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وزارة التعليم العالي والبحث العلمي

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Chapter 3: dynamics of incompressible ideal fluids

1.1 INTRODUCTION

In the first two chapters we learned the basics about fluids and how they behave when they are not moving. Chapter 1 explained what a fluid is, and the difference between perfect and real fluids, and it described simple properties like density and viscosity. Chapter 2 looked at fluids at rest: what pressure is, how pressure changes with depth, Pascal's law, Archimedes' principle, how to calculate pressure forces on flat and curved surfaces, and how to measure pressure with instruments such as barometers and manometers. In Chapter 3 we move to the study of ideal incompressible fluids in motion. We begin with steady flow and the continuity equation, which shows how mass is conserved, then define mass flow rate and volume flow rate and learn how to calculate them. We will study Euler's, then Bernoulli's theorems for cases with and without work exchange and use it to relate pressure, speed, and height. Practical applications will show how devices like Venturi meters, diaphragms, and Pitot tubes measure flow and velocity. This chapter uses simple examples and measurements to build intuition and prepare you for the study of real, viscous flows in the next chapter.

1.2 OBJECTIVES AND GOALS OF THIS CHAPTER

By the end of this chapter, you will be able to:

- Understand **steady flow** when fluid moves smoothly without changes over time

- Use the **continuity equation** to see how fluid speed changes when pipe size changes.
- Calculate **mass flow rate** and **volume flow rate** (how much fluid passes per second).
- Use **Euler's theorem** to calculate forces when fluid changes direction or speed.
- Apply **Bernoulli's theorem** to find the relationship between pressure, speed, and height.
- Solve problems with and without work exchange (like pumps or turbines).
- Understand real **applications**: how Venturi meters, diaphragms, and Pitot tubes measure flow speed and rate.

1.3 STEADY FLOW, IDEAL FLUIDS, AND CONTINUITY EQUATION

1.3.1 Flow visualization

While quantitative study of fluid dynamics requires advanced mathematics, much can be learned from flow visualization (the visual examination of flow field features). Flow visualization is useful not only in physical experiments, but in numerical solutions as well. In fact, the very first thing an engineer does after obtaining a numerical solution is simulate some form of flow visualization, so that he or she can see the “whole picture” rather than merely a list of numbers and quantitative data. Why? Because the human mind is designed to rapidly process an incredible amount of visual information; as they say, a picture is worth a thousand words. There are many types of flow patterns that can be visualized, both physically (experimentally) and/or computationally.

1.3.1.1 Streamline:

A streamline is like an imaginary path drawn in the fluid at a single moment in time. It shows the direction the fluid is flowing right now. If you could freeze the flow and draw arrows showing the direction of every single particle, connecting those arrows would give you streamlines. The velocity field of each fluid element is tangent to the stream line at any given point. In the case of steady flow, streamlines are fixed and does not change. However, when the flow is unsteady, they may change shape with time. Streamlines are obtained analytically by integrating the equations defining lines tangent to the velocity field. As illustrated in the margin of **Fig. (1)**, for two-dimensional flows the slope of the streamline, $\frac{dy}{dx}$, must be equal to the tangent of the angle that the velocity vector makes with the x-axis:

$$\frac{dy}{dx} = \frac{v}{u} \tag{1}$$

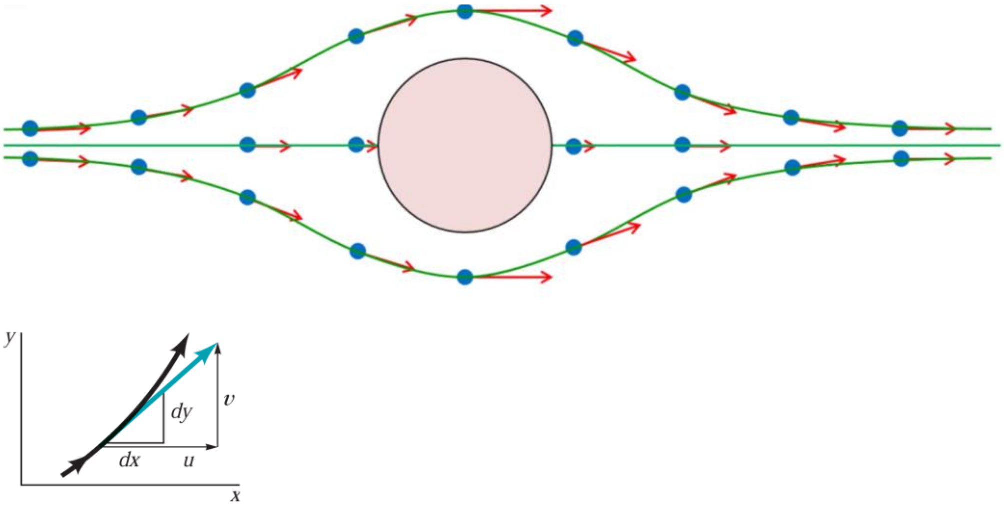

Figure 1 Streamlines across a stationary ball.

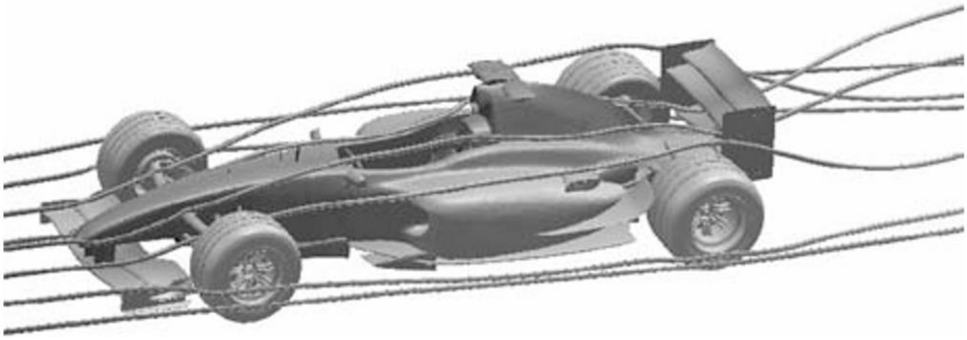

Figure 2 Predicted streamlines for flow past a Formula 1 race car as obtained by using computational fluid dynamics techniques. (Courtesy of Ansys, Inc.)

1.3.1.2 Streaklines:

A streakline is a line made by all the fluid particles that have passed through a specific, fixed point. A simple example is a smoke trail. The smoke you see coming from a chimney is a streakline since it's made of all the tiny particles that have come out of that one chimney opening. A streak line can be represented by fluid elements 1, 2, 3, and 4 produced from a smoke or dye emitted from a tube as in **Fig. (3).** In **Fig. (4),** we can see a streakline of smoke passing over a car.

1.3.1.3 Pathlines:

A pathline is the actual, real path that one single particle of fluid takes as it moves over time. It's like tracking a single leaf floating down a river on a GPS; the line drawn on the map is its pathline. In **Fig. (3),** a pathline can be represented by the red fluid element 1.

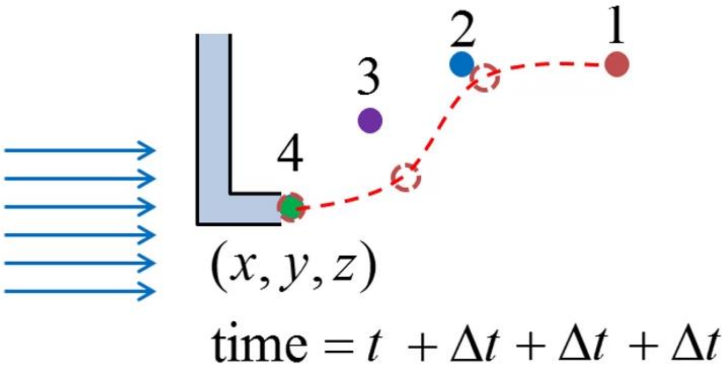

Figure 3 Pathline and streak line presented by fluid elements of smoke or dye.

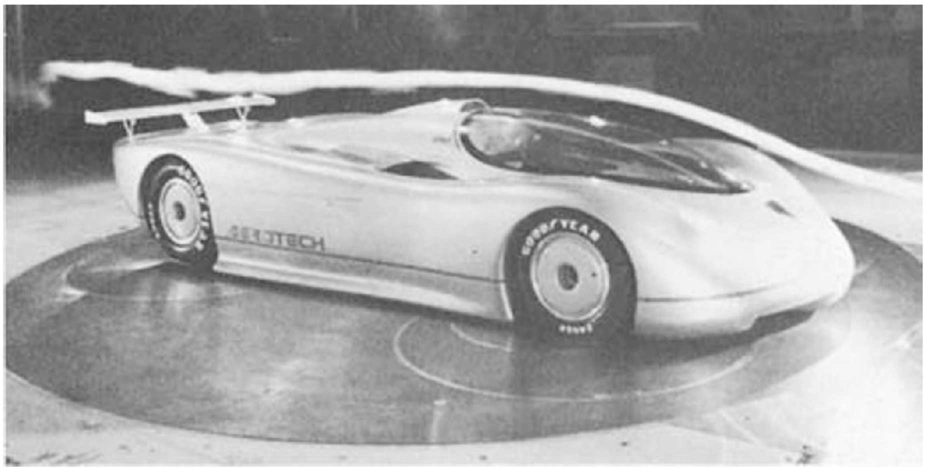

Figure 4 Flow past a full-sized streamlined vehicle in the GM aerodynamics laboratory wind tunnel

1.3.2 Steady flow

Definition: An ideal flow is the laminar movement of a perfect, imaginary fluid that makes calculations simple. It assumes the fluid has no viscosity or stickiness, cannot be compressed, and flows smoothly without turbulence or energy loss. While real fluids like water and air always have some friction and complexity, this simplified model helps engineers understand basic principles and solve problems before adding real-world complications. **Fig. (5)** illustrates the velocity profile differences between an ideal and a real fluid. **Fig. (6)** illustrates the difference between a laminar flow of an ideal fluid compared to fluid experiencing a turbulent flow.

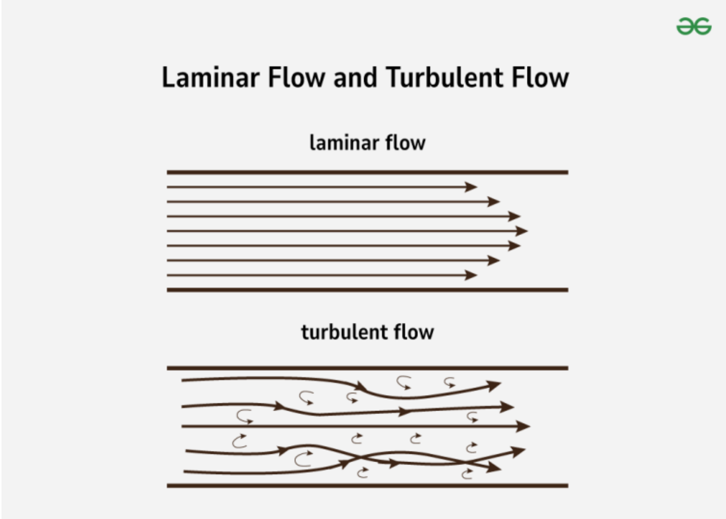

Figure 5 laminar flow compared to turbulent flow

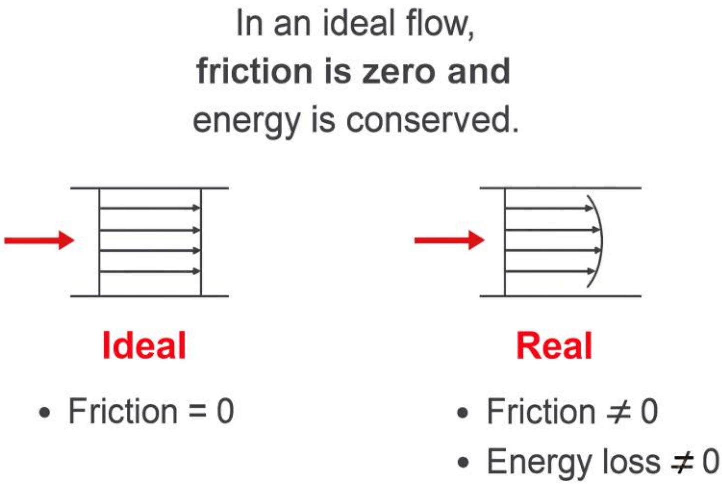

Figure 6 Velocity profile of an ideal fluid compared to a real fluid.

Key points:

- **Inviscid** – the fluid is non-viscous and moves without any friction between its layers.
- **Incompressible** – its density stays constant and it cannot be squeezed.
- **Laminar flow** – no swirling, turbulence, or sudden changes over time.
- **Energy is perfectly conserved** – no energy is lost to heat or friction.

1.3.3 Continuity equation for ideal fluid flow

In simple terms, the continuity equation states that **what goes in must come out**. For an ideal fluid in a steady flow, the mass flow rate must remain constant at every point along a pipe or stream.

Let's consider an ideal fluid flow through a circular pipe which decreases in diameter as in **Fig. (7).** The streamlines show the direction of the fluid flow. As we know in ideal flow the fluid is non-viscous (inviscid) and incompressible, and the flow is laminar and irrotational. Assuming there are no losses of fluid in the circular pipe (meaning there no cracks where fluid could escape from). The only way this could not be true is if the fluid is collecting somewhere in the pipe. However, this is not possible since the fluid is incompressible as mentioned earlier.

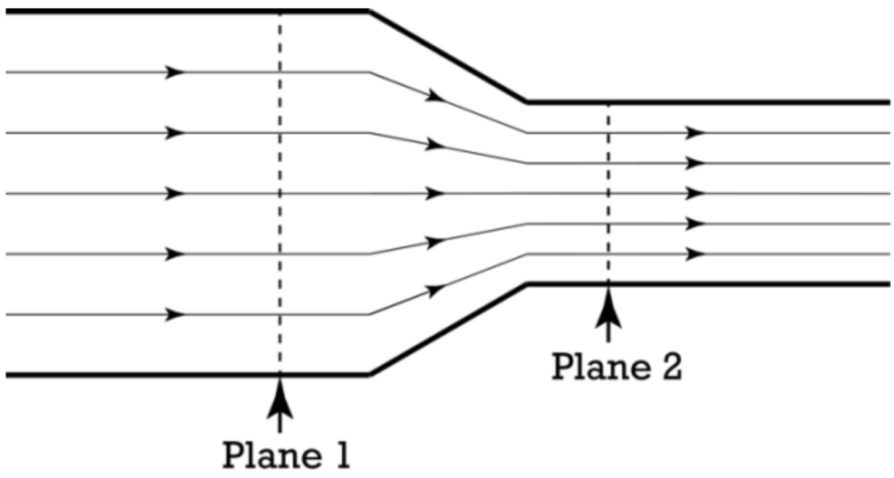

Figure 7 Streamlines of a fluid in a circular pipe.

Because these masses are masses of a fluid flowing through a plane, lets identify these masses as $\Delta\boldsymbol{m_1}$ which is the amount of mass that passes through plane 1 during $\Delta\boldsymbol{t}$**.** Moreover, we know that the mass density is $\boldsymbol{\rho} = \frac{\boldsymbol{m}}{\boldsymbol{V}}$ then $\Delta\boldsymbol{m_1}$ equals the density of the fluid at plane 1 $\boldsymbol{\rho_1}$ multiplied by $\Delta\boldsymbol{V_1}$ which is the volume of the fluid which passes through plane 1 during change in time $\Delta\boldsymbol{t}$**.**

$$\boldsymbol{\rho} = \frac{\boldsymbol{m}}{\boldsymbol{V}} = \frac{\Delta\boldsymbol{m}}{\Delta\boldsymbol{V}} \rightarrow \Delta\boldsymbol{m_1} = \boldsymbol{\rho_1}\Delta\boldsymbol{V_1} \quad (2)$$

During that change in time $\Delta\boldsymbol{t}$, the fluid will through displacement $\Delta\boldsymbol{x_1}$ plane 1, and $\Delta\boldsymbol{x_2}$ at plane 2. We know that the average velocity is $\boldsymbol{v_{avg}} = \frac{\Delta x}{\Delta t}$ which gives:

$$\Delta\boldsymbol{x_1} = \boldsymbol{v_1}\Delta\boldsymbol{t_1}\ \boldsymbol{and}\ \Delta\boldsymbol{x_2} = \boldsymbol{v_2}\Delta\boldsymbol{t_2} \quad (3)$$

In **Fig. (8),** change in time of the mass going through each plane is the same which makes $\Delta\boldsymbol{t_1} = \Delta\boldsymbol{t_2}$. Additionally, the volume of the mass $\Delta\boldsymbol{V}$ flowing through the pipe in time $\Delta\boldsymbol{t}$ is:

$$\Delta\boldsymbol{V_{cylinder}} = (\text{Area})(\text{Length}) = \text{A}\Delta\boldsymbol{x} \quad (4)$$

Which gives:

$$\Delta \boldsymbol{V_1} = \boldsymbol{A_1 v_1 \Delta t} \rightarrow \Delta \boldsymbol{m_1} = \boldsymbol{\rho_1 A_1 v_1 \Delta t} \quad (5)$$

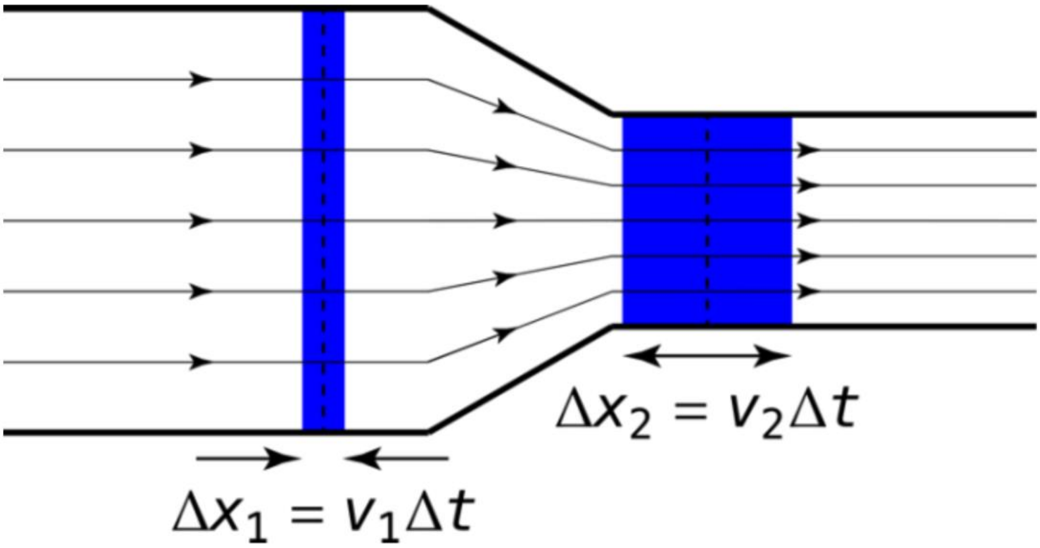

Figure 8 Streamlines of a fluid in a circular pipe with specific $\Delta \boldsymbol{x}$

And because there are no losses, all of the mass that passes through plane 1 will eventually have to pass through plane 2. And therefore, during the same amount of time $\Delta \boldsymbol{t}$ the same mass will have to pass through each plane. In other words, the mass $\Delta \boldsymbol{m_1}$ which passes through plane 1, equals the mass $\Delta \boldsymbol{m_2}$ which passes through plane 2. Now we can substitute $\Delta \boldsymbol{m_1} = \Delta \boldsymbol{m_2}$ in the equation for which we just solved, we get:

$$\Delta \boldsymbol{m_1} = \Delta \boldsymbol{m_2} = \boldsymbol{\rho_1 A_1 v_1 \Delta t} = \boldsymbol{\rho_2 A_2 v_2 \Delta t} \quad (6)$$

We can divide both sides by $\Delta \boldsymbol{t}$ to get:

$$\boldsymbol{\rho_1 A_1 v_1} = \boldsymbol{\rho_2 A_2 v_2} \quad (7)$$

This suggests that $\boldsymbol{\rho A v}$ all at a specific plane is a constant value. Which is called **the continuity equation of fluid flow.** And has the following unit:

$$\boldsymbol{\rho A v} = \left(\frac{\boldsymbol{kg}}{\boldsymbol{m^3}}\right)(\boldsymbol{m^2})\left(\frac{\boldsymbol{m}}{\boldsymbol{s}}\right) = \frac{\boldsymbol{kg}}{\boldsymbol{s}} \quad (8)$$

The continuity equation of fluid flow states that the mass flow rate of a fluid through an enclosed space is constant: $\boldsymbol{\rho A v} = \boldsymbol{constant}$ ($\boldsymbol{units\ are}\ \frac{\boldsymbol{kg}}{\boldsymbol{s}}$).

Mass flow rate is an important term which is the mass of fluid which flows through a plane per second.

Note that in this particular chapter, we are dealing with ideal fluids, and ideal fluids are incompressible, meaning their density $\boldsymbol{\rho}$ remains constant, and if the density of the fluid does not change, it can be removed from the mass flow rate equation which gives:

$$\boldsymbol{Q = A_1 v_1 = A_2 v_2} \tag{9}$$

Which is **the continuity equation of ideal fluid flow (also called the volumetric flow rate)** that has the following unit:

$$\boldsymbol{Av = (m^2)\left(\frac{m}{s}\right) = \frac{m^3}{s}} \tag{10}$$

The continuity equation of ideal fluid flow states that the volumetric flow rate of an ideal fluid through an enclosed space is constant $\boldsymbol{Q = Av = constant\ (units\ are\ \frac{m^3}{s})}$.

Volumetric flow rate is the volume of fluid which flows through a plane per second. We can also write:

$$\boldsymbol{Av = A\frac{\Delta x}{\Delta t} = \frac{\Delta V}{\Delta t} \rightarrow \frac{V}{t} = Av = constant} \tag{11}$$

1.4 DYNAMICS OF IDEAL FLUID FLOW

The concept of fluid in motion can be understood based on fluid dynamics, which is the study of fluid in motion and the forces causing the flow. This entire section of fluid mechanics heavily depends on Newtons 2^{nd} law of motion which is $\boldsymbol{F = ma}$**.** Where $\boldsymbol{F}$ is the sum of forces acting on an object, $\boldsymbol{m}$ is the mass of said object, and $\boldsymbol{a}$ is its velocity rate of change (acceleration).

In the case of Newtons 2^{nd} law of motion being applied to fluids during motion, we can account for a variety of forces such as:

- $\boldsymbol{F_g}$ = Force due to gravity.
- $\boldsymbol{F_p}$ = Force due to pressure.
- $\boldsymbol{F_V}$ = Viscous force.
- $\boldsymbol{F_T}$ = Force due to fluid turbulence.
- $\boldsymbol{F_c}$ = Force due to compressibility.

To study the motion of a fluid particle in the $x - axis$ direction. We will need the sum of all the forces acting on this fluid particle in the $x - axis$ direction using Newtons 2^{nd} law of motion as in the following equation:

$$\sum \boldsymbol{F_x} = \boldsymbol{ma_x} \tag{12}$$

$$(\boldsymbol{F_g})_{\boldsymbol{x}} + (\boldsymbol{F_p})_{\boldsymbol{x}} + (\boldsymbol{F_V})_{\boldsymbol{x}} + (\boldsymbol{F_T})_{\boldsymbol{x}} + (\boldsymbol{F_c})_{\boldsymbol{x}} = \boldsymbol{ma_x} \tag{13}$$

Depending on the present or considered forces, the total force can represent a variety of motion equations.

1) If $(\boldsymbol{F_c})_{\boldsymbol{x}} = \boldsymbol{0}$ or negligible → Incompressible fluid. This equation will be called **Reynold's equation of fluid motion.**
2) If $(\boldsymbol{F_T})_{\boldsymbol{x}} = (\boldsymbol{F_c})_{\boldsymbol{x}} = \boldsymbol{0}$ are negligible or equals zero → Laminar fluid flow. This equation will be called **NAVIER-STOKES equation of fluid motion.**
3) If $(\boldsymbol{F_V})_{\boldsymbol{x}} = (\boldsymbol{F_T})_{\boldsymbol{x}} = (\boldsymbol{F_c})_{\boldsymbol{x}} = \boldsymbol{0}$ are negligible or equals zero → inviscid fluid. This equation will be called **Euler's equation of fluid motion.**

In general, the motion of a fluid particle will be three dimensional and **unsteady** so that three space coordinates and time (x, z, y, t) are needed to describe it. However, in this chapter, we will only consider **steady** flow in two-dimensional motion. Clearly, we could choose to describe the flow in terms of the components of acceleration and forces in the $(x - z)$coordinates directions as shown in **Fig. (9).a**. The resulting equations are frequently referred to as a two-dimensional form of the Euler equations of motion in rectangular Cartesian coordinates. For **steady flows** each particle slides along its path, and its velocity vector is everywhere tangent to the path. The lines that are tangent to the velocity vectors throughout the flow field are called streamlines. For many situations it is easiest to describe the flow in terms of the "streamline" coordinates ($\boldsymbol{s - n\,plane}$**)** based on the streamlines as are illustrated in **Fig. (9).b**. The particle motion is described in terms of its distance, $\boldsymbol{s = s(t)}$, along the streamline from some convenient origin. The distance along the streamline is related to the particle speed by $v = \frac{ds}{dt}$, we will not be concerned with the velocity component in the $n - axis$ since the **Euler's and Bernoulli's equations** results from a force balance along a streamline only**.**

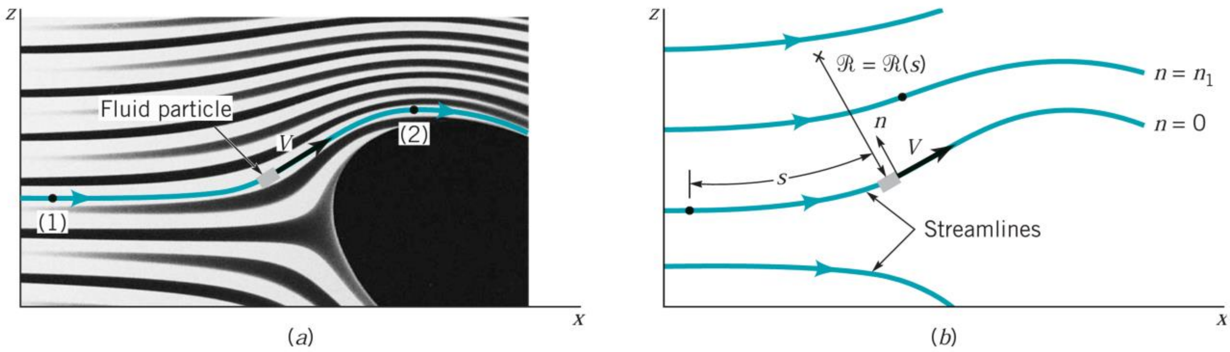

Figure 9 (a) Flow in the x–z plane. (b) Flow in terms of streamline and normal coordinates.

1.4.1 Acceleration of a fluid particle

One may be tempted to think that acceleration is zero in steady flow since acceleration is the rate of change of velocity with time, and in steady flow there is no change with time. Well, a garden hose nozzle in **Fig. (10)** tells us that this understanding is not correct. Even in steady flow and thus constant mass flow rate, water accelerates through the converging geometry of the nozzle. "Steady" indicates that flow properties do not vary with time at a specified position, not that they are uniform throughout the system. Mathematically, this is expressed by treating velocity as a function of both position (s) and time (t), where taking the total differential of $\boldsymbol{v}(\boldsymbol{s},\boldsymbol{t})$ and dividing by dt captures changes due to both temporal fluctuations and distance variations along the flow path.

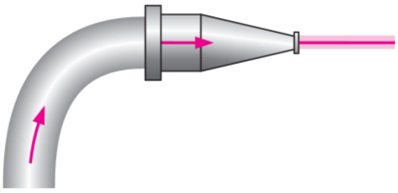

Figure 10

$$\boldsymbol{dv} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial s}}\boldsymbol{ds} + \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial t}}\boldsymbol{dt} \tag{14}$$

$$\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial s}}\frac{\boldsymbol{ds}}{\boldsymbol{dt}} + \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial t}} \tag{15}$$

In steady flow $\frac{\partial v}{\partial t} = 0$ and thus $v = v(s)$, and the acceleration in the s −direction becomes:

$$\boldsymbol{a_s} = \frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial s}}\frac{\boldsymbol{ds}}{\boldsymbol{dt}} = \frac{\boldsymbol{\partial v}}{\boldsymbol{\partial s}}\boldsymbol{v} = \boldsymbol{v}\frac{\boldsymbol{dv}}{\boldsymbol{ds}} \tag{16}$$

Where $\boldsymbol{v} = \frac{\boldsymbol{ds}}{\boldsymbol{dt}}$ if we are following a fluid particle as it moves along a streamline. Therefore, acceleration in steady flow is due to the change of velocity with position.

1.4.2 Euler's equation of fluid motion.

Consider the motion of a fluid particle in a flow field in steady flow as in **Fig. (11).**

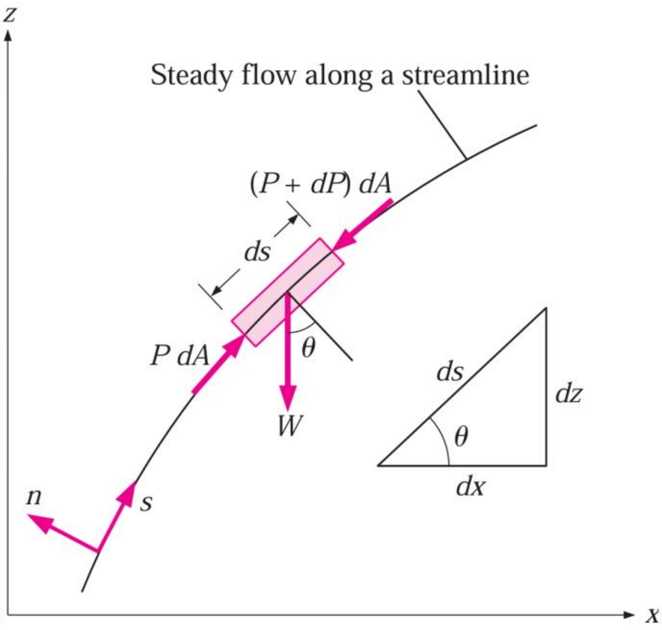

Figure 11 The forces acting on a fluid particle along a streamline.

Applying Newton's second law (which is referred to as the conservation of linear momentum relation in fluid mechanics) in the s-direction on a particle moving along a streamline gives:

$$\sum \boldsymbol{F_s} = \boldsymbol{ma_s} \tag{17}$$

In regions of flow when $(\boldsymbol{F_V})_x = (\boldsymbol{F_T})_x = (\boldsymbol{F_c})_x = \boldsymbol{0}$, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction. Therefore:

$$\boldsymbol{PdA - (P + dP)dA - W\sin\theta = ma_s = mv\frac{dv}{ds}} \tag{18}$$

Where θ is the angle between the normal of the streamline and the vertical axis at that point, $\boldsymbol{m = \rho V = \rho\, dA\, ds}$ is the mass, $\boldsymbol{W = mg = \rho g\, dA\, ds}$ is the weight of the fluid particle, and $\boldsymbol{\sin\theta = \frac{dz}{ds}}$. Substituting,

$$\boldsymbol{-dP\, dA - \rho g\, dA\, ds\, \frac{dz}{ds} = \rho dA\, ds\, v\frac{dv}{ds}} \tag{19}$$

Canceling $\boldsymbol{dA}$ from each term and simplifying:

$$\boldsymbol{-dP - \rho g\, dz = \rho\, v\, dv} \tag{20}$$

Noting that $\boldsymbol{vdv = \frac{1}{2}\, d(v^2)}$ and dividing each term by $\boldsymbol{\rho}$ gives:

$$\frac{dP}{\rho} + g\,dz + \frac{1}{2}\,d(v^2) = 0 \qquad (21)$$

Which is the desired Euler's equation of fluid motion.

1.4.3 The Bernoulli's equation

1.4.3.1 Definition

The Bernoulli equation is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible. Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this section, we demonstrate both the usefulness and limitations of Bernoulli equation. In 1738, Daniel Bernoulli (1700–1782) published his Hydrodynamics in which an equivalent of this famous equation first appeared. To use it correctly, we must constantly remember the basic assumptions used in its derivation:

1) viscous effects are assumed negligible.

2) the flow is assumed to be steady.

3) the flow is assumed to be incompressible.

4) the equation is applicable along a streamline.

In general, this equation is valid for both planar and nonplanar (three-dimensional) flows, provided it is applied along the streamline in certain regions

We will provide many examples to illustrate the correct use of the Bernoulli equation and will show how a violation of the basic assumptions used in the derivation of this equation can lead to erroneous conclusions.

1.4.3.2 Derivation of the Bernoulli Equation

In this section, we derive the Bernoulli equation by integrating **Euler's equation of fluid motion** that was presented earlier in this chapter.

By integrating **Euler's equation of fluid** in equation (21), and assuming **steady flow** we get

$$\int \frac{dP}{\rho} + gz + \frac{1}{2}\,v^2 = c = constant\ (along\ a\ streamline) \qquad (22)$$

Where c is integration constant. In general, it is not possible to integrate the pressure term because the density may not be constant. However, for now we will assume that density and specific weight are constant **(incompressible flow).** We can integrate the pressure term to get:

$$\frac{P}{\rho} + gz + \frac{1}{2}\,v^2 = c = constant\ (along\ a\ streamline) \qquad (23)$$

This is the celebrated **Bernoulli equation**, which is commonly used in fluid mechanics for steady, incompressible flow along a streamline in inviscid regions of flow.

Each term represents a certain type of energy where $\frac{P}{\rho}$ is the flow energy, $\frac{1}{2}\,v^2$ is the kinetic energy, and gz is the potential energy. The value of the constant can be evaluated at any point on the streamline where the pressure, density, velocity, and elevation are known. The Bernoulli equation can also be written between any two points on the same streamline as:

$$\frac{P_1}{\rho} + gz_1 + \frac{1}{2}\,v_1^2 = \frac{P_2}{\rho} + gz_2 + \frac{1}{2}\,v_2^2 \qquad (24)$$

1.4.3.3 Limitations on the Use of the Bernoulli Equation

The Bernoulli equation is one of the most frequently used and misused equations in fluid mechanics. Its versatility, simplicity, and ease of use make it a very valuable tool for use in analysis, but the same attributes also make it very tempting to misuse. Therefore, it is important to understand the restrictions on its applicability and observe the limitations on its use, as explained here:

1. **Steady flow**: The first limitation on the Bernoulli equation is that it is applicable to steady flow. Therefore, it should not be used during the transient start-up and shut-down periods, or during periods of change in the flow conditions. Note that there is an unsteady form of the Bernoulli equation, discussion of which is beyond the scope of the present chapter (see Panton, 1996).

2. **Frictionless flow:** Every flow involves some friction, no matter how small, and frictional effects may or may not be negligible. The situation is complicated even more by the amount of error that can be tolerated. In general, frictional effects are negligible for short flow sections with large cross sections, especially at low flow velocities.

3. **No shaft work:** The Bernoulli equation was derived from a force balance on a particle moving along a streamline. Therefore, the Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles. However, the Bernoulli equation can still be applied to a flow section prior to or past a machine (assuming, of course, that the other restrictions on its use are satisfied).

4. **Incompressible flow**: One of the assumptions used in the derivation of the Bernoulli equation is that ρ is constant and thus the flow is incompressible.

5. **No heat transfer**: The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
6. **Flow along a streamline**: Strictly speaking, the Bernoulli equation is applicable along a streamline, and the value of the constant C, in general, is different for different streamlines. But when a region of the flow is irrotational, and thus there is no vorticity in the flow field, the value of the constant C remains the same for all streamlines, and, therefore, the Bernoulli equation becomes applicable across streamlines as well

1.4.3.4 Static, Dynamic, and Stagnation Pressures

The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenome non can be made more visible by multiplying the Bernoulli equation by the density $\boldsymbol{\rho}$.

$$\boldsymbol{P + \rho g z + \frac{\rho}{2}\, v^2 = constant\ (along\ a\ streamline)} \tag{25}$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

- $\boldsymbol{P}$ is the **static pressure (**it does not incorporate any dynamic effects**):** Static pressure is the true thermodynamic pressure of the fluid which is a result of the molecular-level force exerted by fluid particles colliding with each other and container walls. It exists whether the fluid is moving or not.
- $\boldsymbol{\frac{\rho}{2}\, v^2}$ is the **dynamic pressure:** Dynamic pressure is not a physical pressure you can measure directly at a point. It is a concept that represents the pressure rise you would get if you could isentropically (i.e., without losses) bring the moving fluid to a complete stop.
- $\boldsymbol{\rho g z}$ is the **hydrostatic pressure:** which is not pressure in a real sense since its value depends on the reference level selected. This term accounts for the pressure change due to the weight of the fluid above a certain point.

The sum of the **static**, **dynamic**, and **hydrostatic** pressures is called the **total pressure**. Therefore, the Bernoulli equation states that the total pressure along a streamline is constant.

$$\boldsymbol{P_{stg} = P + \frac{\rho}{2}\, v^2} \qquad (26)$$

1.4.3.5 Velocity, pressure, and flow measuring devices

The static, dynamic, and stagnation pressures are shown in **Fig. (12).**

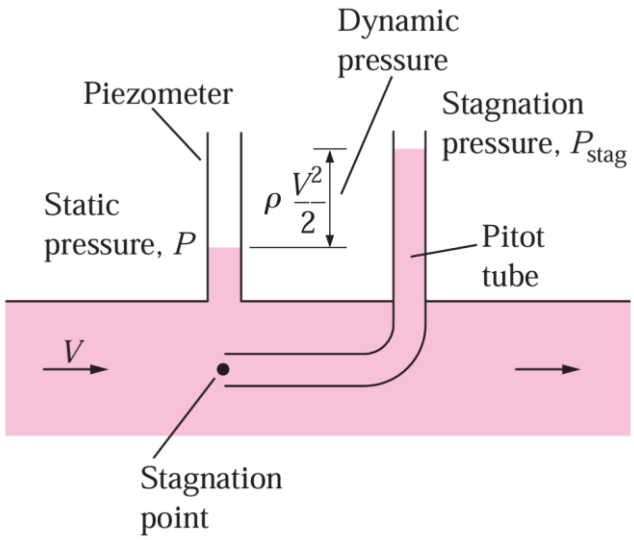

Figure 12 The static, dynamic, and stagnation pressures

When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated from:

$$\boldsymbol{v = \sqrt{\frac{2(P_{stg} - P)}{\rho}}} \qquad (27)$$

Equation (27) is useful in the measurement of flow velocity when a com bination of a static pressure tap and a Pitot tube is used, as illustrated in **Fig. (12)**. Each pressure can be measures using various devices some of which are:

- **Static pressure tap:** A static pressure tap is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure.
- **Pitot tube:** A Pitot tube is a small tube with its open end aligned into the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure.

- **Piezometer tube:** In situations in which the static and stagnation pressure of a flowing liquid are greater than atmospheric pressure, a vertical trans parent tube called a piezometer tube (or simply a piezometer) can be attached to the pressure tap and to the Pitot tube, as sketched in **Fig. (12)**. If the pressures to be measured are below atmospheric, or if measuring pressures in gases, piezometer tubes do not work.
- **Pitot-static probe:** Sometimes it is convenient to integrate static pressure holes on a Pitot probe. The result is a Pitot-static probe. The actual shape and size of Pitot-static tubes vary considerably. A typical Pitot-static probe used to determine aircraft airspeed is shown in **Fig. (13).**
- **Venturi meter:** A Venturi meter is a device used to measure the flow rate of a fluid through a pipe. It is based on a classic principle of fluid dynamics known as the Venturi Effect**.**

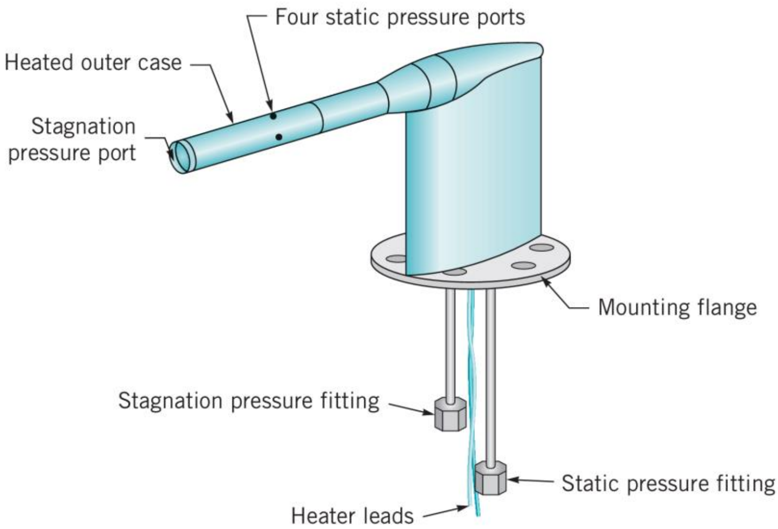

Figure 13 Airplane Pitot-static probe.

To measure the static pressure, we can drill a hole in a flat surface and fasten a piezometer tube as indicated by the location of **point (3)** in **Fig. (14).** The pressure at **point (1)** is $\boldsymbol{P_1 = P_3 + \gamma h_{3-1}}$**,** we know that $\boldsymbol{P_3 = P_{atm} + \gamma h_{4-3}}$**.** Thus, since $\boldsymbol{h_{3-1} + h_{4-3} = h}$**,** it follows that $\boldsymbol{P_1 = P_{atm} + \gamma h}$**.**

Moreover, as indicated by **Fig. (14).** The **dynamic pressure** interpretation can be seen by considering the pressure at the end of a **Pitot tube** inserted into the flow and pointing upstream. After the initial transient motion has died out, the liquid will fill the tube to a height of $\boldsymbol{H}$ as shown. The fluid in the tube, including that at its tip, **(2)**, will be stationary. That is, $\boldsymbol{v_2 = 0}$**,** or **point (2)** is a **stagnation point.** If we apply the Bernoulli equation between points **(1)** and **(2)** we get:

$$\boldsymbol{P_1 + \rho g z_1 + \frac{\rho}{2}\, v_1^2 = P_2 + \rho g z_2 + \frac{\rho}{2}\, v_2^2} \tag{28}$$

Using $\boldsymbol{v_2 = 0}$, and $\boldsymbol{z_1 = z_2}$ we find that:

$$\boldsymbol{P_2 = P_1 + \frac{\rho}{2}\, v_1^2} \tag{29}$$

Hence, the pressure at the stagnation point is greater than the static pressure, $\boldsymbol{P_1}$, by an amount $\boldsymbol{\frac{\rho}{2}\, v_1^2}$, which is the dynamic pressure.

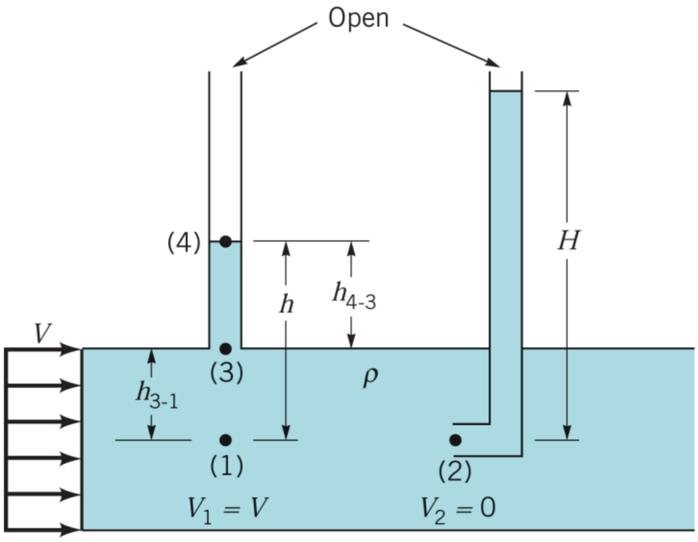

Figure 14 Measurement of static and stagnation pressures.

Consider the venturi meter in **Fig. (15)**, we take **point (1)** at the main flow section and **point (2)** at the throat along the centerline of the venturi meter.

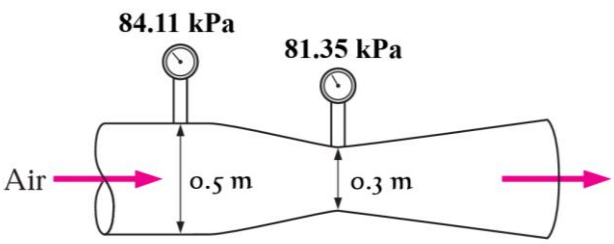

Figure 15 venturi meter measurement device.

Noting that $\boldsymbol{z_1 = z_2}$, the application of the Bernoulli equation between points **1** and **2** gives:

$$\boldsymbol{\frac{P_1}{\rho} + g z_1 + \frac{1}{2}\, v_1^2 = \frac{P_2}{\rho} + g z_2 + \frac{1}{2}\, v_2^2 \rightarrow P_1 - P_2 = \rho \frac{v_2^2 - v_1^2}{2}} \tag{30}$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\boldsymbol{Q = A_1 v_1 = A_2 v_2} \tag{31}$$

Substituting into equation (30) we get:

$$\boldsymbol{P_1 - P_2 = \rho \frac{\left(\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_1}\right)^2}{2} = \frac{\rho Q^2}{2A_2^2}(1 - \frac{A_2^2}{A_1^2})} \tag{32}$$

Solving for gives the desired relation for the flow rate:

$$\boldsymbol{Q = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho[1 - \left(\frac{A_2}{A_1}\right)^2]}}} \tag{33}$$

1.4.4 Mechanical energy and efficiency

Fluid systems are mechanisms that either moves fluid like water or air, from one place to another using pumps or fans, or that creates power from moving fluids using turbines. These systems do not use chemical or nuclear energy, and their temperature stays the same. Because of this, we can understand how they work by only looking at mechanical energy and the energy lost through friction.

Definition of fluids mechanical energy: Mechanical energy in fluid systems refers to the forms of energy that can be fully and directly converted into useful mechanical work by an ideal device, such as an ideal turbine.

Kinetic and potential energies are the familiar forms of mechanical energy. Thermal energy is not mechanical energy, however, since it cannot be converted to work directly and completely (the second law of thermodynamics). A pump transfers mechanical energy to a fluid by raising its pressure, and a turbine extracts mechanical energy from a fluid by dropping its pressure. Therefore, the pressure of a flowing fluid is also associated with its mechanical energy. In fact, the pressure unit $\boldsymbol{Pa}$ is equivalent to $\boldsymbol{Pa = N/m^2 = N \times m/m^3 = J/m^3,}$ which is energy per unit volume, and the term $\boldsymbol{\frac{P}{\rho}}$ has the unit $\boldsymbol{J/kg}$, which is energy per unit mass. Note that pressure itself is not a form of energy. But a pressure force acting on a fluid through a distance produces work, called flow work. Flow work is expressed in terms of fluid properties, and it is convenient to view it as part of the energy of a flowing fluid and call it flow energy. Therefore, the mechanical energy of a flowing fluid can be expressed on a unit-mass basis as in:

$$\boldsymbol{e_{mech} = \frac{P}{\rho} + gz + \frac{1}{2}\, v^2} \tag{34}$$

As mentioned before, $\boldsymbol{\frac{P}{\rho}}$ is the flow energy, $\boldsymbol{\frac{1}{2}\, v^2}$ is the kinetic energy, and $\boldsymbol{gz}$ is the potential energy. The mechanical energy change of a fluid during incompressible flow becomes:

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{(v_2^2 - v_1^2)}{2} \quad (\frac{kJ}{kg}) \tag{35}$$

Therefore, the mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant. In the absence of any losses, the mechanical energy change represents the mechanical work supplied to the fluid.

Consider the container with a turbine at its bottom as in **Fig. (16).** if we take point (1) as a reference, and with ($v_2^2 = v_1^2$) and $z_2 = z_1$**.** The change of the mechanical energy or work per unit mass of the fluid after passing through the turbine from point (1) to point (2) can be written as:

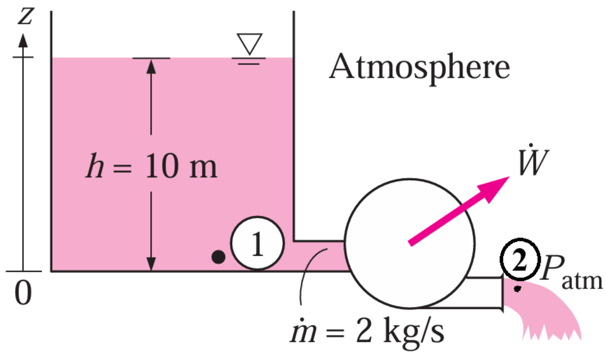

Figure 16

$$|\Delta e_{mech}| = |\Delta E_{mech}| = W_{max} = \frac{P_1 - P_2}{\rho} = \frac{P_{atm} + \rho g h - P_{atm}}{\rho} = gh \tag{36}$$

The mechanical work $\dot{W}_{max}$ supplied to the turbine is then:

$$|\dot{W}_{max}| = |\Delta \dot{E}_{mech}| = M \times W_{max} = Mgh = \left(2\frac{kg}{s}\right)\left(9.81\frac{m}{s^2}\right)(10\,m) \tag{37}$$

$$= 196\,W$$

$\dot{W}_{max}$ can either be supplied to a turbine or received from a pump.

Consider a container of height h filled with water, as shown in **Fig. (17)**, with the reference level selected at the bottom surface. The gage pressure and the potential energy per unit mass are, respectively, $P_A = 0$ and $pe_A = gh$ at point **A** at the free surface, and $P_B = \rho gh$ and $pe_B = gh$ at point **B** at the bottom of the container. An ideal hydraulic turbine would produce the same work per unit mass $W_{turbine} = gh$ whether it receives water (or any other fluid with constant density) from the top or from the bottom of the container. Note that we are also assuming ideal flow (no irreversible losses) through the pipe leading from the tank to the turbine. Therefore, the total mechanical energy of water at the bottom is equivalent to that at the top.

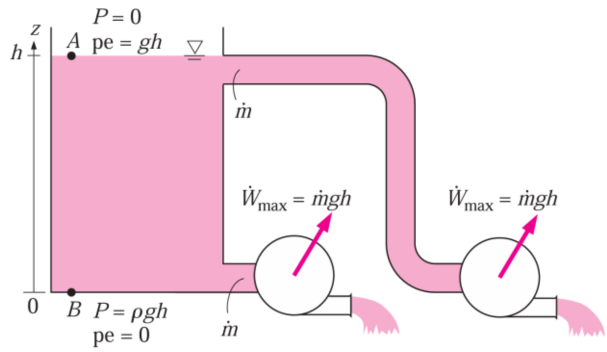

Figure 17

The transfer of mechanical energy is usually accomplished by a rotating shaft, and thus mechanical work is often referred to as shaft work. A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses). A turbine, on the other hand, converts the mechanical energy of a fluid to shaft work. In the absence of any irreversibilities such as friction, mechanical energy can be converted entirely from one mechanical form to another, and the **mechanical efficiency** of a device or process can be defined as:

$$\eta_{mech} = \frac{Mechanical\ energy\ output}{Mechanical\ energy\ input} = \frac{E_{mech,out}}{E_{mech,in}} \tag{38}$$

A conversion efficiency of less than 100 percent indicates that conversion is less than perfect and some losses have occurred during conversion.

Consider the fan in **Fig. (18)**, with $v_1 = 0$, $v_2 = 12\ m/s$, $P_1 - P_2$, and $z_2 = z_1$, the mechanical efficiency can be written as:

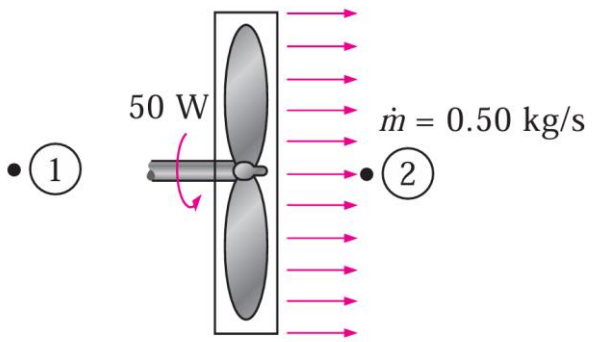

Figure 18

$$\eta_{mech} = \frac{|\Delta\dot{E}_{mech,fluid}|}{\dot{W}_{shaft,in}} = M \times \frac{v_2^2}{2} \times \left(\frac{1}{\dot{W}_{shaft,in}}\right) = \frac{(0.5\frac{kg}{s})(12\frac{m}{s^2})/2}{50\ W} \tag{39}$$

$$= 0.72$$

In fluid systems, we are usually interested in increasing the pressure, velocity, and/or elevation of a fluid. This is done by supplying mechanical energy to the fluid by a pump, a fan, or a compressor (we will refer to all of them as pumps). Or we are interested in the reverse process of extracting mechanical energy from a fluid by a turbine and producing mechanical power in the form of a rotating shaft that can drive a generator or any other rotary device. The degree of perfection of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by the pump efficiency and turbine efficiency, defined as

$$\eta_{pump} = \frac{Mechanical\ energy\ added\ to\ the\ fluid}{Mechanical\ energy\ input} = \frac{|\Delta\dot{E}_{mech,fluid}|}{\dot{W}_{shaft,in}} \tag{40}$$

And:

$$\eta_{turbine} = \frac{Mechanical\ energy\ output}{Mechanical\ energy\ extracted\ from\ to\ the\ fluid} = \frac{\dot{W}_{shaft,out}}{|\Delta\dot{E}_{mech,fluid}|} \tag{41}$$

The mechanical efficiency should not be confused with the motor efficiency and the generator efficiency, which are defined as:

$$\textbf{Motor}: \eta_{motor} = \frac{\textit{mechanical power output}}{\textit{electrical power input}} = \frac{\dot{W}_{shaft,out}}{\dot{W}_{elect,in}} \quad (42)$$

And:

$$\textbf{Generator}: \eta_{gen} = \frac{\textit{electrical power output}}{\textit{mechanical power input}} = \frac{\dot{W}_{elect,out}}{\dot{W}_{shaft,in}} \quad (43)$$

A pump is usually packaged together with its motor, and a turbine with its generator. Therefore, we are usually interested in the combined or overall efficiency of pump–motor and turbine–generator combinations **Fig. (19)**, which are defined as:

$$\eta_{pump-motor} = \eta_{pump}\eta_{motor} = \frac{|\Delta\dot{E}_{mech,fluid}|}{\dot{W}_{elect,in}} \quad (44)$$

And:

$$\eta_{turbine-gen} = \eta_{turbine}\eta_{gen} = \frac{\dot{W}_{elect,out}}{|\Delta\dot{E}_{mech,fluid}|} \quad (45)$$

All the efficiencies just defined range between 0 and 100 percent. The lower limit of 0 percent corresponds to the conversion of the entire mechanical or electric energy input to thermal energy, and the device in this case functions like a resistance heater.

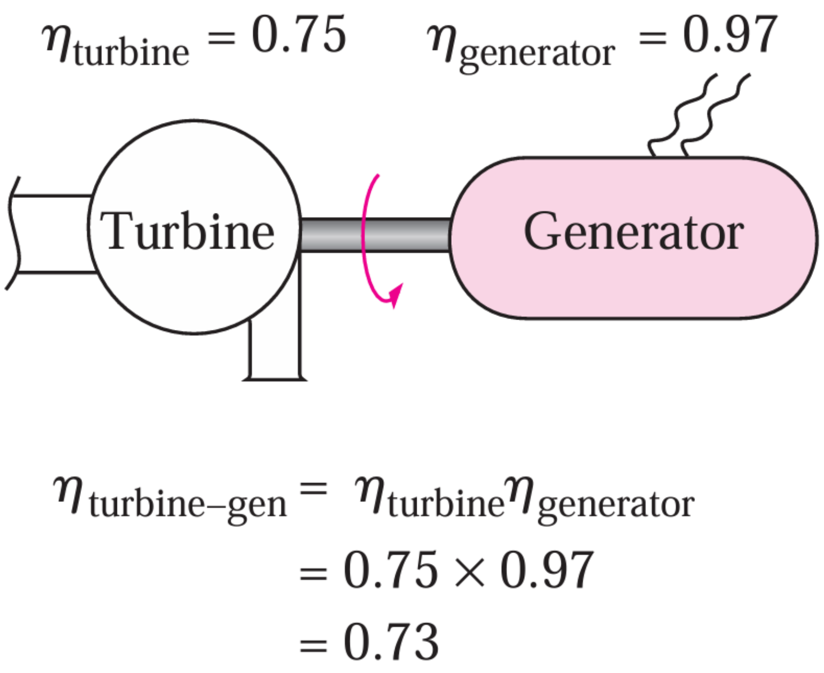

Figure 19 The overall efficiency of a turbine generator is the product of the efficiency of the turbine and the efficiency of the generator, and represents the fraction of the mechanical energy of the fluid converted to electric energy.