Chapter one: Mathematical reminder on complex numbers CN

① ~~De~~ **Definition**: $z \in \mathbb{C}$

* $z = x + jy$ with: $x \in \mathbb{R}$, $y \in \mathbb{R}$ and $j^2 = -1$
* The real part of z is the real number x

We write: $Re(z) = x$

* The imaginary part of z is the real number y

We write: $Im(z) = y$

* $z = x + jy$ is the algebric or the rectangular forme of z.

② Mathematical operations on CN.

We have: $z = x + jy$ and $z' = x' + jy'$ two CNs

* $z + z' = x + x' + j(y + y')$.
* $zz' = xy - yy' + j(xy' + x'y)$.
* $z = z' \Longleftrightarrow x = x'$ and $y = y'$.

③ Conjugate of CN.

$z = x + jy \in \mathbb{C}$. The Conjugate of z is the ~~number~~ complex number $\bar{z} = x - jy$

04 Modulus, of complex number: $z \in \mathbb{C}$

$z = x + jy$

The modulus or absolute value $|z|$ of z is

$$|z| = \sqrt{x^2+y^2} = \sqrt{z\bar{z}}, \quad \text{So: } z\bar{z} = |z|^2$$

04-1 Properties of modulus: $(z, z') \in \mathbb{C}^2$

- $|zz'| = |z||z'|$ • $|z| = 0 \Rightarrow z = 0$
- $\left|\dfrac{z}{z'}\right|_{z' \neq 0} = \dfrac{|z|}{|z'|}$ • $\forall n \in \mathbb{Z}: |z^n| = |z|^n$

05 Polar form of CN:

$z \in \mathbb{C}^*, r \in \mathbb{R}_+^*, \theta \in \mathbb{R}$.

$z = x + jy \Rightarrow r = |z| = \sqrt{x^2+y^2}$

and $\theta = Arg(z) = \tan^{-1}\dfrac{y}{x} \equiv 2\pi$

$z = r(\cos\theta + j\sin\theta)$ is the polar form of z.

So: $z = x + jy = r(\cos\theta + j\sin\theta) = r\angle\theta$.

-5-1 Properties of arguments:

$(z, z') \in \mathbb{C}^2, n \in \mathbb{N}$.

- $Arg(zz') = Arg(z) + Arg(z') \equiv 2\pi$ • $Arg(\bar{z}) = -Arg(z) \equiv 2\pi$
- $Arg(z)^n = n\,Arg(z) \equiv 2\pi$ • $Arg\left(\dfrac{z}{z'}\right)_{z' \neq 0} = Arg(z) - Arg(z') \equiv 2\pi$
- $Arg(1/z)_{z \neq 0} = -Arg(z) \equiv 2\pi$

(06) De Moivre's Theorem

If $z = r(\cos\theta + j\sin\theta)$ and n is a positive integer

Then: $z^n = r^n(\cos n\theta + j\sin n\theta)$

(07) Euler's theorem: $\forall\ \theta \in \mathbb{R}$.

$e^{j\theta} = \cos\theta + j\sin\theta$ So. $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ and,

(08) Complex Numbers applications

See Supervised work ~~activities~~ activity

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(08) Exponential form of a CN

$z \in \mathbb{C}$, $r = |z|$, $\theta = Arg(z)$.

$z = re^{j\theta}$ is the exponential form

of z.

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