

Exercise 1

Air passes through a duct. The air state and its mach number are specified ($T=373.2^\circ\text{K}$, $P=200\text{kPa}$, $Ma=0.8$). If we consider that air is an ideal gas of constant specific heat (at ambient temperature are $R_s=0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$ and $\gamma=1.4$), calculate:

- 1°)-The speed and density of the air at the inlet
- 2°)-The temperature, pressure and total density at the inlet

Exercise 2

An air compressor stage works with a pressure of 1bar and 25°C which requires an input work of 17 kJ/kg with an isentropic efficiency of 0.9. The air inlet and outlet speed are the same. Assuming that the properties of the air are unchanged as it passes through the compressor stage.

- 1°)-Draw the enthalpy diagram indicating the static and total values?
- 2°)- Calculate the stage outlet pressure?

We give $C_p=1,01\text{ kJ/kg}\cdot\text{K}$, $\gamma=1,4$ for an ideal gas at 20°C

Exercise 3

Steam at $p_1=6000\text{kPa}$ and $T_1=400^\circ\text{C}$ expanded adiabatically through a steam turbine at a pressure $p_3=60\text{kPa}$ with an isentropic efficiency of 90%.

- 1°)-Draw the shape of the turbine enthalpy diagram (static and total states)?
- 2°)-Calculate the steam strength at the outlet?
- 3°)-Calculate the ideal and real work developed if we neglect the variation in kinetic energy?

We give from the thermodynamic tables for $(6000\text{kPa}, 400^\circ\text{C})$ we have $h_1=3177,0\text{ kJ/kg}$, $S_1=6,5404\text{ kJ/kg}\cdot\text{K}$ and for 60 kPa we have ($h_f = 359,79\text{ kJ/kg}$, $h_{fg} = 293,11\text{ kJ/kg}$, $S_f=1,145\text{ kJ/kg}\cdot\text{K}$, $S_g=7,5314\text{ kJ/kg}\cdot\text{K}$)

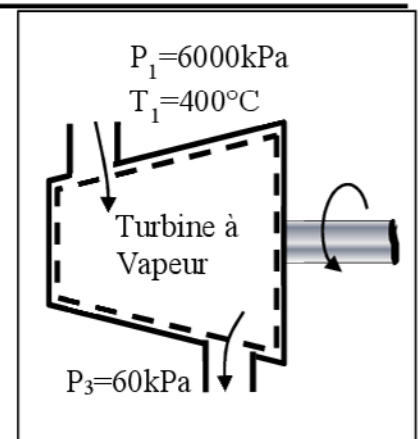


Fig.1-Turbine à Vapeur

Solution to exercise n°1

1°)-The speed and density of the air at the inlet

The speed of sound in air at specific conditions is: $a = \sqrt{\gamma r T} = \sqrt{1,4 \cdot 0,287 \cdot 373,2 \cdot 1000} = 387,2 \text{ m/s}$

The speed and density of the air at the inlet:

$$C = M \cdot a = 0,8 \cdot 387,2 = 310 \text{ m/s}$$

$$\rho = \frac{P}{R_s T} = \frac{200}{0,287 \cdot 373,2} = 1,867 \text{ kg/m}^3$$

2°)-The temperature, pressure and total density at the inlet

$$T_0 = T \left[1 + \frac{\gamma - 1}{2} M^2 \right] = 373,2 \left(1 + \frac{(1,4 - 1)(0,8)^2}{2} \right) = 421 \text{ K}$$

$$P_0 = P \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma - 1}} = 200 \left(\frac{421}{373,2} \right)^{\frac{1,4}{1,4 - 1}} = 305 \text{ kPa}$$

$$\rho_0 = \rho \left(\frac{T_0}{T} \right)^{\frac{1}{\gamma - 1}} = 1,867 \left(\frac{421}{373,2} \right)^{\frac{1}{1,4 - 1}} = 2,52 \text{ kg/m}^3$$

Solution to exercise n°2

1°) - the enthalpy diagram indicating the static and total values (see Figure 2):

2°)- Calculate the stage outlet pressure?

►The work of the input is specific and since the input speed is equal to that of output $V_1 = V_3$ therefore:

$$W_A = h_3 - h_1 = 17 \text{ kJ/kg}$$

►The air is considered perfect therefore: $C_p = 1,01 \text{ kJ/kg.K}$

$$C_p \Delta T = \Delta h \Rightarrow \Delta T = \frac{\Delta h}{C_p}$$

$$\Rightarrow T_3 - T_1 = \frac{17}{1,01} = 16,83 \text{ K}$$

►The compressor efficiency of an ideal gas can be expressed in temperatures:

$$\eta = \frac{h_{3s} - h_1}{h_3 - h_1} = \frac{T_{3s} - T_1}{T_3 - T_1} = 0,9$$

$$\Rightarrow T_{3s} - T_1 = 0,9(T_3 - T_1) = 15,15 \text{ K}$$

►For isentropic compression: $\frac{T_{3s}}{T_1} = \left(\frac{P_3}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}$

And for a temperature of 20°C we have $\gamma = 1,4$

$$\frac{P_3}{P_1} = \left(\frac{298 + 15,15}{298} \right)^{\frac{1,4}{0,4}} = 1,19$$

⇒ the stage outlet pressure is: $P_3 = 1,19 \cdot P_1 = 1,19 \text{ bar}$

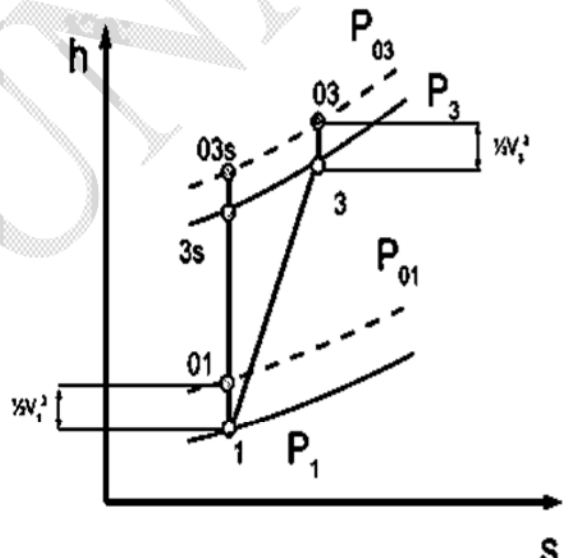


Fig.2-Diagramme h-s de compression

Solution to exercise n°3

1°)-The appearance of the turbine enthalpy diagram (static and total states):

2°)-The steam title:

$$x_3 = \frac{s_3 - s_f}{s_g - s_f} = \frac{6,5404 - 1,1451}{7,5314 - 1,1451} = 0,8448$$

3°)-To obtain the enthalpy value at the outlet:

$$h_{3s}(60) = h_f + x_3 \cdot h_{fg}$$

$$= 359,79 + 0,8448 * 2293,11$$

$$= 2297,0 \text{ kJ/kg}$$

The isentropic work developed is:

$$W_s = (h_1 - h_{3s}) = (3177 - 2297) = 880 \text{ kJ/kg}$$

The actual work developed is:

$$W_r = (h_1 - h_3) = \eta_s (h_1 - h_{3s}) = 0,9880 =$$

kg

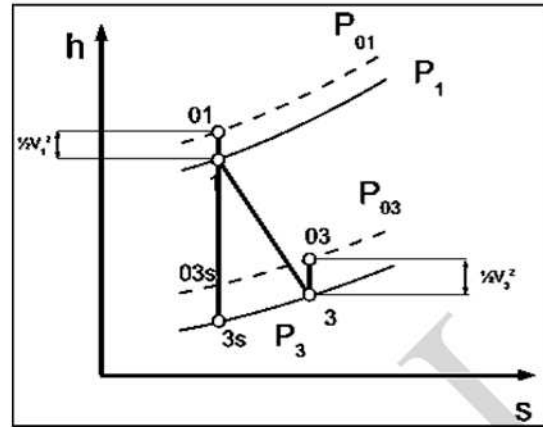


Fig.3-Diagramme h-s de détente

T (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg K)
p = 60 bar				
276.62	0.03244	2589.3	2783.9	5.8886
280	0.03317	2604.7	2803.7	5.9245
320	0.03874	2719.0	2951.5	6.1830
360	0.04330	2810.6	3070.4	6.3771
400	0.04739	2892.7	3177.0	6.5404
440	0.05121	2970.2	3277.4	6.6854
480	0.05487	3045.3	3374.5	6.8179

p bar	T °C	$v_f \cdot 10^3$ m³/kg	v_g m³/kg	u_f kJ/kg	u_g kJ/kg	h_f kJ/kg	h_g kJ/kg	s_f kJ/kg K	s_g kJ/kg K
0.60	85.92	1.0330	2.7351	359.73	2488.8	359.79	2652.9	1.1451	7.5314
0.70	89.93	1.0359	2.3676	376.56	2493.8	376.64	2659.5	1.1917	7.4793
0.80	93.48	1.0385	2.0895	391.51	2498.1	391.60	2665.3	1.2327	7.4342
0.90	96.69	1.0409	1.8715	405.00	2502.0	405.09	2670.5	1.2693	7.3946
1.00	99.61	1.0431	1.6958	417.30	2505.5	417.40	2675.1	1.3024	7.3592

Fig.4 Tables thermodynamiques pour la vapeur d'eau

From the thermodynamic table or the Mollier diagram: $h_1=3177.0 \text{ kJ/kg}$ (6000kPa, 400°C) and $S_1=6.5404 \text{ kJ/kgK}$ and by leading isentropically $S_1=S_3$ up to 60kPa ($S_f=1, 1451 \text{ kJ/kgK}$, $S_g=7.5314 \text{ kJ/kgK}$)