Chapter 4-Nozzle Study

4.1- Definitions: (Fig.4-2):

→A nozzle is a device where the kinetic energy of a fluid is increased during an adiabatic evolution. This increase in energy causes a drop in pressure and is achieved through an appropriate modification of the flow section.

→A diffuser is a device which has the opposite function (increasing pressure by reducing flow speed).

4-2-Equation of energy and adiabatic losses in a nozzle:

► For a fixed channel we have:

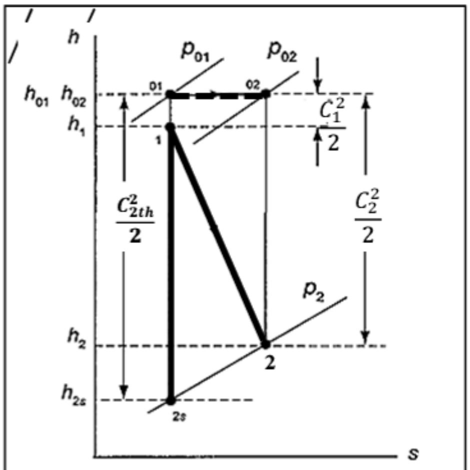

Fig.9-

$$W + Q = \left(h_2 + \frac{C_2^2}{2}\right) - \left(h_1 + \frac{C_1^2}{2}\right) = 0$$

$$\Rightarrow h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \Rightarrow h_{01} = h_{02}$$

And the same for: $h_1 + \frac{C_1^2}{2} = h_{2s} + \frac{C_{2th}^2}{2} \Rightarrow h_{01} = h_{02s}$

► Speed slowdown coefficient: $\boldsymbol{\varphi = \frac{C_2}{C_{2th}}}$;

► Calculation of adiabatic losses: $\boldsymbol{\pi_s = h_2 - h_{2s} = \frac{C_{2th}^2}{2}(1 - \varphi^2)}$

► Coefficient of pressure loss in the stator: $\boldsymbol{\zeta_s = 1 - \frac{P_{02}}{P_{01}}}$

4-3-Efficiency of a nozzle:

► The isentropic efficiency for a nozzle is:

$$\Rightarrow \quad \eta_{ty} = \frac{W_{rD}}{W_{sD}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{C_2^2 - C_1^2}{2(h_1 - h_{2s})} = \frac{C_2^2 - C_1^2}{C_{2th}^2 - C_1^2}$$

► If the fluid enters from a large reservoir therefore C1=0, the efficiency becomes:

$$\Rightarrow \quad \eta_{ty} = \frac{C_2^2 - C_1^2}{C_{2th}^2 - C_1^2} = \frac{C_2^2}{C_{2th}^2} = \boldsymbol{\varphi^2}$$

a)- Speed of sound: $a = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma r T}$

b)- Flow regime as a function of the number of Mach M: (Fig.4-1)

The different flow regimes are:

- *0.3 <M < 0.8: the flow is subsonic (M < 1),*
- *M=1: the flow is sonic,*
- *0.8 <M < 1.2: the flow is transonic,*
- *1.2 <M < 3: the flow is supersonic (M > 1),*
- *M > 3: the flow is hypersonic*

4-5- Effect of section variation on isentropic flow: (Fig.4-3):

From the equation of (PCM) and (PCE) plus (BSV) we obtain the Hugoniot relation:

$$\frac{dA}{A}=\frac{dP}{\rho C^2}[1-M^2]=-\frac{dC}{C}[1-M^2] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Note: The interpretation of equation (1) is given by table 4-1:

▸the equation between two points 1 and 2 from (PCM) and (BSV): $\frac{A_1}{A_2}=\frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

By putting A* for M=1 as a reference we will therefore have: $\frac{A}{A^*}=\frac{1}{M}\left\{\left(\frac{2}{\gamma+1}\right)\left[1+\frac{\gamma-1}{2}M^2\right]\right\}^{\frac{\gamma+1}{2(\gamma-1)}}$

Note: The interpretation of the equation is given by the figure below.

4.6-Isentropic flow rate in a converging nozzle:

We consider the flow of an ideal gas in a converging nozzle. In the isentropic case, we can write the mass flow as q_m=ρ.C.A, or:

$$\frac{q_m}{A}=\frac{P}{rT}aM=\frac{P}{rT}\sqrt{\gamma rT}M=\frac{P}{\sqrt{T}}\sqrt{\frac{\gamma}{r}}M=\sqrt{\frac{\gamma}{r}}\frac{P_0}{\sqrt{T_0}}M\frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}}$$

The condition $M=1$, we obtain d A=0 or A = ctene can therefore only be applied to the neck this with a maximum mass flow, we then say that the nozzle is in sonic blocking condition.

▸By substituting the condition $M=1$ into (11.35), we obtain: $\frac{q_m}{A^*}=\sqrt{\frac{\gamma}{r}}\frac{P_0}{\sqrt{T_0}}\frac{1}{\left(1+\frac{\gamma-1}{2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}}$

4.7-Pressure Distribution and choked flow in a Converging Nozzle: (Fig.4-6-and Fig.4-7)

(i)▸ At $P_E=P_B=P_0$, no flow in the nozzle

(ii)▸ $\mathbf{P_B}$ ↓, $\mathbf{q_m}$↑ and $P_E=P_B$ until the maximum flow condition is reached.

(iii)▸ $\mathbf{P_B}$ ↓, the pressure P_E will remain $P_E=P^*$ even if PB can decrease. The maximum value of q_m at M= 1 is known as the blocked flow (choked flow or sonic choking phenomenon).

(iv)▸ For $P_B<P^*$, additional relaxation at the outlet therefore a series of shock waves which form at the outlet of the nozzle.

4.8-Isentropic flow in a convergent-divergent nozzle (T-Laval): (Fig.4-8)

We consider a flow in a converging-diverging nozzle (see Figure 7). The upstream stopping conditions are assumed to be constant;

(i)▸By opening the valve very slowly, a completely subsonic flow appears.

(ii)▸If $\mathbf{P_B}$ ↓so as to achieve sonic conditions at the neck and maximum mass flow.

(iii)▸ $\mathbf{P_B}$ ↓, No effect on flow. Supersonic flow in the divergent section and appearance of a normal shock wave.

(iv)▸ $\mathbf{P_B}$ ↓, The position of this shock wave moves to the exit of the nozzle.

(v)▸ $\mathbf{P_B}$ ↓, The flow in the entire divergent part up to the outlet is supersonic and appearance of oblique shock waves outside the nozzle.

4-9- Normal shock wave (Right): (Fig.4-4)

Shock waves represent very strong localized irreversibility in the flow. Over a distance of the order of the mean free path, the flow goes from a supersonic state to a subsonic state, the speed suddenly decreases and the pressure increases significantly. . **($P_1<P_2$, $\rho_1<\rho_2$, $T_1<T_2$, $M_1>M_2$).**

Normal (or straight) shock waves are perpendicular to the flow and oblique shock waves are inclined at a specific angle.

4.-9-1-Calculation of Flow Properties through a Normal Shock:

—Conservation of energy: $h_1+\frac{C_1^2}{2}=h_2+\frac{C_2^2}{2}$ We go through total conservation because the shock breaks isentropic continuity, we cannot therefore use Saint-Venant. Only the total energy is conserved. For an ideal gas: T01=T02

—Conservation of mass: On a fixed shock surface A, we have $q_m=\rho_1 C_1 A=\rho_2 C_2 A$

We have $\frac{\rho_1}{\rho_2}=\frac{C_2}{C_1}=\frac{a_1 M_1}{a_2 M_2}$ or $\frac{P_2}{P_1}=\frac{M_1}{M_2}\left(\frac{T_2}{T_1}\right)^{1/2}$ by the state law

The energy equation using the shutdown temperature relationship: $\frac{T_2}{T_1}=\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$

—Conservation of momentum (or dynamics): (Fig.4-5)

$$q_m(C_2-C_1)=(P_1-P_2).A=\rho_2 C_2^2-\rho_1 C_1^2$$

We introduce the Mach number: $\frac{P_2}{P_1}=\frac{(1+\gamma M_1^2)}{(1+\gamma M_2^2)}$

—The intersection between the Fanno line and that of Rayleigh gives:

▸The first is trivial, M2= M1 which means that there is no shock

▸The other solution is: $M_2^2=\frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}$

We can also combine these equations to obtain:

- $T_{01}=T_{02}$
- $\frac{\rho_2}{\rho_1}=\frac{C_1}{C_2}=\frac{a_1 M_1}{a_2 M_2}=\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$
- $\frac{T_2}{T_1}=[(\gamma-1)M_1^2+2]\frac{2\gamma M_1^2-(\gamma-1)}{(\gamma+1)^2 M_1^2}$
- $\frac{P_2}{P_1}=\frac{2\gamma}{\gamma+1}M_1^2-\frac{\gamma-1}{\gamma+1}$
- $\frac{P_{02}}{P_{01}}=\left(\frac{P_{02}}{P_2}\right)\left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_{01}}\right)=\left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}}.\left[\frac{(\gamma+1)}{2\gamma M_1^2-(\gamma-1)}\right]^{-\frac{1}{\gamma-1}}=\frac{\rho_{02}}{\rho_{01}}$

Note: These relationships are a function of M_1 and γ only these relationships are tabulated on shock wave tables.

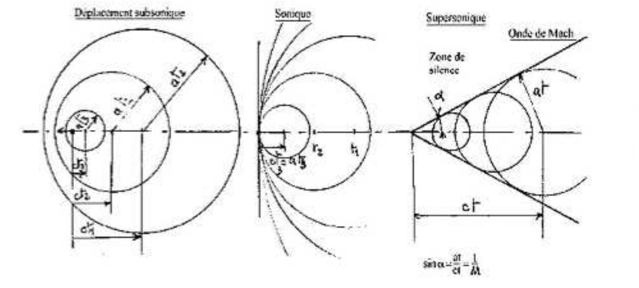

Fig.4-1- Différents Régimes d'écoulement

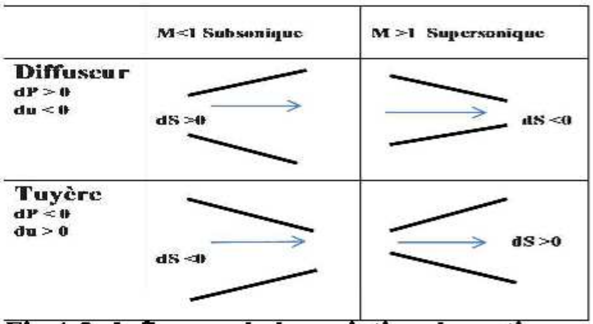

Fig.4-2- Influence de la variation de section

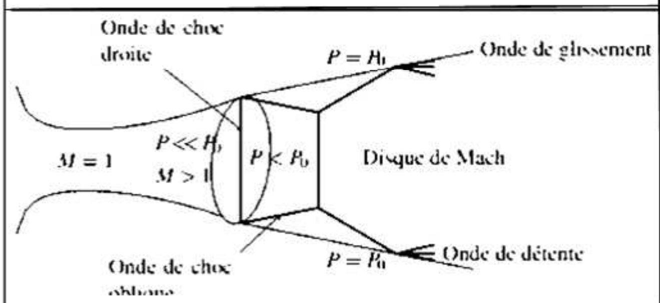

Fig.4-4- Différentes types d'ondes de choc

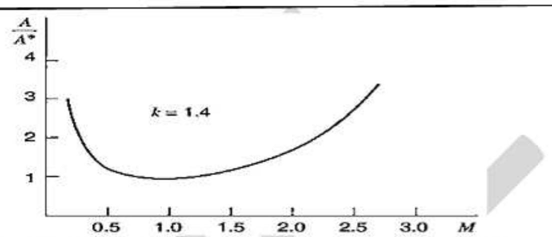

Fig.4-3- Variation de section en fonction de M

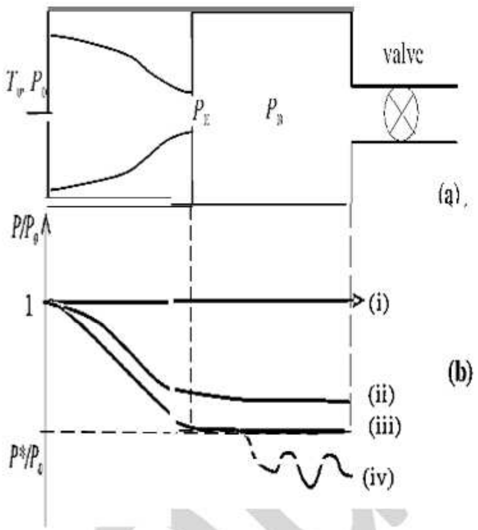

Fig.4-6- Ecoulement compressible à travers une tuyère convergente

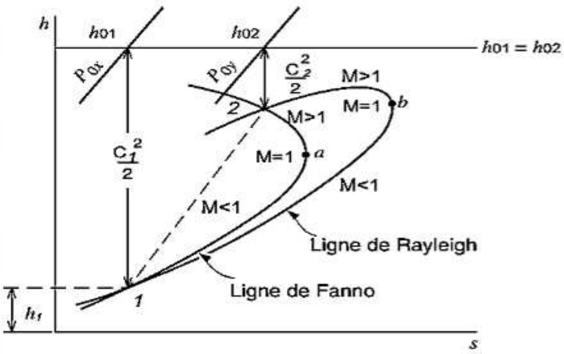

Fig.4-5- Diagramme h-s pour une onde de choc droite a une dimension

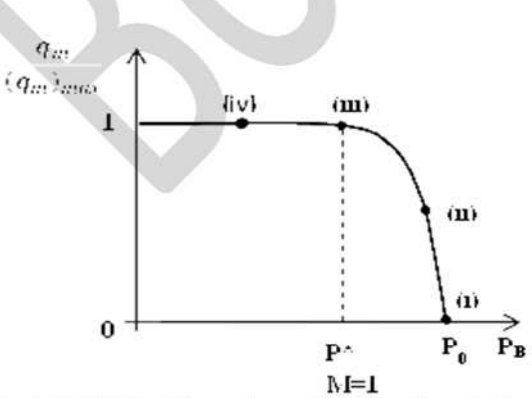

Fig.4-7- Débit adimensionnel en fonction de la pression

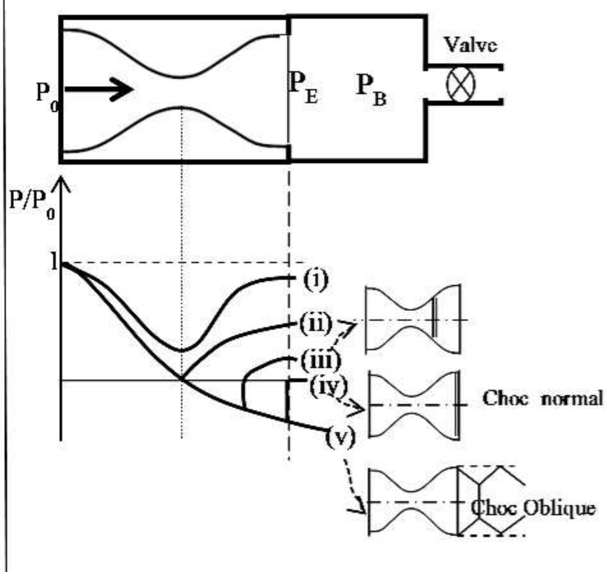

Fig.4-8- Distribution de pression dans une tuyère de Laval pour différentes pression P_B