

First Series of Exercises :

AC/DC Converters (Rectifiers)

Exercise 1:

- 1- Consider the rectifier represented in Figure 1, which is connected to a power supply $V_s(t) = 240V$, 50Hz.
 - a. For a purely resistive load $R = 10 \Omega$, plot the waveforms of the current (I_L), the output voltage (V_L), and the voltage across the diode (V_D).
 - b. Finding the Performance Parameters of the Rectifier (V_{dc} , I_{dc} , V_{rms} , I_{rms} , P_{dc} , P_{ac} , η , FF, RF, TUF and PF).
 - c. Repeat question (a) for a load consisting of an inductance $L = 0.1 H$ in series with a resistance $R = 10 \Omega$.
- 2- Replace the diode in Figure 1 with a thyristor.
 - a. What is the difference between a diode and a thyristor?
 - b. For a purely resistive load $R = 10 \Omega$, plot the waveforms of the current (I_L), the output voltage (V_L), then calculate the average value V_{dc} for a firing angle of 0° , 45° , 90° , 135° , 180° . What is your conclusion?
- 3- Consider the rectifier represented in Figure 2
 - a. What is the role of the diode?
 - b. Calculate V_{dc} for a firing angle of 0° , 45° , 90° .

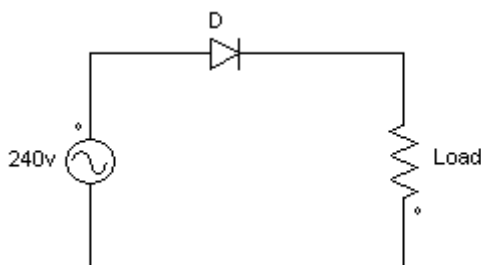


Figure 1

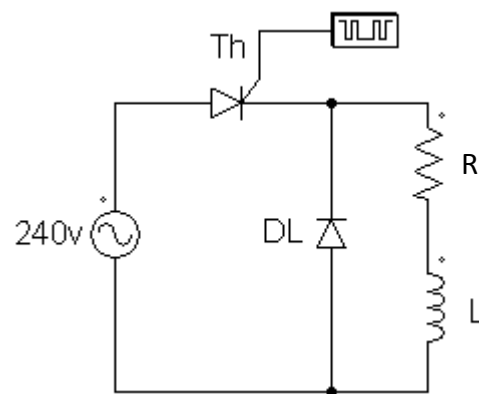
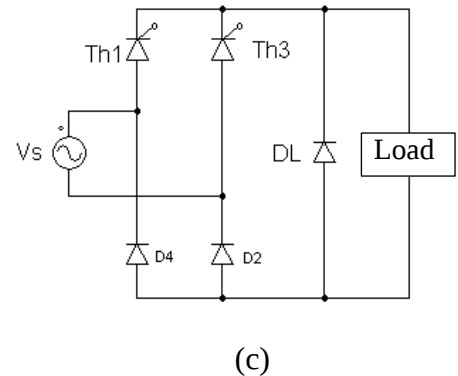
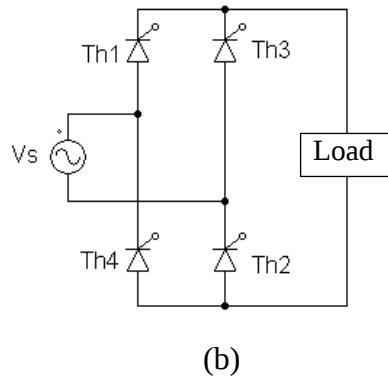
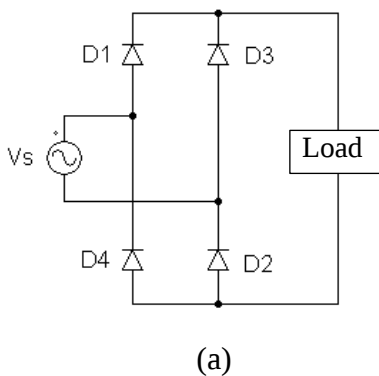


Figure 2

Exercise 2:

A voltage of 120V is applied to a single-phase bridge rectifier for:

1. A diode rectifier (Figure (a)), determine the average load voltage, neglecting the diode voltage drop.
2. A fully controlled rectifier (Figure (b)),
 - Determine the average voltage of the load for the following firing angles: $\alpha = 0^\circ$, 45° and 90° .
The voltage drop across each thyristor is 1.5v.
 - Calculate the peak reverse voltage of each thyristor.
3. A semi-controlled rectifier (Figure (c)),
 - Determine the characteristics of the components for a highly inductive load consumes 25A.
Neglect voltage drops.
4. Repeat question 3 for $\alpha = 90^\circ$ and assume that the voltage drops of the thyristors and diodes are respectively 1.5v and 0.7v.



Exercise 3 :

Given data

- Supply voltage:
 $v_s(t) = V_m \sin(\omega t)$
 $V_{\text{rms}} = 240 \text{ V} \Rightarrow V_m = \sqrt{2} 240 \approx 339.4 \text{ V}$
- Frequency: $f = 50 \text{ Hz} \Rightarrow \omega = 2\pi f = 314 \text{ rad/s}$
- Diode: ideal
- Load resistance: $R = 10 \Omega$

1. Waveforms description

Output voltage (load voltage V_L):

$$V_L(t) = \begin{cases} V_m \sin(\omega t), & \text{if diode ON} \\ 0, & \text{if diode OFF} \end{cases}$$

Diode ON: $0 < \omega t < \pi$ (only positive half-cycle in theory), but actually **half-wave**: diode conducts from 0 to π each cycle, then off from π to 2π .

So:

$$V_L(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq \omega t \leq \pi \\ 0, & \pi < \omega t < 2\pi \end{cases}$$



Load current $I_L(t)$:

Since purely resistive:

$$I_L(t) = \frac{V_L(t)}{R} = \begin{cases} \frac{V_m}{R} \sin(\omega t), & 0 \leq \omega t \leq \pi \\ 0, & \pi < \omega t < 2\pi \end{cases}$$

$$V_m/R = 339.41/10 \approx 33.941 \text{ A peak.}$$

Diode voltage $V_D(t)$:

Define $V_D = V_{\text{anode}} - V_{\text{cathode}}$ (usual convention for diode forward drop is ignored in ideal analysis).

- **When diode ON:** $V_D \approx 0$ (ideal diode).
- **When diode OFF:** Cathode potential $V_L = 0$, Anode potential $V_s(t)$, so $V_D(t) = V_s(t) - 0 = V_m \sin(\omega t)$, which is negative during negative half-cycle? Wait — careful.

(a) Purely resistive load ($R = 10 \Omega$)

Operating principle

- Diode conducts only during positive half-cycles of the input voltage.
- For $0 < \omega t < \pi$:

$$v_L(t) = v_s(t), \quad i_L(t) = \frac{v_s(t)}{R}$$

- For $\pi < \omega t < 2\pi$:

$$v_L(t) = 0, \quad i_L(t) = 0$$

Part (b) – Performance parameters for resistive load

Half-wave rectifier formulas:

Average (DC) voltage:

$$V_{dc} = \frac{V_m}{\pi} \quad V_{dc} \approx \frac{339.41}{\pi} \approx 108.04 \text{ V}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R} \quad I_{dc} = \frac{V_{dc}}{R} \approx \frac{108.04}{10} = 10.804 \text{ A}$$

RMS voltage:

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^\pi V_m^2 \sin^2 \theta d\theta}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

So,

$$\int_0^\pi \sin^2 \theta d\theta = \int_0^\pi \frac{1 - \cos 2\theta}{2} d\theta = \left. \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right|_0^\pi = \frac{\pi}{2}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi} \cdot \frac{\pi}{2}} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2}$$

$$V_{rms} \approx \frac{339.41}{2} \approx 169.705 \text{ V}$$

RMS current:

$$I_{rms} = \frac{V_{rms}}{R} \approx 16.9705 \text{ A}$$

DC power: $P_{dc} = V_{dc} I_{dc} \approx (108.04)(10.804) \approx 1167.5 \text{ W}$

AC power (input power to load):

$$P_{ac} = V_{rms} I_{rms} \quad P_{ac} = I_{rms}^2 R = (16.9705)^2 \times 10 \approx 2880 \text{ W}$$

Efficiency η :

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\% = \frac{4}{\pi^2} \times 100\% \approx 40.5\%$$

Form Factor:

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} \approx 1.571$$

Ripple Factor:

$$RF = \sqrt{FF^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} \approx \sqrt{2.4674 - 1} = \sqrt{1.4674} \approx 1.211$$

Transformer Utilization Factor (TUF):

For half-wave:

$$TUF \approx 0.287$$

$$TUF = \frac{P_{dc}}{V_{rms(supply)} \cdot I_{rms(supply)}}$$

$$TUF \approx \frac{1167.5}{240 \times 16.9705} \approx \frac{1167.5}{4072.92} \approx 0.286$$

Power Factor

$$PF = \frac{P_{ac}}{V_{s,rms} I_{s,rms}} \approx$$