

Exercise n°1

A stationary descending glider, whose mass (with its pilot) is 300 kg, has a wing area of 15 m². The glider is initially at a height of 1000 m in air without vertical or horizontal currents.

1°)-Determine the operating point on the glider's polar (figure 1), the angle of attack of the wing, the horizontal and vertical speeds, the path traveled and the flight time in the following two situations:

- the pilot wants to land his glider as far away as possible,
- the pilot wants to have the minimum absolute speed without stalling.

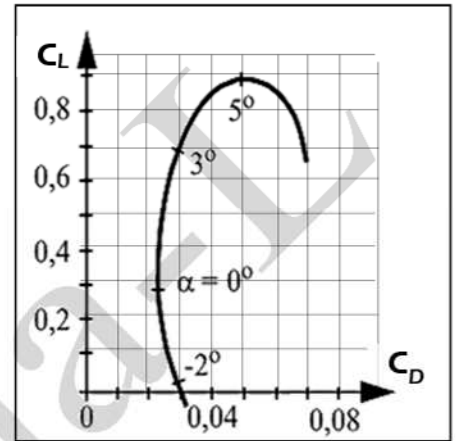


Fig.1-Polaire

Exercise n°2

For a blade grille working under nominal conditions and whose camber line is a circular arc, with a ratio (pitch/chord) equal to 0.8 and the air entry and exit angle respectively 40° and 52.5°, assuming zero incidence on the blade and that $\delta=0.23\theta$, then calculate:

- the nominal angle of deflection?
- the camber angle of the blade?
- the angle of deviation?
- the pitch angle of the blade if we give $\theta_1 = \frac{\theta}{2}$?

Exercise n°3

A Kaplan turbine has four blades on the rotor (See fig.2), each with a span of 0.62 m operating at a loss angle of 3.9° and a speed of 75 rpm. The average diameter of the pitch circle is 3.5 m. The chord of the blade has a length of 2.5 m and its inclination towards the tangent of the blade is 30° in a condition where the axial flow speed is 5.3 m.s⁻¹. The lift coefficient is 0.76, Calculate:

- The tangential component of the force on the blade?
- The power developed by the turbine in kW?
- The efficiency of the turbine?

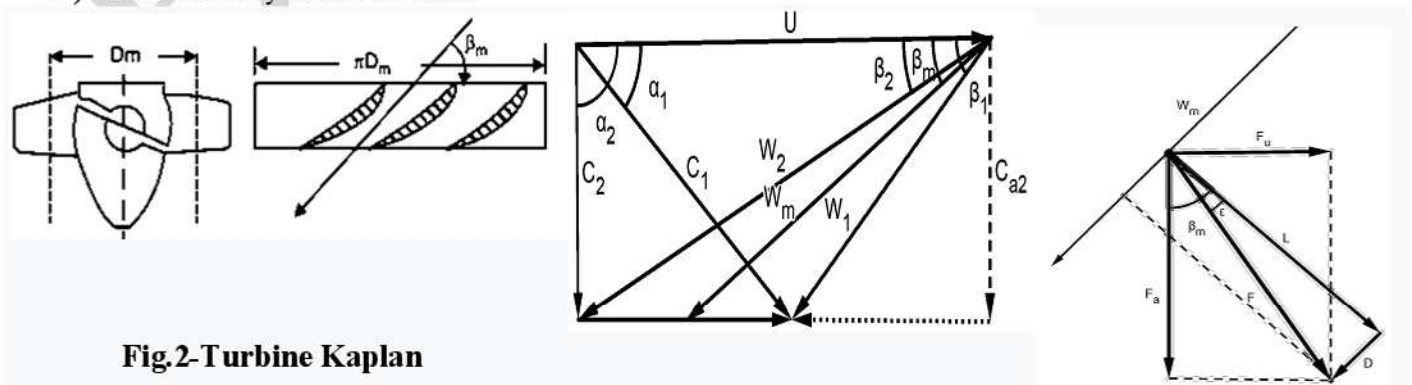


Fig.2-Turbine Kaplan

Solution to Exercise n°1

1°)- The operating point on the glider's polar (see figure.1) in the following two situations:

a)-To go as far as possible, $\tan \epsilon$ must be minimal.

•The operating point on the polar corresponds to the minimum angle $\epsilon_{\min}$:

$$\text{Here: } \alpha = 3^\circ; C_L = 0,7 \text{ et } C_D = 0,03 \Rightarrow \epsilon_{\min} = 2,45^\circ$$

•Horizontal and vertical speeds:

$$v = \sqrt{\frac{2mg \cos \epsilon}{\rho S C_L}} = \sqrt{\frac{2 \cdot 300 \cdot 9,81 \cdot \cos(2,45)}{1,23 \cdot 15 \cdot 0,7}} = 21,33 \text{ m/s}$$

$$\Rightarrow v_{\text{hor}} = v \cos \epsilon_{\min} \approx 21,3 \text{ m/s} \quad v_{\text{ver}} = v \sin \epsilon_{\min} \approx 0,9 \text{ m/s}$$

•The horizontal distance traveled is thus: $D = \frac{h}{\tan \epsilon} = \frac{h C_L}{C_D} \approx 23,3 \text{ km}$

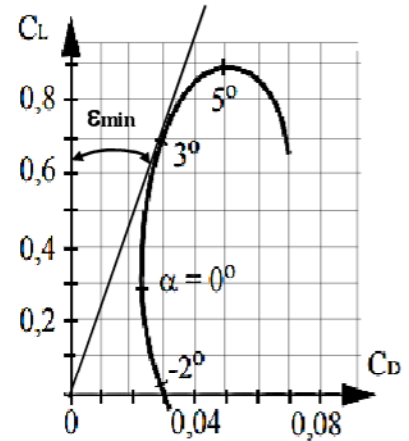
•Flight time vol $t = \frac{D}{v_{\text{hor}}} \approx 18 \text{ minutes}$.

b)-To obtain the minimum absolute speed for normal flight conditions without stalling,

$$\alpha = 5^\circ; C_L = 0,9; C_D = 0,05; \epsilon = 3,18^\circ; v_{\text{hor}} \approx 18,8 \frac{\text{m}}{\text{s}}; v_{\text{ver}} \approx 1,05 \frac{\text{m}}{\text{s}};$$

the angle ϵ is small $\Rightarrow \cos \epsilon \approx 1 \Rightarrow v = \sqrt{\frac{2mg}{\rho S C_p}} = 18,82 \text{ m/s}$ and therefore v is minimal for maximum C_p :

$$D \approx 18 \text{ km et } t \approx 15,95 \text{ min} = 15 \text{ min } 57 \text{ s.}$$



Solution to Exercise n°2

$$\frac{t}{l} = 0,8 \quad \alpha_1 = 40^\circ \quad \alpha_2 = 52,5^\circ$$

(a)- the angle of deflection: $\varphi = \varphi_n = \alpha_1 + \alpha_2 = 40^\circ + 52,5^\circ = 92,5^\circ$

(b)- the camber angle of the blade: -Since: $\delta = 0,26\theta(0,8)^{1/2} = 0,233\theta$

$$\text{Camber angle: } \theta = \beta_1 + \beta_2 = \theta_1 + \theta_2 \rightarrow \beta_2 = \theta_2 + \phi = \delta + \alpha_2 \text{ et } \beta_1 = \theta_1 - \phi = \alpha_1 - i$$

$$\text{Since the incidence } i=0 \text{ therefore } \beta_1 = \alpha_1 \rightarrow \theta = \alpha_1 + \alpha_2 + 0,233\theta \rightarrow \theta = \frac{40^\circ + 52,5^\circ}{1 - 0,23} = 120,13^\circ$$

Or we can solve it another way:

$$\text{-Deflection angle: } \varphi = \alpha_1 + \alpha_2 = \theta + i - \delta = \theta + i - 0,233\theta \Rightarrow \theta = \frac{\varphi}{1 - 0,23} = 120,13^\circ$$

(c)- the angle of deviation: $\delta = 0,233\theta = 0,233 \cdot 120,13^\circ = 27,99^\circ$

(d) - the pitch angle of the blade ϕ : since the arc of the camber is circular

$$\phi = -\beta_1 + \frac{\theta}{2} = -\alpha_1 + \frac{\theta}{2} = -40^\circ + \frac{120,13^\circ}{2} = 20^\circ$$

Solution to Exercise n°3

1°)-According to Fig. 2, Velocity triangles, we see that: $W_m = W_{a2} / \sin \beta_m = 5,3 / \sin 30 = 10,6 \text{ m.s}^{-1}$

$$\text{-The tangential component of force on the blade is: } F_u = C_L A \frac{\rho W_m^2 \sin(\beta_m - \epsilon)}{2 \cos \epsilon} = 29180,18 \text{ N}$$

2°)-Tangential speed:

$$u = \pi D_m N / 60 = (\pi)(3,5 \times 75) / 60 = 13,74 \text{ m.s}^{-1}$$

-The power developed:

$$P = Z * u * F_u = (4)(13,74)(29180,18) = 1603742,7 \text{ W} = 1603,7 \text{ kW}$$

3°)-The efficiency of the turbine:

$$\eta_a = \frac{1 - \tan \epsilon \cdot \tan \beta_m}{1 + \tan \epsilon \cdot \cot \beta_m} = \frac{1 - \tan(3,9) \cdot \tan(30)}{1 + \tan(3,9) \cdot \cot(30)} = \frac{0,96}{1,118} = 0,8586 = 85,8\%$$