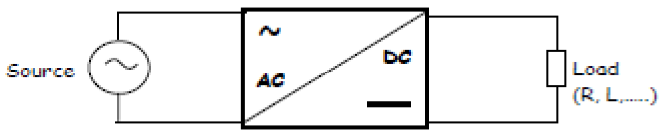
CHAPTER 2

AC-DC CONVERSION (RECTIFIERS)

1. Introduction

Rectifier circuits are power electronic converters that perform the conversion of alternating current (AC) into direct current (DC). When supplied with a single-phase or three-phase AC voltage source, they provide DC current to the load connected at their output.

Rectifiers are employed whenever there is a need for direct current while the electrical energy source is in alternating current form. Rectifiers find application in a wide range of areas due to their versatility and capability to convert AC to DC power effectively.



AC-DC conversion

Diode rectifiers, or uncontrolled rectifiers, do not allow for varying the ratio between the input alternating voltage and the output direct voltage. Furthermore, they are irreversible, meaning that power can only flow from the alternating side to the direct side.

Thyristor rectifiers, or controlled rectifiers, allow, for a fixed input alternating voltage, to vary the output direct voltage. Additionally, they are reversible; when they transfer power from the direct side to the alternating side, they are referred to as non-autonomous inverters.

2. Basic definitions

Certain terms will be frequently used in this lesson and subsequent lessons while characterizing different types of rectifiers. Such commonly used terms are defined in this section.

2.1. Mean value

$$\langle V(t) \rangle = \overline{V(t)} = \frac{1}{T}\int_{0}^{T} v(t)dt$$

2.2. Root mean square value (RMS)

$$V_{eff}{}^{2} = \frac{1}{T}\int_{0}^{T} v(t)^{2}dt$$

2.3. Form factor

The form factor in electrical engineering refers to a dimensionless ratio that characterizes the shape or quality of a waveform, often used in the context of alternating current (AC) voltage or current. It's a measure of how closely the waveform resembles a pure sinusoidal waveform. The closer the form factor is to 1, the closer the waveform resembles a pure sinusoidal waveform. A form factor close to 1 indicates a smoother, less distorted waveform, which is desirable for many electrical applications. This coefficient is used for comparing different rectifier configurations.

The formula for calculating the form factor is typically as follows:

$$F = \frac{V_{eff}}{\bar{V}}$$

F=1, the closer the obtained voltage is to a continuous quantity.

2.4. Ripple Factor

The ripple factor is a measure of the amount of alternating current (AC) component or voltage ripple present in a direct current (DC) output. It is typically expressed as a ratio or percentage and is used to evaluate the quality of the DC output from a rectifier or power supply. A lower ripple factor indicates a smoother, more stable DC output, while a higher ripple factor suggests a less stable and noisier output. The ripple factor is an important parameter in power electronics and electrical engineering. By definition, the ripple factor is defined as:

$$k = \frac{V_{max} - V_{min}}{2\bar{V}}$$

k=0, the closer the rectified voltage is to a continuous quantity.

2.5. Distortion factor of a rectifier (DF)

The ratio of the RMS value of the fundamental frequency to the total RMS value is the distortion factor:

$$DF = \frac{I_{1rms}}{I_{rms}}$$

The distortion factor represents the reduction in power factor due to the non-sinusoidal property of the current.

2.6. Displacement factor of a rectifier (DPF)

If *V* and *I* are the per phase input voltage and input current of a rectifier respectively, then the displacement factor of a rectifier is defined as:

$$DPF = \cos\varphi$$

2.7. Power factor of rectifier (PF)

As for any other equipment, the definition of the power factor of a rectifier is:

$$PF = \frac{Actual\ power\ input\ to\ the\ rectifier}{Apparent\ power\ input\ to\ the\ rectifier}$$

If the per phase input voltage and current of a rectifier are V and I respectively then

$$PF = \frac{V_1 I_1 \cos\varphi}{V_{rms} I_{rms}}$$

If the rectifier is supplied from an ideal sinusoidal voltage source then:

$$\text{So, } PF = \frac{I_1}{I_{rms}} \cos\varphi = DF.DPF$$

3. Uncontrolled rectifiers (Using diodes)

3.1. Single-phase half-wave rectifier

The half-wave rectifier is a simplest electronic circuit that converts alternating current (AC) into pulsating direct current (DC) using a single diode. It's one of the simplest rectification circuits and is typically used when a relatively low Dc output voltage is required. The output voltage and current of this rectifier are strongly influenced by the type of the load. In this section, operation of this rectifier with resistive and inductive loads will be discussed. A thorough understanding of the half-wave rectifier circuit will enable the student to advance to the analysis of more complicated circuits with a minimum of effort.

- **Resistive load (R)**

A basic half-wave rectifier with a resistive load is shown in Fig. The source is AC, and the objective is to create a load voltage that has a nonzero DC component. The diode is a basic electronic switch that allows current in one direction only. For the positive half-cycle of the source in this circuit, the diode is on forward biased.

Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive. For the negative half-cycle of the source, the diode is reverse-biased, making the current zero The voltage across the reverse-biased diode is the source voltage, which has a negative value. Therefore, the voltage across the resistive load is:

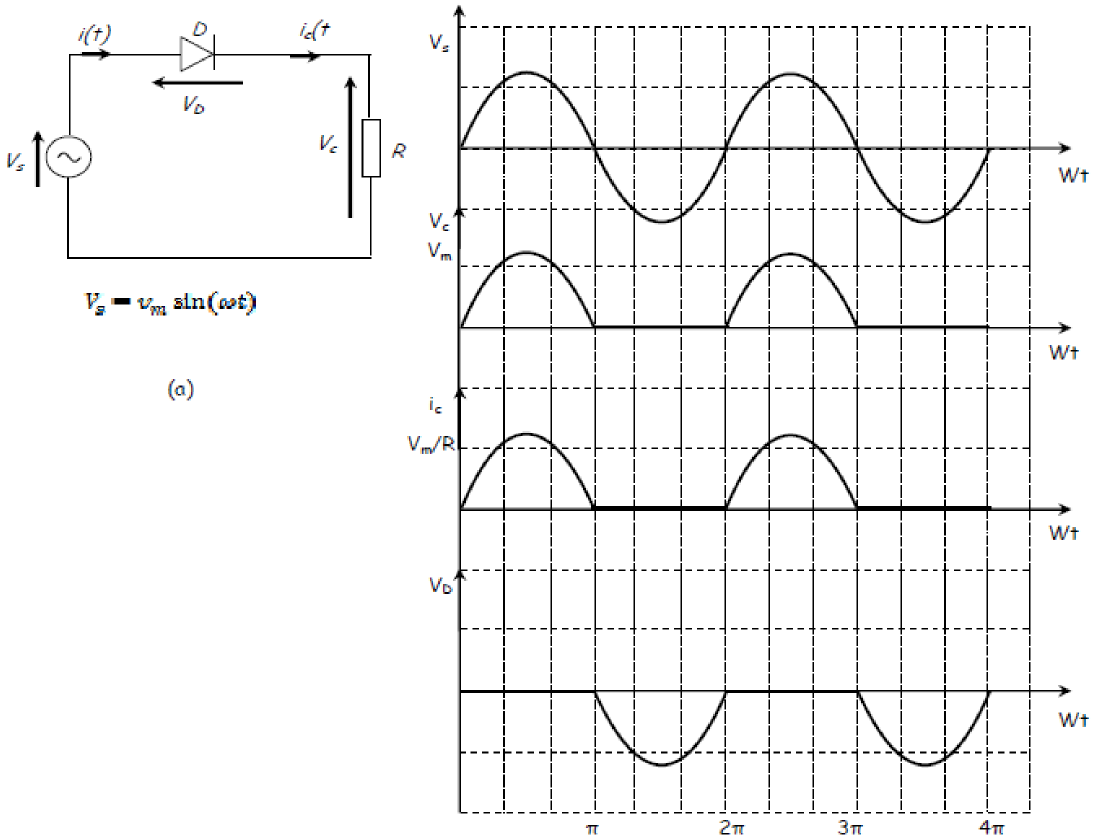
For $0 \leq \omega t \leq \pi$:

$$V_c = V_s = v_m \sin(\omega t), \quad V_D = 0, \quad i_c = \frac{V_c}{R}$$

For $\pi \leq \omega t \leq 2\pi$:

$$V_c = 0, \quad V_D = V_s = v_m \sin(\omega t), \quad i_c = 0$$

The voltage waveforms across the source load, and diode:



The average value of the rectified voltage:

$$\langle V_c \rangle = \frac{1}{T}\int_0^T V_c(t)dt = \frac{1}{T}\int_0^{T/2} V_m \sin(\omega t)\, dt = \frac{V_m}{\pi} = 0.318V_m$$

The root mean square (RMS) value of the rectified voltage:

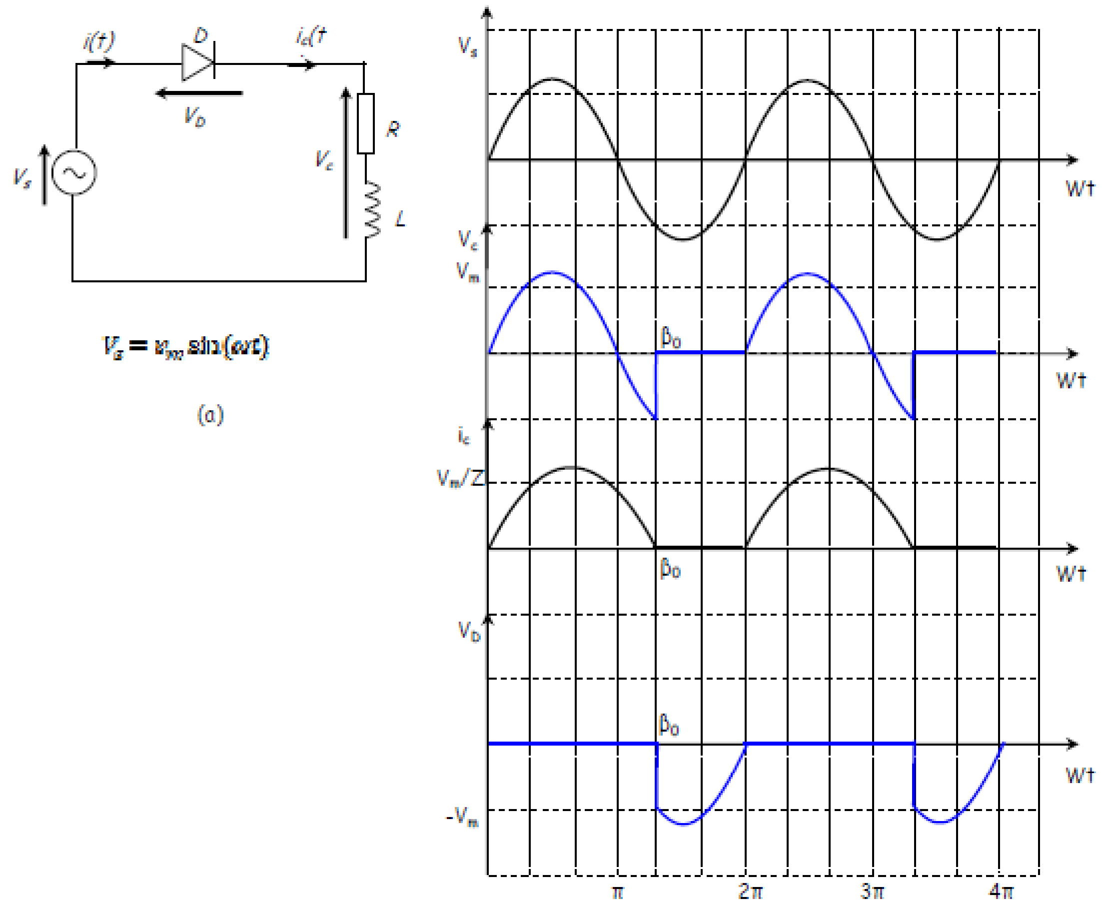
$$V_{ceff} = \sqrt{\frac{1}{T}\int_0^T V_c(t)^2 dt} = \sqrt{\frac{1}{T}\int_0^{T/2} (V_m \sin(\omega t))^2 dt} = \sqrt{\frac{V_m{}^2}{T}\int_0^{T/2} (1-\cos(2\omega t))dt} = \frac{V_m}{2} = 0.5V_m$$

The form factor:

$$F = \frac{V_{ceff}}{\langle V_c \rangle} = \frac{\frac{V_m}{2}}{\frac{V_m}{\pi}} = \frac{\pi}{2} = 1.57$$

- **Inductive load (RL)**

Industrial loads typically contain inductance as well as resistance. As the source voltage goes through zero, becoming positive in the circuit, the diode becomes forward-biased. For an inductive load, the diode remains conductive as long as the voltage Vs is positive and the current ic (t) is not zero. Therefore, the diode is in forced conduction until the current ic (t) becomes zero at the instant β 0. The voltage waveforms across the source, load, and diode:



For $0 \le \omega t \le \beta_0$:

$$V_c = V_s = v_m \sin(\omega t), \quad V_D = 0, \quad i_c = \frac{V_c}{Z}$$

For $\beta_0 \le \omega t \le 2\pi$:

$$V_c = 0, \quad V_D = V_s = v_m \sin(\omega t), \quad i_c = 0$$

The Kirchhoff voltage law equation that describes the current in the circuit for the forward-biased ideal diode is:

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt}$$

The solution can be obtained by expressing the current as the sum of the forced response and the natural response:

$$i(t) = i_f(t) + i_n(t)$$

The forced response for this circuit is the current that exists after the natural response has decayed to zero. In this case, the forced response is the steady-state sinusoidal current that would exist in the circuit if the diode were not present. This steady-state current can be found from phasor analysis, resulting in:

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta)$$

Where $Z = \sqrt{R^2 + (\omega L)^2}$ and $\theta = tan^{-1}\left(\frac{\omega L}{R}\right)$

The natural response is the transient that occurs when the load is energized. It is the solution to the homogeneous differential equation for the circuit without the source or diode:

$$Ri(t) + L\frac{di(t)}{dt} = 0$$

For this first-order circuit, the natural response is:

$$i_n(t) = Ae^{-\frac{R}{L}t}$$

Adding the forced and natural responses gets the complete solution:

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\frac{R}{L}t}$$

The constant *A* is evaluated by using the initial condition for current. The initial condition of current in the inductor is zero because it was zero before the diode started conducting and it cannot change instantaneously.

The final result is:

$$i(t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)\, e^{-\frac{R}{L}t}\right]$$

3.2 Single-phase full-wave rectifier

The primary goal of a full-wave rectifier (Bridge rectifier) is to generate a voltage or current that is purely direct current (DC) or contains a specified DC component. Although the fundamental purpose of the fullwave rectifier aligns with that of the half-wave rectifier, full-wave rectifiers offer notable advantages. In the full-wave rectifier, the average current from the alternating current (AC) source is reduced to zero, thus mitigating issues related to non-zero average source currents, particularly in transformers. Furthermore, the output of the full-wave rectifier exhibits inherently lower ripple compared to that of the half-wave rectifier.

The single-phase full-wave rectifier circuit (Bridge rectifier), employing diodes, comprises four diodes interconnected in pairs in reverse.

- **Resistive load**

The voltage is positive, with diodes D_1 and D_4 conducting, while D_2 and D_3 are blocked.

$$V_c = V_s = v_m \sin(\omega t)$$
$$V_{D1} = 0,\ V_{D4} = 0$$
$$i_c = \frac{V_c}{R}$$

The voltage is negative, with diodes D_2 and D_3 conducting, while D_1 and D_4 are blocked.

$$V_c = -V_s = -v_m \sin(\omega t)$$
$$V_{D2} = 0,\ V_{D3} = 0$$
$$V_{D1} = V_s,\ V_{D4} = V_s$$
$$i_c = \frac{V_c}{R}$$

The DC component of the output voltage is the average value of the rectified voltage:

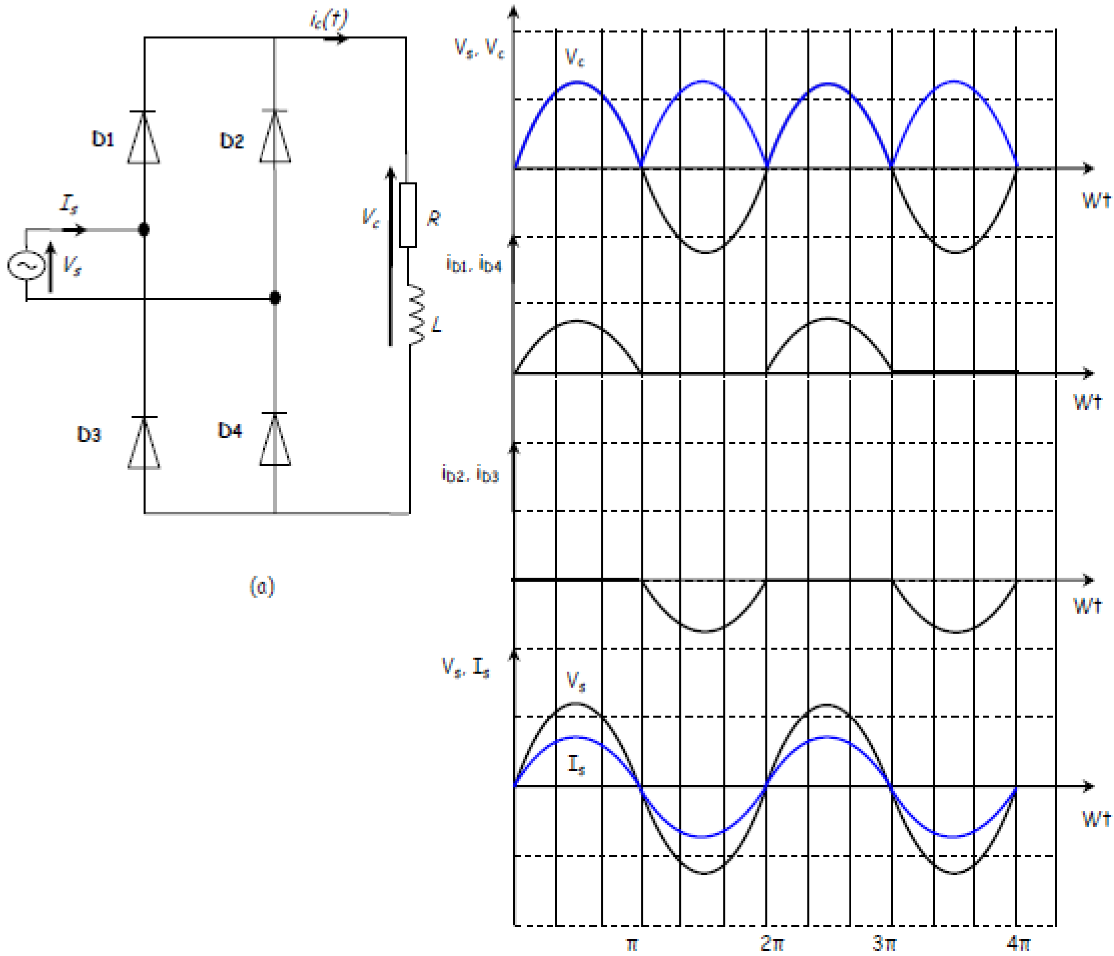
$$\langle V_c \rangle = \frac{2}{T}\int_0^{T/2} V_c(t)dt = \frac{2}{T}\int_0^{T/2} V_m \sin(\omega t)\, dt = \frac{2V_m}{\pi} = 0.636V_m$$

The root mean square (RMS) value of the rectified voltage:

$$V_{ceff} = \sqrt{\frac{1}{T}\int_0^{T} V_c(t)^2 dt} = \sqrt{\frac{2}{T}\int_0^{T/2} (V_m \sin(\omega t))^2 dt} = \sqrt{\frac{2V_m{}^2}{T}\int_0^{T/2} (1 - \cos(2\omega t))dt} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

The form factor :

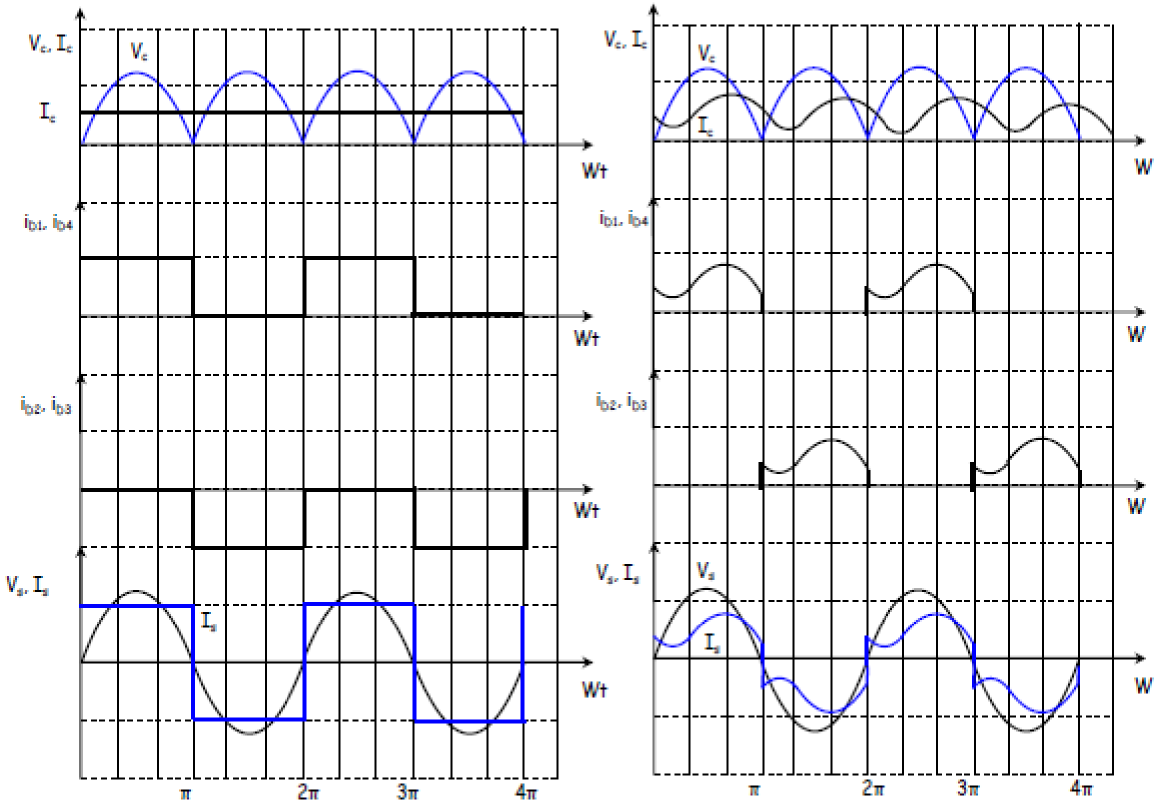
$$F = \frac{V_{ceff}}{\langle V_c \rangle} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1.11$$



- **Inductive load**

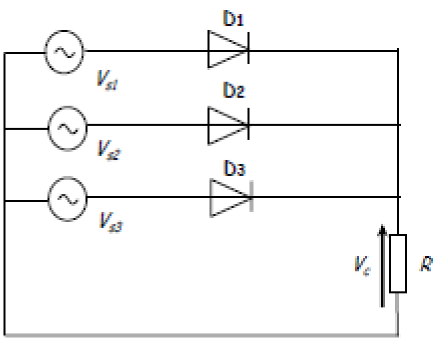
The load now consists of a pure inductance *L* in series with a resistance *R*. The inductance *L* opposes variations in the current $i_c(t)$ and smoothes it out. If a sufficient value is provided, the current in the load becomes continuous (the current does not pass through zero), and this is known as continuous conduction mode.

If *L* is significant enough (L>>), the current $i_c(t)$ can be considered constant and equal to I_c.

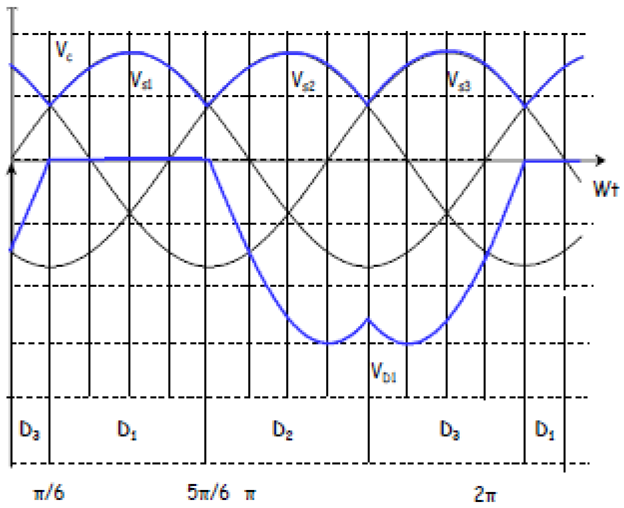


3.3 Three-phase half-wave rectifier

It is the diode connected to the phase with the highest instantaneous voltage. The output voltage varies from $V_m/2$ to V_m three times during each input cycle.



(a)



(b)

The average or DC value of the output voltage is:

$$\langle V_c \rangle = \frac{3}{T}\int_{T/12}^{5T/12} V_c(t)dt = \frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_m \sin(\omega t)\, d\omega t = \frac{3\sqrt{3}V_m}{2\pi} = 0.827V_m$$

The root mean square (RMS) value of the rectified voltage:

$$V_{ceff} = \sqrt{\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6}(V_m \sin(\omega t))^2 d\omega t} = \sqrt{\frac{3V_m^2}{4\pi}\int_{\pi/6}^{5\pi/6}(1-\cos(2\omega t))d\omega t} = 0.8407V_m$$

The form factor :

$$F = \frac{V_{ceff}}{\langle V_c \rangle} = \frac{0.8407V_m}{0.827V_m} = 1.016$$

3.4 Three-phase fall-wave rectifier

The three-phase full-bridge rectifier, Kirchhoff's voltage la around any path shows that:

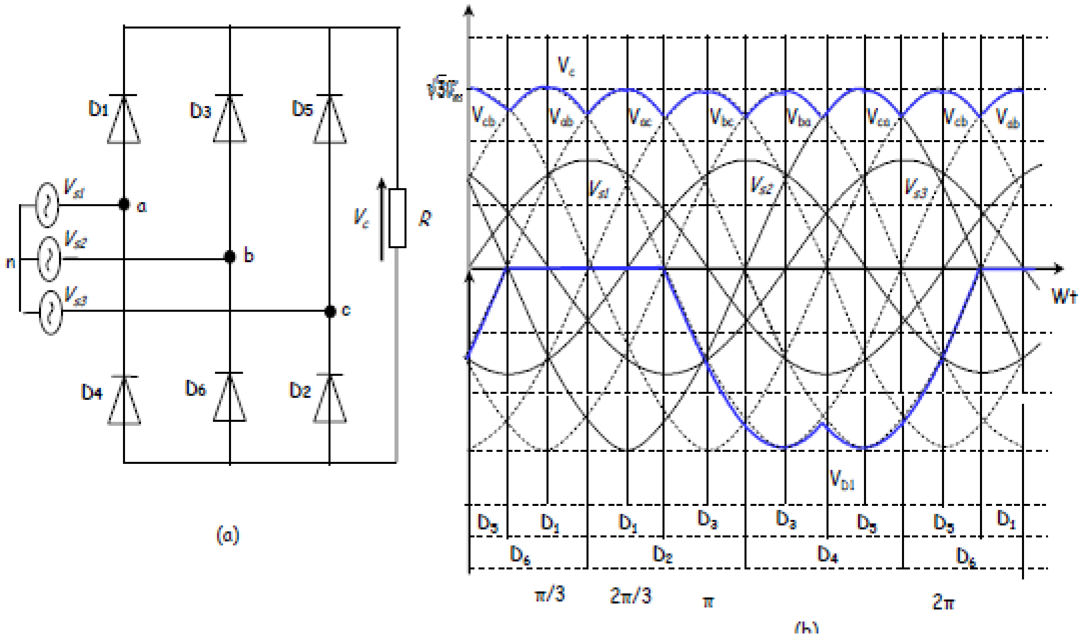
1-only one diode in the top half of the bridge may conduct at one time (D1, D3, or D5). The diode that is conducting will have its anode connected to the phase voltage that is highest at that instant.

2-only one diode in the bottom half of the bridge may conduct at one time (D2, D4, or D6). The diode that is conducting will have its cathode connected to the phase voltage that is lowest at that instant.

3-as a consequence of items 1 and 2, diodes on the same leg (D1 and D4, D3 and D6, D2 and D5) cannot conduct at the same time.

4-The output voltage across the load is one of the line-to-line voltages of the source. For example, when D1 and D2 are on, the output voltage is Vac. Furthermore, the diodes that are on are determined by which line-to-line voltage is the highest at that instant. For example, when Vac is the highest line-to-line voltage, the output is Vac.

5-The fundamental frequency of the output voltage is 6w, where w is the frequency of the three-phase source.



If we define the three line-neutral voltages as follows:

$$\begin{cases} v_{s1} = V_m \sin \omega t \\ v_{s2} = V_m \sin\left(\omega t - \dfrac{2\pi}{3}\right) \\ v_{s3} = V_m \sin\left(\omega t + \dfrac{2\pi}{3}\right) \end{cases}$$

The corresponding line-to-line voltages are:

$$\begin{cases} v_{ab} = v_{an} - v_{bn} = \sqrt{3}V_m \sin\left(\omega t + \dfrac{\pi}{6}\right) \\ v_{bc} = v_{bn} - v_{cn} = \sqrt{3}V_m \sin\left(\omega t - \dfrac{\pi}{2}\right) \\ v_{ca} = v_{cn} - v_{an} = \sqrt{3}V_m \sin\left(\omega t + \dfrac{5\pi}{6}\right) \end{cases}$$

The average or DC value of the output voltage is:

$$\langle V_c \rangle = \frac{6}{T}\int_{T/12}^{3T/12} V_c(t)dt = \frac{3}{\pi}\int_{\pi/6}^{\pi/2} V_{ab}(t)d\omega t = \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \left(V_m \sin(\omega t) - V_m \sin\left(\omega t - \frac{2\pi}{3}\right)\right) d\omega t = 1.654V_m$$

The root mean square (RMS) value of the rectified voltage:

$$V_{ceff} = \sqrt{\frac{3}{\pi}\int_{\pi/6}^{\pi/2} \left(V_m \sin(\omega t) - V_m \sin\left(\omega t - \frac{2\pi}{3}\right)\right)^2 d\omega t} = 1.6554V_m$$

The form factor :

$$F = \frac{V_{ceff}}{\langle V_c \rangle} = \frac{1.6554V_m}{1.654V_m} = 1.0008$$