angle equal to the local latitude, L. As can be seen, with this tilt angle the collector is parallel to the axis of the earth. During an equinox, at *solar noon*, when the sun is directly over the local meridian (line of longitude), the sun's rays will strike the collector at the best possible angle; that is, they are perpendicular to the collector face. At other times of the year the sun is a little high or a little low for normal incidence, but on the average it would seem to be a good tilt angle.

Solar noon is an important reference point for almost all solar calculations. In the Northern Hemisphere, at latitudes above the Tropic of Cancer, solar noon occurs when the sun is due south of the observer. South of the Tropic of Capricorn, in New Zealand for example, it is when the sun is due north. And in the tropics, the sun may be either due north, due south, or directly overhead at solar noon.

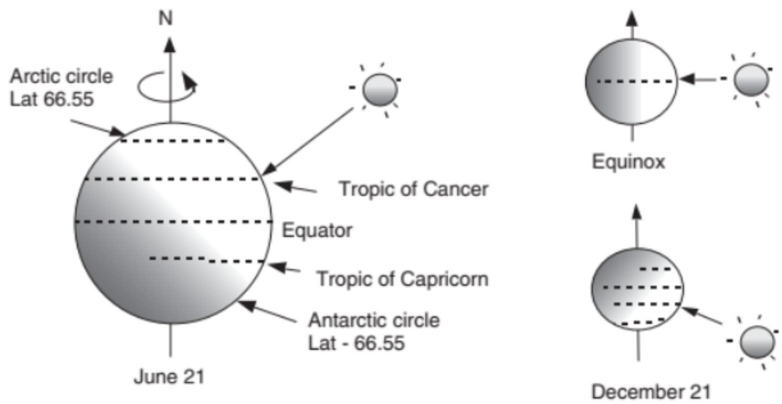

Figure II.7– Defining the earth's key latitudes is easy with the simple version of the earth–sun system.

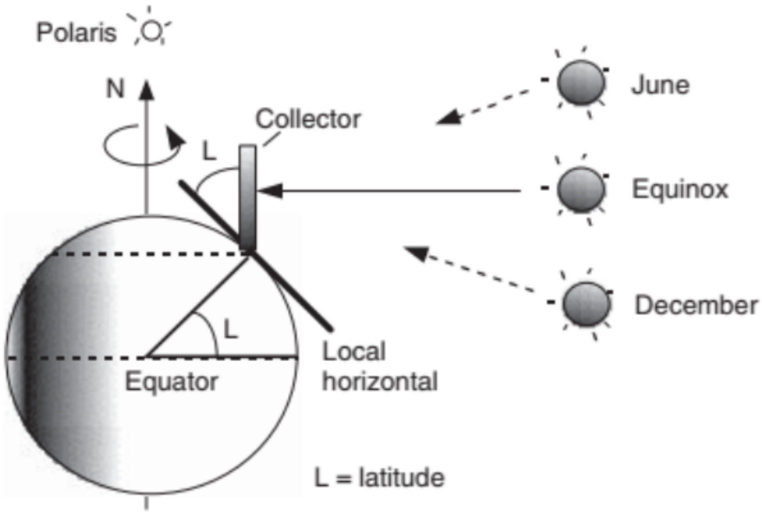

Figure II.8– A south-facing collector tipped up to an angle equal to its latitude is perpendicular to the sun's rays at solar noon during the equinoxes.

On the average, facing a collector toward the equator (for most of us in the Northern Hemisphere, this means facing it south) and tilting it up at an angle equal to the local latitude is a good rule-of-thumb for annual performance. Of course, if you want to emphasize winter collection, you might want a slightly higher angle, and vice versa for increased summer efficiency.

Having drawn the earth-sun system as shown in Fig. 2.6 also makes it easy to determine a key solar angle, namely the *altitude angle* β_N of the sun at solar noon. The altitude angle is the angle between the sun and the local horizon directly beneath the sun. From Fig. 2.9 we can write down the following relationship by inspection:

$$\beta_N = 90^\circ - L + \delta \tag{II.7}$$

Where L is the latitude of the site. Notice in the figure the term *zenith* is introduced, which refers to an axis drawn directly overhead at a site.

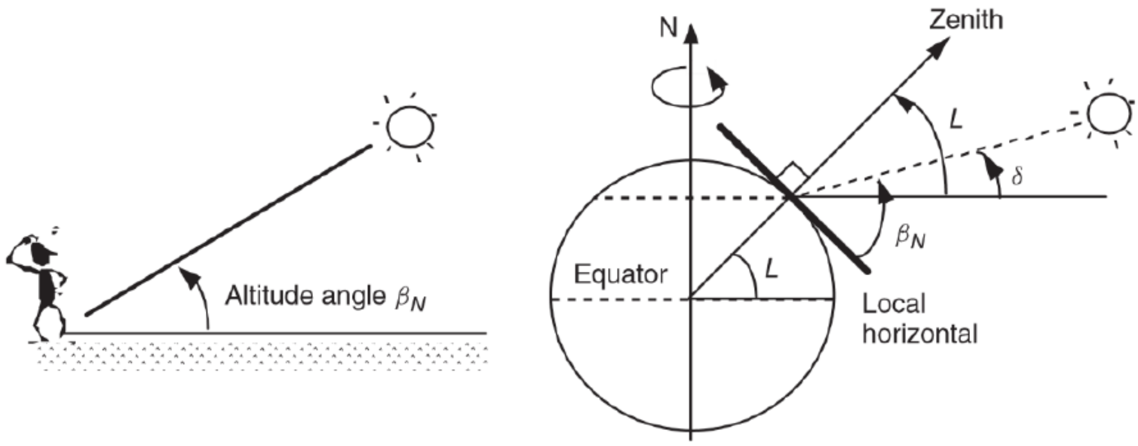

Figure II.9– The altitude angle of the sun at solar noon.

II.5 Solar position at any time of day

The location of the sun at any time of day can be described in terms of its altitude angle β and its azimuth angle φ_s as shown in Fig. 2.10. The subscript s in the azimuth angle helps us remember that this is the azimuth angle of the sun. Later, we will introduce another azimuth angle for the solar collector and a different subscript c will be used. *By convention, the azimuth angle is positive in the morning with the sun in the east and negative in the afternoon with the sun in the west.* Notice that the azimuth angle shown in Fig. 2.10 uses true south as its reference, and this will be the assumption in this text unless otherwise stated. For solar work in the Southern Hemisphere, azimuth angles are measured relative to north.

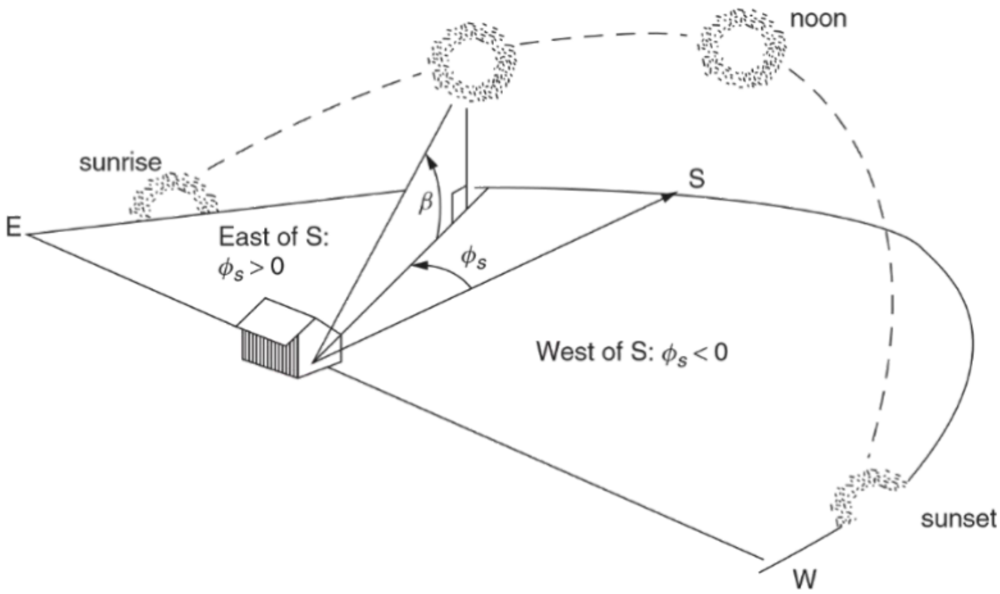

Figure II.10–The sun's position can be described by its altitude angle β and its azimuth angle φs. By convention, the azimuth angle is considered to be positive before solar noon.

The azimuth and altitude angles of the sun depend on the latitude, day number, and, most importantly, the time of day. For now, we will express time as the number of hours before or after solar noon. Thus, for example, 11 A.M. *solar time* is one hour before the sun crosses your local meridian (due south for most of us). Later we will learn how to make the adjustment between solar time and local clock time. The following two equations allow us to compute the altitude and azimuth angles of the sun. For a derivation see, for example, T. H. Kuen et al. (1998):

$$\mathrm{Sin}(\beta) = \cos(L)\cos(\delta)\cos(H) + \sin(L)\sin(\delta) \tag{II.8}$$

$$\sin\phi_S = \frac{\cos\delta \sin H}{\cos\beta} \tag{II.9}$$

Notice that time in these equations is expressed by a quantity called the *hour angle*, H. The hour angle is the number of degrees that the earth must rotate before the sun will be directly over your local meridian (line of longitude). As shown in Fig. 2.11, at any instant, the sun is directly over a particular line of longitude, called the sun's meridian. The difference between the local meridian and the sun's meridian is the hour angle, with positive values occurring in the morning before the sun crosses the local meridian.

Considering the earth to rotate 360° in 24 h, or 15°/h, the hour angle can be described as follows:

$$H = \left(\frac{15^\circ}{\text{hour}}\right) \cdot \tag{II.10}$$

Thus, the hour angle H at 11:00 A.M. solar time would be +15° (the earth needs to rotate another 15°, or 1 hour, before it is solar noon). In the afternoon, the hour angle is negative, so, for example, at 2:18 P.M. solar time H would be −34.5°.

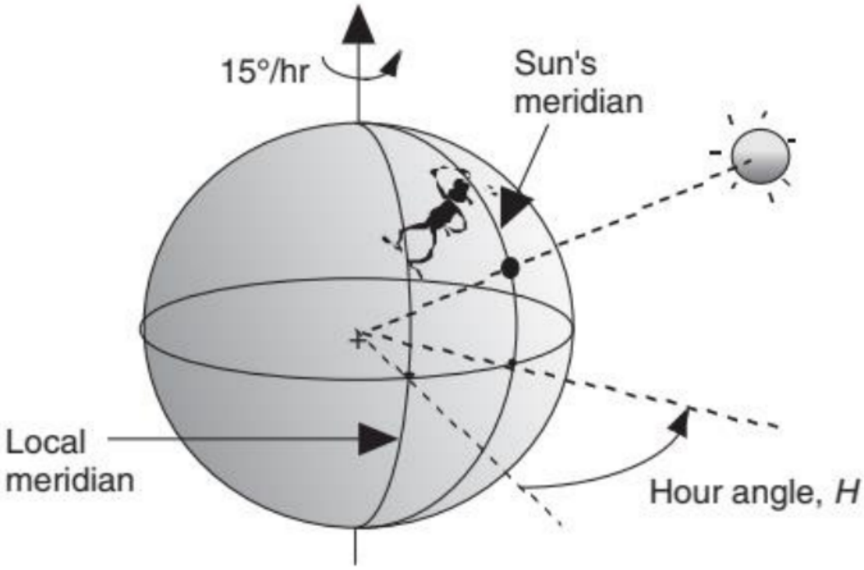

Figure II.11– The hour angle is the number of degrees the earth must turn before the sun is directly over the local meridian. It is the difference between the sun's meridian and the local meridian.

There is a slight complication associated with finding the azimuth angle of the sun from (2.9). During spring and summer in the early morning and late afternoon, the magnitude of the sun's azimuth is liable to be more than 90° away from south (that never happens in the fall and winter). Since the inverse of a sine is ambiguous, sin x = sin (180 − x), we need a test to determine whether to conclude the azimuth is greater than or less than 90° away from south. Such a test is:

$$\text{if} \quad cos\left(H \geq \frac{\tan\delta}{\tan L}\right), \quad \text{then } |\varphi_S| \leq 90^\circ; \;\; \text{otherwise } |\varphi_S| > 90^\circ \qquad \text{(II.11)}$$

Solar altitude and azimuth angles for a given latitude can be conveniently portrayed in graphical form, an example of which is shown in Fig. 2.12. Similar sun path diagrams for other latitudes are given in Appendix B. As can be seen, in the spring and summer the sun rises and sets slightly to the north and our need for the azimuth test given in (2.11) is apparent; at the equinoxes, it rises and sets precisely due east and due west (everywhere on the planet); during the fall and winter the azimuth angle of the sun is never greater than 90°.

II.6 Sun path diagrams for shading analysis

Not only do sun path diagrams, such as that shown in Fig. 2.12, help to build one's intuition into where the sun is at any time, they also have a very practical application in the field when trying to predict shading patterns at a site—a very important consideration for photovoltaics, which are very shadow sensitive. The concept is simple. What is needed is a sketch of the azimuth and altitude angles for trees, buildings, and other obstructions along the southerly horizon that can be drawn on top of a sun path diagram. Sections of the sun path diagram that are covered by the obstructions indicate periods of time when the sun will be behind the obstruction and the site will be shaded.

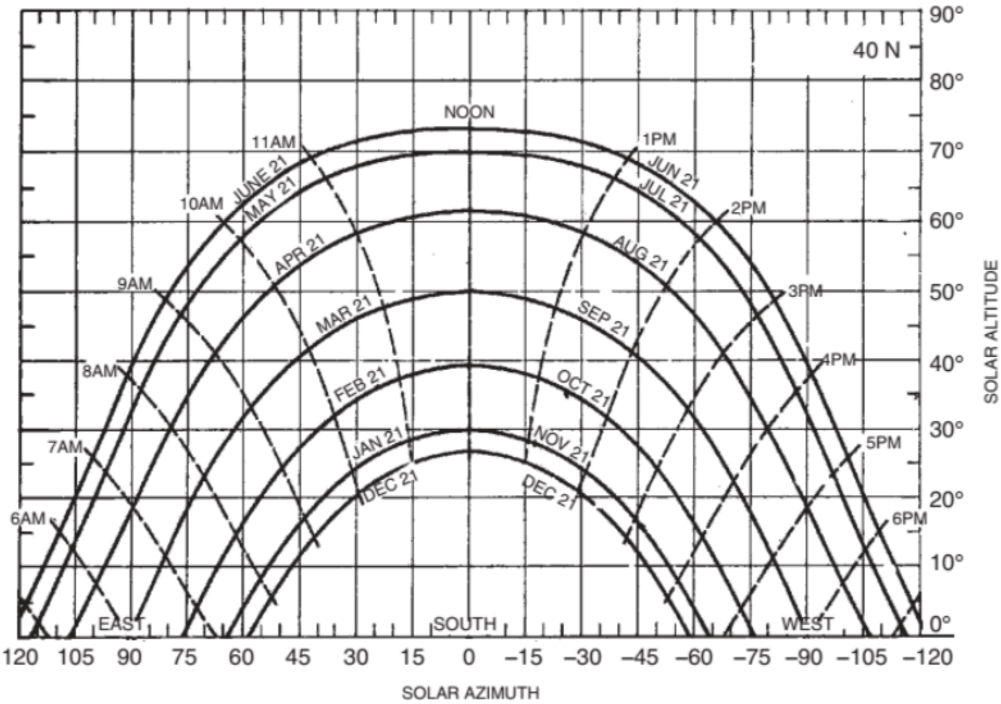

Figure II.12– A sun path diagram showing solar altitude and azimuth angles for 40◦ latitude.

There are several site assessment products available on the market that make the superposition of obstructions onto a sun path diagram pretty quick and easy to obtain. You can do just as good a job, however, with a simple compass, plastic protractor, and plumb bob, but the process requires a little more effort. The compass is used to measure azimuth angles of obstructions, while the protractor and plumb bob measure altitude angles.

Begin by tying the plumb bob onto the protractor so that when you sight along the top edge of the protractor the plumb bob hangs down and provides the altitude angle of the top of the obstruction. Figure 2.13 shows the idea. By standing at the site and scanning the southerly horizon, the altitude angles of major obstructions can be obtained reasonably quickly and quite accurately.

The azimuth angles of obstructions, which go along with their altitude angles, are measured using a compass. Remember, however, that a compass points to magnetic north rather than true north; this difference, called the magnetic declination or deviation, must be corrected for. In the continental United States, this deviation ranges anywhere from about 22°E in Seattle (the compass points 22° east of true north), to essentially zero along the east coast of Florida, to 22°W at the northern tip of Maine. Figure 2.14 shows this variation in magnetic declination and shows an example, for San Francisco, of how to use it.

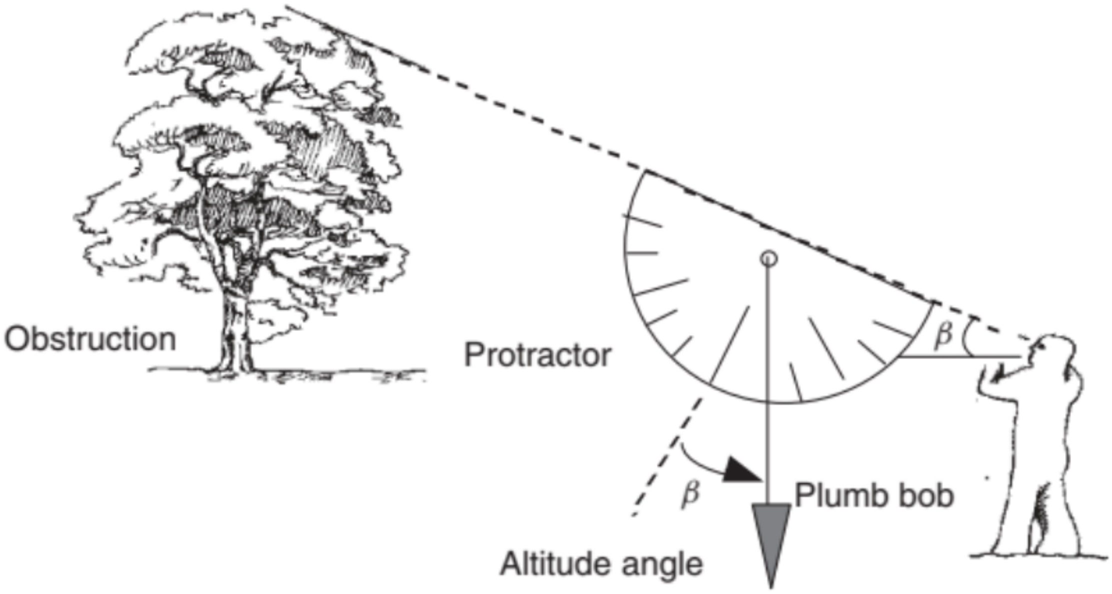

Figure II.13– Measuring the altitude angle of a southerly obstruction using a plumb bob and protractor.

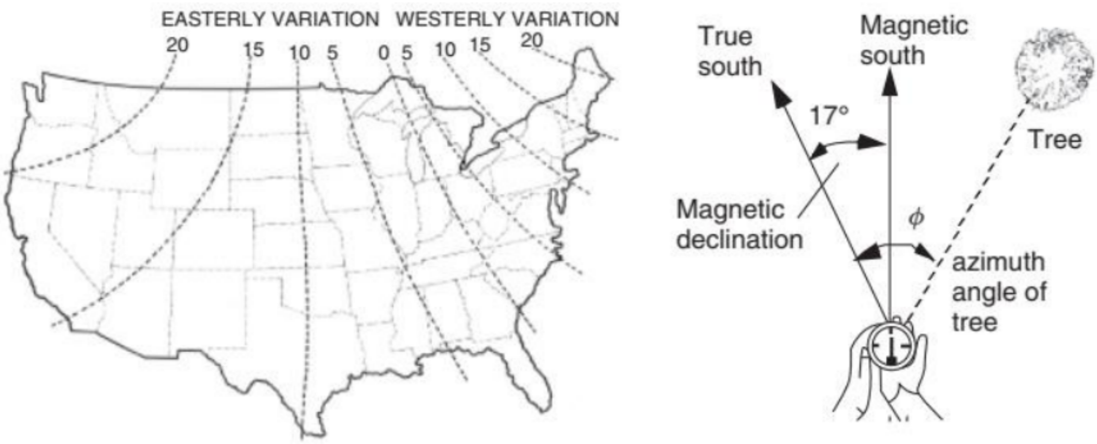

Figure II.14– Lines of equal magnetic declination across the United States. The example shows the correction for San Francisco, which has a declination of 17°E.

Figure 2.15 shows an example of how the sun path diagram, with a superimposed sketch of potential obstructions, can be interpreted. The site is a proposed solar house with a couple of trees to the southeast and a small building to the southwest. In this example, the site receives full sun all day long from February through October. From November through January, the trees cause about one hour's worth of sun to be lost from around 8:30 A.M. to 9:30 A.M., and the small building shades the site after about 3 o'clock in the afternoon.

When obstructions plotted on a sun path diagram are combined with hour-by-hour insolation information, an estimate can be obtained of the energy lost due to shading. Table 2.3 shows an example of the hour-by-hour insolations available on a clear day in January at 40° latitude for south-facing collectors with fixed tilt angle, or for collectors mounted on 1-axis or 2-axis tracking systems. Later in this chapter, the equations that were used to compute this table will be presented, and in Appendix C there is a full set of such tables for a number of latitudes.

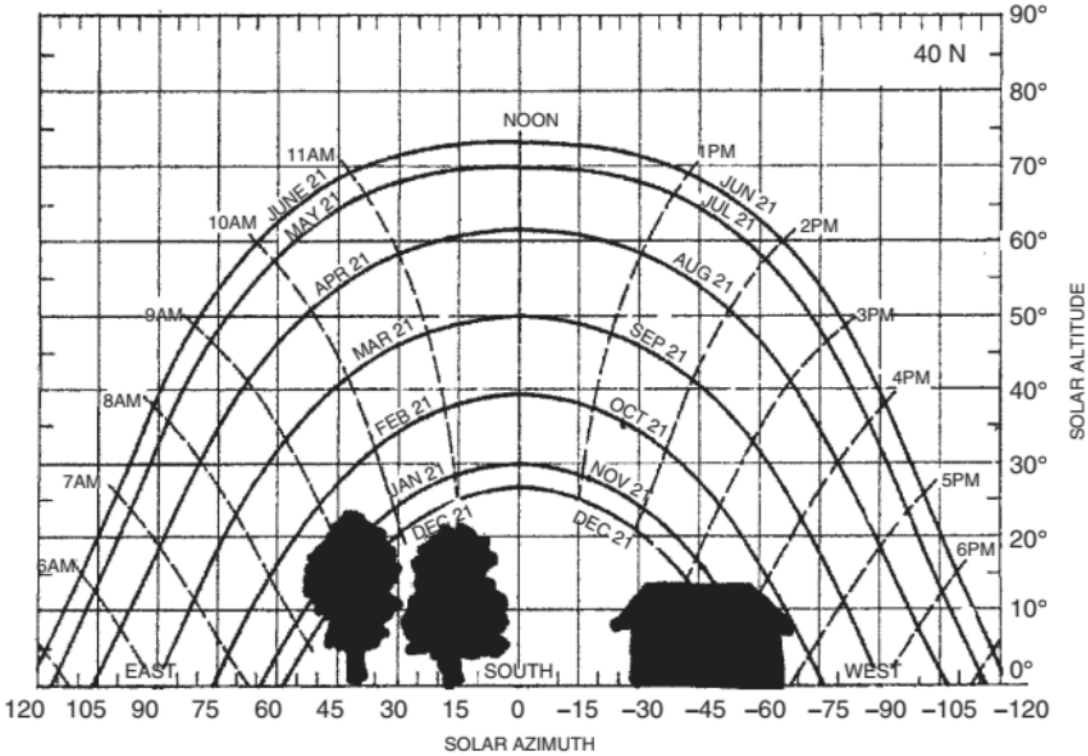

Figure II.15– A sun path diagram with superimposed obstructions makes it easy to estimate periods of shading at a site.

Tableau II.3 – Clear Sky Beam Plus Diffuse Insolation at 40° Latitude in January for South-Facing Collectors with Fixed Tilt Angle and for Tracking Mounts (hourly W/m² and daily kWh/m²-day).

Solar Time	Tracking		Tilt Angles				Latitude 40°		
	One-Axis	Two-Axis	0	20	30	40	50	60	90
			January 21					(W/m²)	
7, 5	0	0	0	0	0	0	0	0	0
8, 4	439	462	87	169	204	232	254	269	266
9, 3	744	784	260	424	489	540	575	593	544
10, 2	857	903	397	609	689	749	788	803	708
11, 1	905	954	485	722	811	876	915	927	801
12	919	968	515	761	852	919	958	968	832
kWh/d:	6.81	7.17	2.97	4.61	5.24	5.71	6.02	6.15	5.47

II.7 Solar time and civil (clock) time

For most solar work it is common to deal exclusively in solar time (ST), where everything is measured relative to solar noon (when the sun is on our line of longitude). There are occasions, however, when local time, called civil time or clock time (CT), is needed. There are two adjustments that must be made in order to connect local clock time and solar time. The first is a longitude adjustment that has to do with the way in which regions of the world are divided into time zones. The second is a little fudge factor that needs to be thrown in to account for the uneven way in which the earth moves around the sun.

Obviously, it just wouldn't work for each of us to set our watches to show noon when the sun is on our own line of longitude. Since the earth rotates 15° per hour (4 minutes per degree), for every degree of longitude between one location and another, clocks showing solar time would have to differ by 4 minutes. The only time two clocks would show the same time would be if they both were due north/south of each other.

To deal with these longitude complications, the earth is nominally divided into 24 1-hour time zones, with each time zone ideally spanning 15° of longitude. Of course, geopolitical boundaries invariably complicate the boundaries from one zone to another. The intent is for all clocks within the time zone to be set to the same time. Each time zone is defined by a *Local Time Meridian* located, ideally, in the middle of the zone, with the origin of this time system passing through Greenwich, England, at 0° longitude. The local time meridians for the United States are given in Table 2.4.

Tableau II.4 – Local Time Meridians for U.S. Standard Time Zones (Degrees West of Greenwich).

Time Zone	LT Meridian
Eastern	75°
Central	90°
Mountain	105°
Pacific	120°
Eastern Alaska	135°
Alaska and Hawaii	150°

The longitude correction between local clock time and solar time is based on the time it takes for the sun to travel between the local time meridian and the observer's line of longitude. If it is solar noon on the local time meridian, it will be solar noon 4 minutes later for every degree that the observer is west of that meridian. For example, San Francisco, at longitude 122°, will have solar noon 8 minutes after the sun crosses the 120° Local Time Meridian for the Pacific Time Zone.

The second adjustment between solar time and local clock time is the result of the earth's elliptical orbit, which causes the length of a *solar day* (solar noon to solar noon) to vary throughout the year. As the earth moves through its orbit, the difference between a 24-hour day and a solar day changes following an expression known as the *Equation of Time*, E:

$$E = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B) \quad \text{[minutes]} \qquad \text{(II.12)}$$

Where:

$$B = \frac{360}{364}(n - 81) \quad \text{[degrees]} \qquad \text{(II.13)}$$

As before, n is the day number. A graph of (2.12) is given in Fig. 2.16.

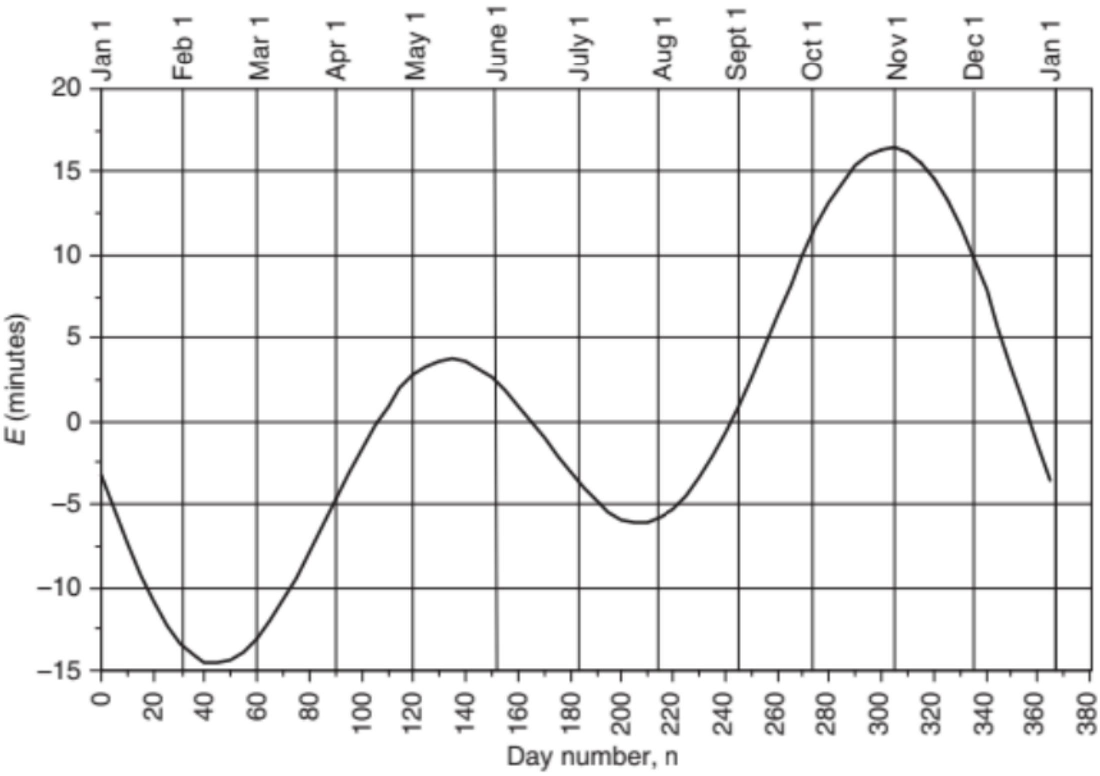

Figure II.16– The Equation of Time adjusts for the earth's tilt angle and noncircular orbit.

Putting together the longitude correction and the Equation of Time gives us the final relationship between local standard clock time (CT) and solar time (ST).

$$Solar\ Time\ (ST) = Clock\ Time\ (CT) + \frac{4min}{degree}(Local\ Time\ Meridian - Local\ longitude)^\circ + E(min) \tag{II.14}$$

When Daylight Savings Time is in effect, one hour must be added to the local clock time ("Spring ahead, Fall back").

II.8 Sunrise and sunset

A sun path diagram, such as was shown in Fig. 2.12, can be used to locate the azimuth angles and approximate times of sunrise and sunset. A more careful estimate of sunrise/sunset can be found from a simple manipulation of (2.8). At sunrise and sunset, the altitude angle β is zero, so we can write:

$$\sin\beta = \cos L \cos\delta \cos H + \sin L \sin\delta = 0 \tag{II.15}$$

$$\cos H = -\frac{\sin L \sin\delta}{\cos L \cos\delta} = -\tan L \tan\delta \tag{II.16}$$

Solving for the hour angle at sunrise, H_{SR}, gives:

$$H_{SR} = \cos^{-1}(-\tan L \tan \delta) \qquad (+ \text{ for sunrise}) \tag{II.17}$$

Notice in (2.17) that since the inverse cosine allows for both positive and negative values, we need to use our sign convention, which requires the positive value to be used for sunrise (and the negative for sunset).

Since the earth rotates *15°/h*, the hour angle can be converted to time of sunrise or sunset using

$$\text{Sunrise(geometric)} \quad = 12{:}00 - \frac{H_{SR}}{15°/h} \tag{II.18}$$

Equations (2.15) to (2.18) are geometric relationships based on angles measured to the center of the sun, hence the designation *geometric sunrise* in (2.18). They are perfectly adequate for any kind of normal solar work, but they won't give you exactly what you will find in the newspaper for sunrise or sunset. The difference between weather service sunrise and our geometric sunrise (2.18) is the result of two factors. The first deviation is caused by atmospheric refraction; this bends the sun's rays, making the sun *appear* to rise about 2.4 min sooner than geometry would tell us and then set 2.4 min later. The second is that the weather service definition of sunrise and sunset is the time at which the upper limb (top) of the sun crosses the horizon, while ours is based on the center crossing the horizon. This effect is complicated by the fact that at sunrise or sunset the sun pops up, or sinks, much quicker around the equinoxes when it moves more vertically than at the solstices when its motion includes much more of a sideward component. An adjustment factor Q that accounts for these complications is given by the following:

$$Q = \frac{3.467}{\cos L \cos \delta \sin H_{SR}} \qquad (min) \tag{II.19}$$

Since sunrise is earlier when it is based on the top of the sun rather than the middle, Q should be subtracted from geometric sunrise. Similarly, since the upper limb sinks below the horizon later than the middle of the sun, Q should be added to our geometric sunset. A plot of (2.19) is shown in Fig. 2.17. As can be seen, for mid-latitudes, the correction is typically in the range of about 4 to 6 min.

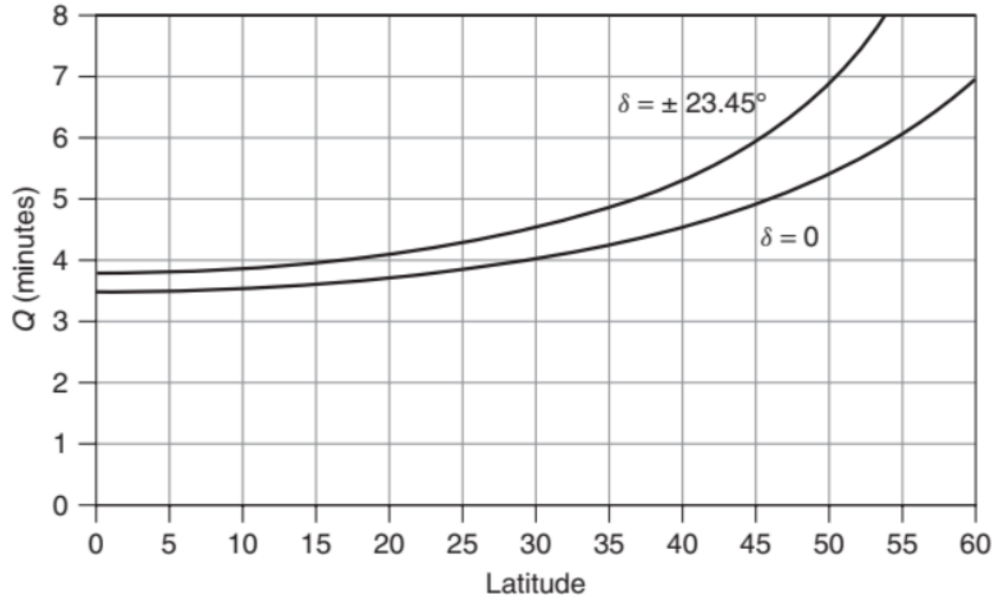

Figure II.17– Sunrise/sunset adjustment factor to account for refraction and the upper-limb definition of sunrise. The range of solar declinations is shown.

II.9 Clear sky direct-beam radiation

Solar flux striking a collector will be a combination of *direct-beam* radiation that passes in a straight line through the atmosphere to the receiver, *diffuse* radiation that has been scattered by molecules and aerosols in the atmosphere, and *reflected* radiation that has bounced off the ground or other surface in front of the collector (Fig. 2.18). The preferred units, especially in solar–electric applications, are watts (or kilowatts) per square meter. Other units involving British Thermal Units, kilocalories, and langleys may also be encountered. Conversion factors between these units are given in Table 2.5.

Solar collectors that focus sunlight usually operate on just the beam portion of the incoming radiation since those rays are the only ones that arrive from a consistent direction. Most photovoltaic systems, however, do not use focusing devices, so all three components beam, diffuse, and reflected can contribute to energy collected. The goal of this section is to be able to estimate the rate at which just the beam portion of solar radiation passes through the atmosphere and arrives at the earth's surface on a clear day. Later, the diffuse and reflected radiation will be added to the clear day model. And finally, procedures will be presented that will enable more realistic average insolation calculations for specific locations based on empirically derived data for certain given sites.

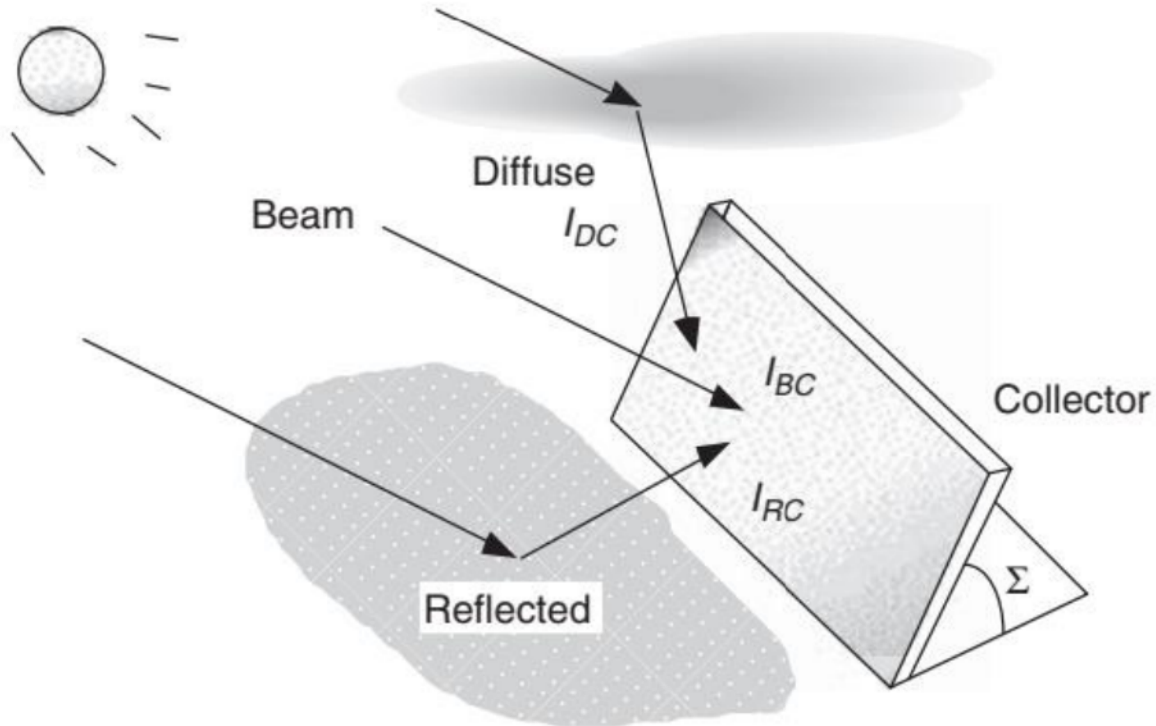

Figure II.18– Solar radiation striking a collector, I_C, is a combination of direct beam, I_{BC}, diffuse, I_{DC}, and reflected, I_{RC}.

Tableau II.5 – Conversion Factors for Various Insolation Units.

1 kW/m^2	=	316.95 Btu/h-ft^2
	=	1.433 langley/min
1 kWh/m^2	=	316.95 Btu/ft^2
	=	85.98 langleys
	=	3.60×10^6 joules/m^2
1 langley	=	1 cal/cm^2
	=	41.856 kjoules/m^2
	=	0.01163 kWh/m^2
	=	3.6878 Btu/ft^2

The starting point for a clear sky radiation calculation is with an estimate of the extraterrestrial (ET) solar insolation, I_0, that passes perpendicularly through an imaginary surface just outside of the earth's atmosphere as shown in Fig. 2.19. This insolation depends on the distance between the earth and the sun, which varies with the time of year. It also depends on the intensity of the sun, which rises and falls with a fairly predictable cycle. During peak periods of magnetic activity on the sun, the surface has large numbers of cooler, darker regions called *sunspots*, which in essence block solar radiation, accompanied by other regions, called *faculae*, that are brighter than the surrounding surface. The net effect of sunspots that dim the sun, and faculae that brighten it, is an increase in solar intensity during periods of increased numbers of sunspots. Sunspot activity seems to follow an 11-year cycle. During sunspot peaks, the most recent of which was in 2001, the extraterrestrial insolation is estimated to be about 1.5% higher than in the valleys (U.S. Department of Energy, 1978).

Ignoring sunspots, one expression that is used to describe the day-to-day variation in extraterrestrial solar insolation is the following:

$$I_0 = \mathrm{SC} \cdot \left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] \qquad (\mathrm{W/m^2}) \tag{II.20}$$

where SC is called the *solar constant* and n is the day number. The solar constant is an estimate of the average annual extraterrestrial insolation. Based on early NASA measurements, the solar constant was often taken to be 1.353 kW/m^2, but 1.377 kW/m^2 is now the more commonly accepted value.

As the beam passes through the atmosphere, a good portion of it is absorbed by various gases in the atmosphere, or scattered by air molecules or particulate matter. In fact, over a year's time, less than half of the radiation that hits the top of the atmosphere reaches the earth's surface as direct beam. On a clear day, however, with the sun high in the sky, beam radiation at the surface can exceed 70% of the extraterrestrial flux.

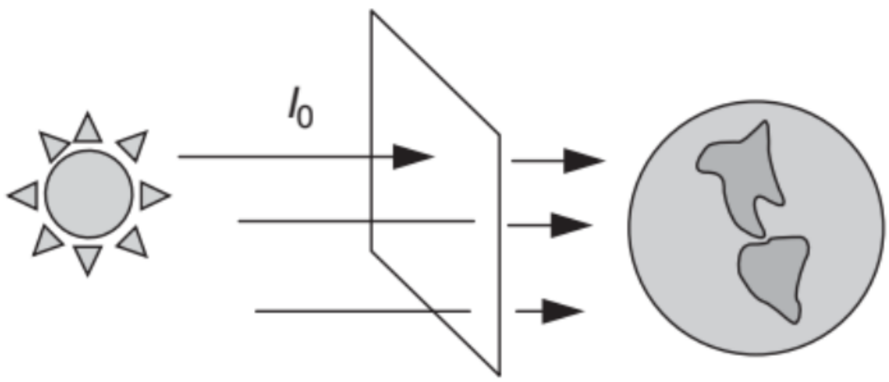

Figure II.19– The extraterrestrial solar flux.

Attenuation of incoming radiation is a function of the distance that the beam has to travel through the atmosphere, which is easily calculable, as well as factors such as dust, air pollution, atmospheric water vapor, clouds, and turbidity, which are not so easy to account for. A commonly used model treats attenuation as an exponential decay function:

$$I_B = Ae^{-km}\ [w/m^2] \tag{II.21}$$

Where I_B is the beam portion of the radiation reaching the earth's surface (normal to the rays), A is an "apparent" extraterrestrial flux, and k is a dimensionless factor called the *optical depth*. The air mass ratio m was introduced earlier as (2.4).

For computational purposes, it is handy to have an equation to work with rather than a table of values. Close fits to the values of optical depth k and apparent extraterrestrial (ET) flux A given in Table 2.6 are as follows:

$$A = 1160 + 75 \sin\left[\frac{360}{365}(n - 275)\right]\ [w/m^2] \tag{II.22}$$

$$k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right] \tag{II.23}$$

Where again *n* is the day number.

Table 2.6 gives values of A and k that are used in the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE) Clear Day Solar Flux Model. This model is based on empirical data collected by Threlkeld and Jordan (1958) for a moderately dusty atmosphere with atmospheric water vapor content equal to the average monthly values in the United States. Also included is a diffuse factor, C, that will be introduced later.

Tableau II.6 – Optical Depth k, Apparent Extraterrestrial Flux A, and the Sky Diffuse Factor C for the *21st* Day of Each Month.

Month:	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
A (W/m^2):	1230	1215	1186	1136	1104	1088	1085	1107	1151	1192	1221	1233
k:	0.142	0.144	0.156	0.180	0.196	0.205	0.207	0.201	0.177	0.160	0.149	0.142
C:	0.058	0.060	0.071	0.097	0.121	0.134	0.136	0.122	0.092	0.073	0.063	0.057

II.10 Total clear sky insolation on a collecting surface

Reasonably accurate estimates of the clear sky, direct beam insolation are easy enough to work out and the geometry needed to determine how much of that will strike a collector surface is straightforward. It is not so easy to account for the diffuse and reflected insolation but since that energy bonus is a relatively small fraction of the total, even crude models are usually acceptable.

II.10.1 Direct-Beam Radiation

The translation of direct-beam radiation I_B (normal to the rays) into beam insolation striking a collector face I_{BC} is a simple function of the angle of incidence θ between a line drawn normal to the collector face and the incoming beam radiation (Fig. 2.20). It is given by:

$$I_{BC} = I_B \cos(\theta) \ [w/m^2] \tag{II.24}$$

For the special case of beam insolation on a horizontal surface I_{BH} :

$$I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin(\beta) \ [w/m^2] \tag{II.25}$$

The angle of incidence θ will be a function of the collector orientation and the altitude and azimuth angles of the sun at any particular time. Figure 2.21 introduces these important angles. The solar collector is tipped up at an angle and faces in a direction described by its azimuth angle φ_C (measured relative to due south, *with positive values in the southeast direction and negative values in the southwest*). The incidence angle is given by:

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma \tag{II.26}$$

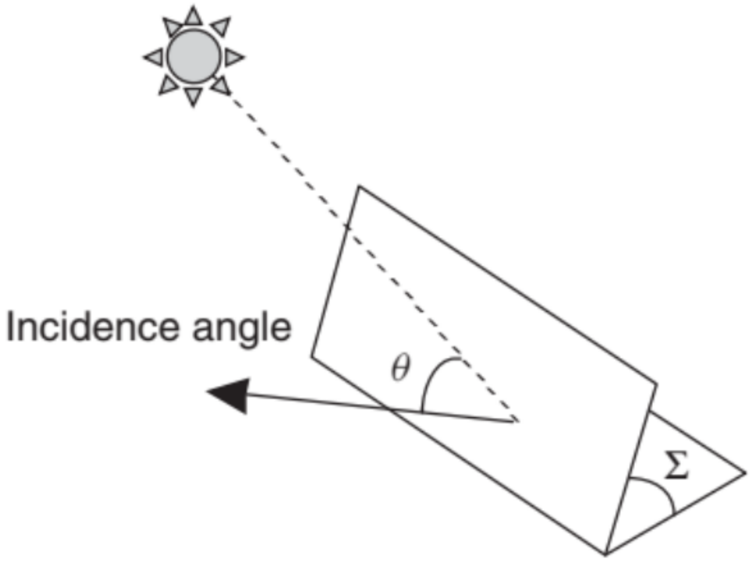

Figure II.20– The incidence angle θ between a normal to the collector face and the incoming solar beam radiation.

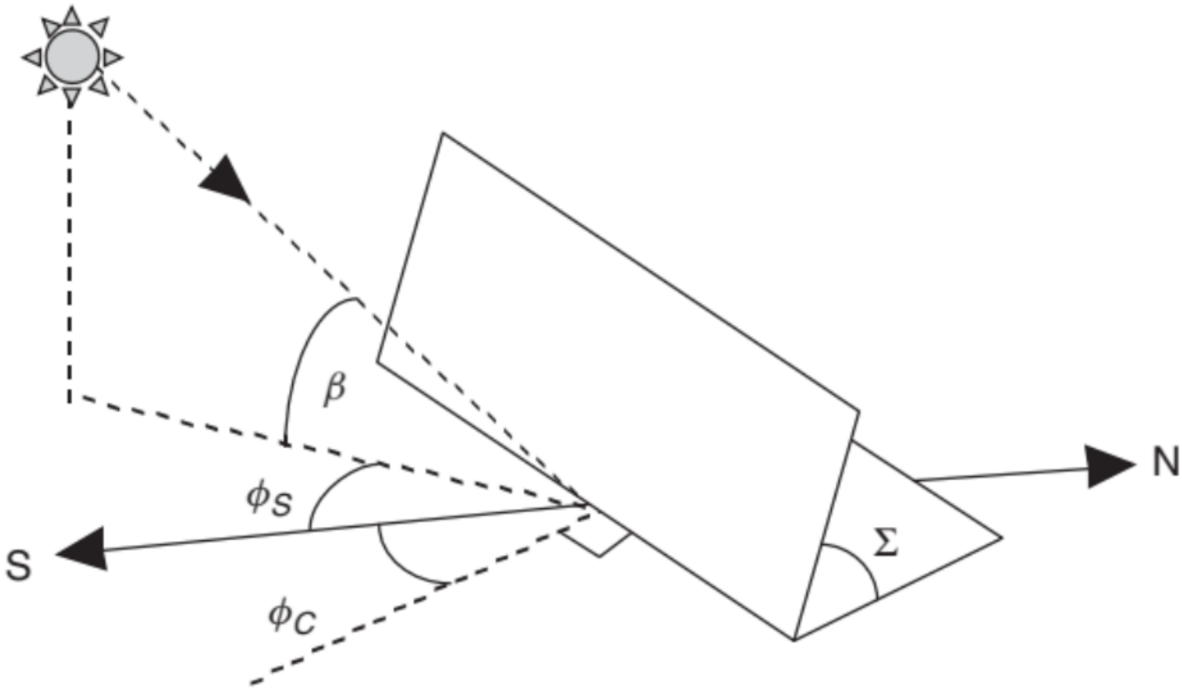

Figure II.21– Illustrating the collector azimuth angle φ_C and tilt angle along with the solar azimuth angle φ_S and altitude angle β. Azimuth angles are positive in the southeast direction and are negative in the southwest.

II.10.2 Diffuse Radiation

The diffuse radiation on a collector is much more difficult to estimate accurately than it is for the beam. Consider the variety of components that make up diffuse radiation as shown in Fig. 2.22. Incoming radiation can be scattered from atmospheric particles and moisture, and it can be reflected by clouds. Some is reflected from the surface back into the sky and scattered again back to the ground. The simplest models of diffuse radiation assume it arrives at a site with equal intensity from all directions; that is, the sky is considered to be *isotropic*. Obviously, on hazy or overcast days the sky is considerably brighter in the vicinity of the sun, and measurements show a similar phenomenon on clear days as well, but these complications are often ignored.

The model developed by Threlkeld and Jordan (1958), which is used in the ASHRAE Clear-Day Solar Flux Model, suggests that diffuse insolation on a horizontal surface I_{DH} is proportional to the direct beam radiation I_B no matter where in the sky the sun happens to be:

$$I_{DH} = C\, I_B \; [w/m^2] \tag{II.27}$$

Where C is a sky diffuse factor. Monthly values of C are given in Table 2.6, and a convenient approximation is as follows:

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right] \tag{II.28}$$

Applying (2.27) to a full day of clear skies typically predicts that about 15% of the total horizontal insolation on a clear day will be diffuse.

What we would like to know is how much of that horizontal diffuse radiation strikes a collector so that we can add it to the beam radiation. As a first approximation, it is assumed that diffuse radiation arrives at a site with equal intensity from all directions. This means that the collector will be exposed to whatever fraction of the sky the face of the collector points to, as shown in Fig. 2.23. When the tilt angle of the collector is zero—that is, the panel is flat on the ground—the panel sees the full sky and so it receives the full horizontal diffuse radiation, I_{DH} . When it is a vertical surface, it sees half the sky and is exposed to half of the horizontal diffuse radiation, and so forth. The following expression for diffuse radiation on the collector, I_{DC}, is used when the diffuse radiation is idealized in this way:

$$I_{DC} = I_{DH}\left(\frac{1 + \cos\Sigma}{2}\right) = C\, I_B\left(\frac{1 + \cos\Sigma}{2}\right) [w/m^2] \tag{II.29}$$

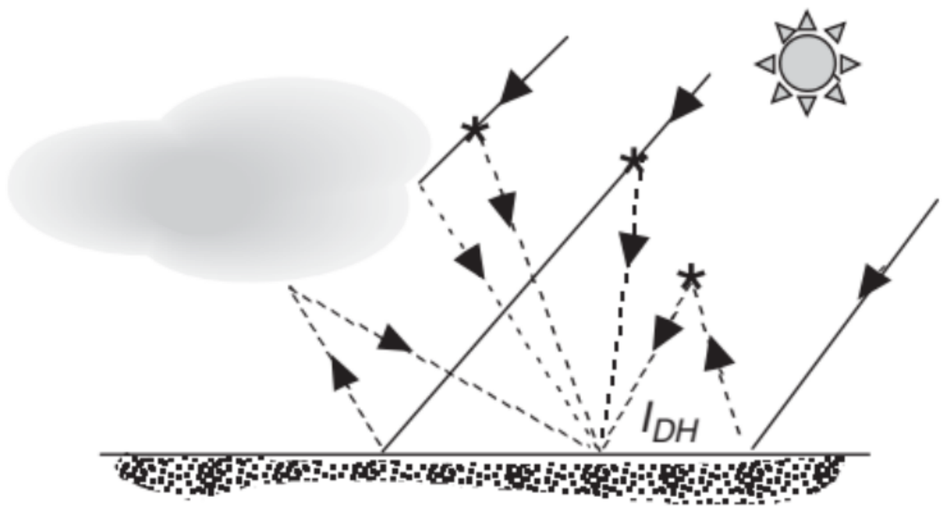

Figure II.22– Diffuse radiation can be scattered by atmospheric particles and moisture or reflected from clouds. Multiple scatterings are possible.

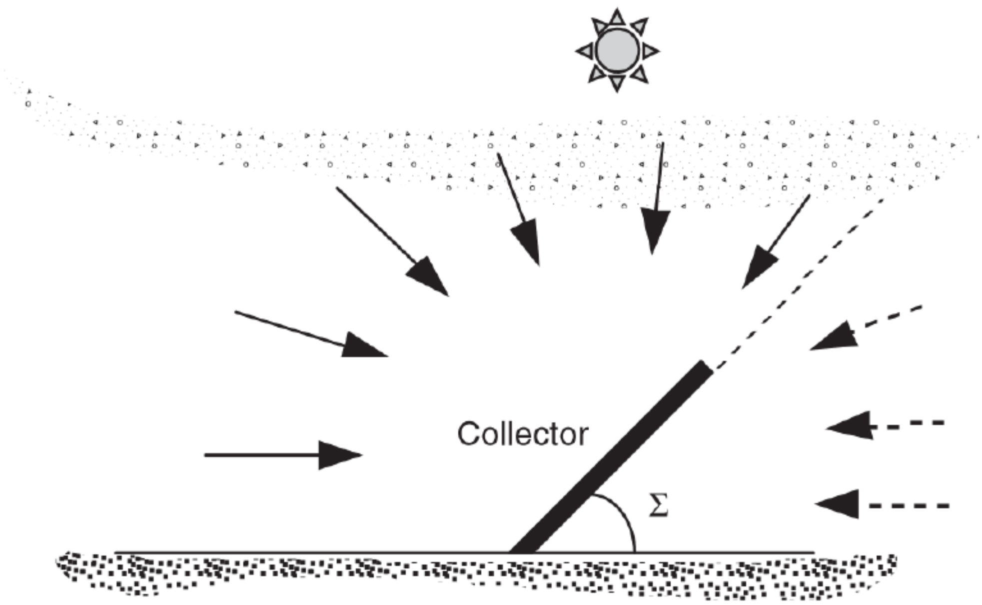

Figure II.23– Diffuse radiation on a collector assumed to be proportional to the fraction of the sky that the collector "sees".

II.10.3 Reflected Radiation

The final component of insolation striking a collector results from radiation that is reflected by surfaces in front of the panel. This reflection can provide a considerable boost in performance, as for example on a bright day with snow or water in front of the collector, or it can be so modest that it might as well be ignored. The assumptions needed to model reflected radiation are considerable, and the resulting estimates are very rough indeed. The simplest model assumes a large horizontal area in front of the collector, with a reflectance ρ that is diffuse, and it bounces the reflected radiation in equal intensity in all directions, as shown in Fig. 2.24. Clearly this is a very gross assumption, especially if the surface is smooth and bright.

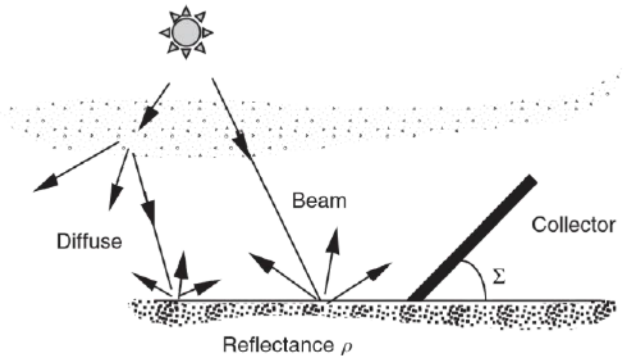

Figure II.24– The ground is assumed to reflect radiation with equal intensity in all directions.

Estimates of ground reflectance range from about 0.8 for fresh snow to about 0.1 for a bituminous-and-gravel roof, with a typical default value for ordinary ground or grass taken to be about 0.2. The amount reflected can be modeled as the product of the total horizontal radiation (beam I_{BH} , plus diffuse I_{DH}) times the ground reflectance ρ. The fraction of that ground-reflected energy that will be intercepted by the collector depends on the slope of the panel , resulting in the following expression for reflected radiation striking the collector I_{RC}:

$$I_{RC} = \rho(I_{BH} + I_{DH})\left(\frac{1-\cos\Sigma}{2}\right) [w/m^2] \tag{II.30}$$

For a horizontal collector (∑=0), Eq. (2.30) correctly predicts no reflected radiation on the collector; for a vertical panel, it predicts that the panel "sees" half of the reflected radiation, which also is appropriate for the model.

Substituting expressions (2.25) and (2.27) into (2.30) gives the following for reflected radiation on the collector:

$$I_{RC} = \rho I_B(\sin\beta + C)\left(\frac{1-\cos\Sigma}{2}\right) [w/m^2] \tag{II.31}$$

II.11 Tracking systems

Thus far, the assumption has been that the collector is permanently attached to a surface that does not move. In many circumstances, however, racks that allow the collector to track the movement of the sun across the sky are quite cost effective. Trackers are described as being either *two-axis trackers*, which track the sun both in azimuth and altitude angles so the collectors are always pointing directly at the sun, or *single-axis trackers*, which track only one angle or the other.

II.11.1 Two-Axis Tracking

Calculating the beam plus diffuse insolation on a two-axis tracker is quite straightforward (Fig. 2.25). The beam radiation on the collector is the full insolation I_B normal to the rays calculated using (2.21). The diffuse and reflected radiation are found using (2.29) and (2.31) with a collector tilt angle equal to the complement of the solar altitude angle, that is, $90^\circ - \beta$.

$$I_{BC} = I_B \quad [w/m^2] \tag{II.32}$$

$$I_{DC} = C I_B \left[\frac{1 + \cos(90^\circ - \beta)}{2} \right] [w/m^2] \tag{II.33}$$

$$I_{RC} = \rho (I_{BH} + I_{DH}) \left[\frac{1 - \cos(90^\circ - \beta)}{2} \right] [w/m^2] \tag{II.34}$$

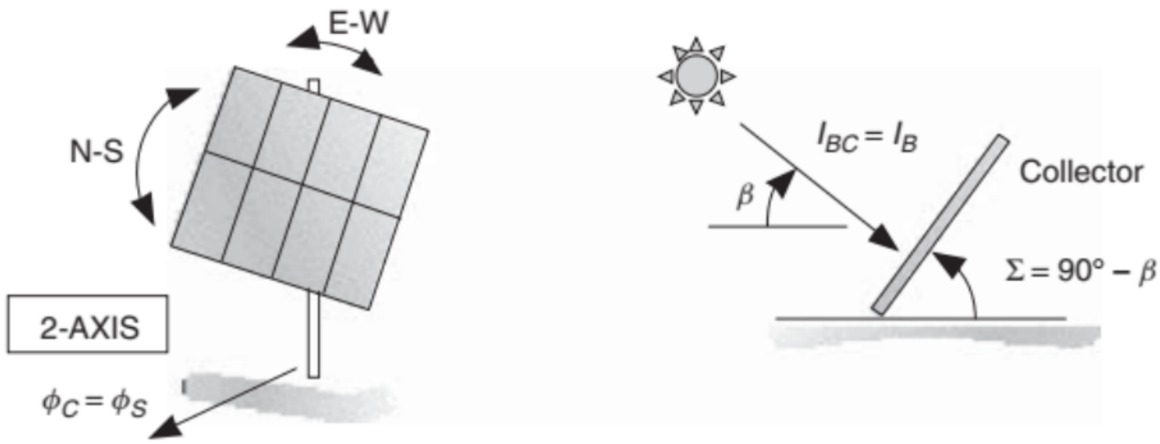

Figure II.25– Two-axis tracking angular relationships.

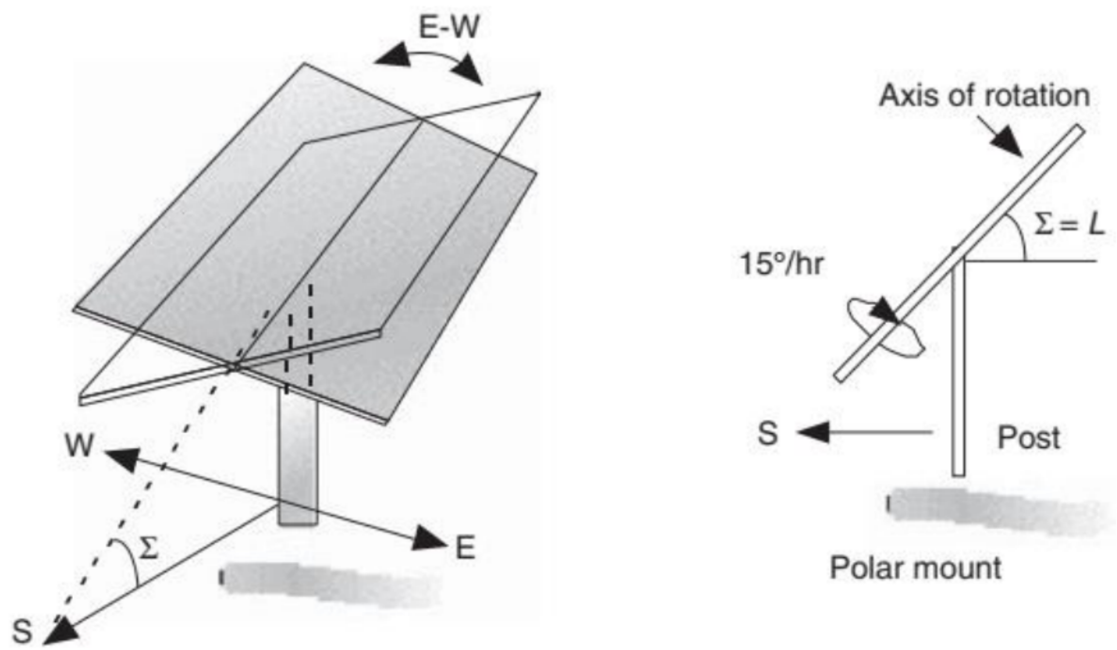

Figure II.26– A single-axis tracking mount with east–west tracking. A polar mount has the axis of rotation facing south and tilted at an angle equal to the latitude.

II.11.2 Single-Axis Tracking

Single-axis tracking for photovoltaics is almost always done with a mount having a manually adjustable tilt angle along a north-south axis, and a tracking mechanism that rotates the collector array from east-to-west, as shown in Fig. 2.26. When the tilt angle of the mount is set equal to the local latitude (called a *polar mount*), not only is that an optimum angle for annual collection, but the collector geometry and resulting insolation are fairly easy to evaluate as well.

As shown in Fig. 2.27, if a polar mount rotates about its axis at the same rate as the earth turns, 15°/h, then the centerline of the collector will always face directly into the sun. Under these conditions, the incidence angle θ between a normal to the collector and the sun's rays will be equal to the solar declination δ. That makes the $\Sigma = L$ direct-beam insolation on the collector just I_B cos δ. To evaluate diffuse and reflected radiation, we need to know the tilt angle of the collector. As can be seen in Fig. 2.26, while the axis of rotation has a fixed tilt of, unless it is solar noon, the collector itself is cocked at an odd angle with respect to the horizontal plane. The effective tilt, which is the angle between a normal to the collector and the horizontal plane, is given by:

$$\Sigma_{\text{effective}} = 90 - \beta + \delta \tag{II.35}$$

The beam, diffuse, and reflected radiation on a polar mount, one-axis tracker are given by:

$$I_{BC} = I_B \cos\delta \ \text{[w/m}^2\text{]} \tag{II.36}$$

$$I_{DC} = C I_B \left[\frac{1+\cos(90^\circ - \beta + \delta)}{2}\right] \text{[w/m}^2\text{]} \tag{II.37}$$

$$I_{RC} = \rho(I_{BH} + I_{DH}) \left[\frac{1-\cos(90^\circ - \beta + \delta)}{2}\right] \text{[w/m}^2\text{]} \tag{II.38}$$

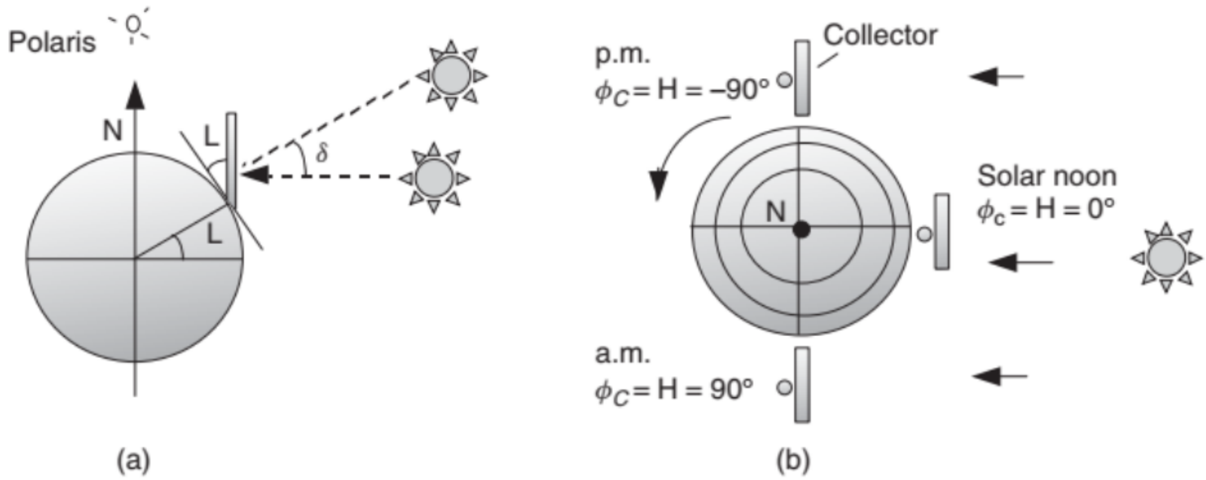

Figure II.27– (a) Polar mount for a one-axis tracker showing the impact of a 15◦/h angular rotation of the collector array. (b) Looking down on North Pole.