Chapter 3. General equations of turbomachines

3-1-Basic principle in turbomachines: (Fig3-1and 3-2)

With: Wm: mechanical losses (friction on the bearings)

W_T: work exchanged between the fluid and the operator (piston, fins, etc.) or Work indicated, engine, technique, etc.

Qe: heat supplied or received by the fluid between the inlet and outlet (adiabatic system Qe=0)

C_1, C_2: absolute speed of the fluid at the inlet and outlet

Q_f: Heat or friction work linked to the viscosity of the fluid (reversible transformation Q_f=0)

P,T: state of the fluid (Pressure, temperature)

Note: negligible work of gravitational forces for compressible fluids.

▸Principle of conservation of mass (PCM): $\rho_1 S_1 V_{1d} = \rho_2 S_2 V_{2d}$

▸Fundamental principle of dynamics (Eq-D'Euler) (PFD):

$$W_T = [\Delta(UC_u)] = \left(\frac{U_2^2 - U_1^2}{2} + \frac{C_2^2 - C_1^2}{2} + \frac{W_1^2 - W_2^2}{2}\right)$$

▸First law of thermodynamics (1st P): $W_T + Q_e = \Delta h + \frac{\Delta C^2}{2} + \underbrace{g\Delta z}_{=0}$

▸Second law of thermodynamics (2nd P): $d(Q_f + Q_e) = TdS$

▸Kinetic energy theorem (GIBS eq) (PEC): $W_T = -\int \frac{dP}{\rho} - \int TdS + \frac{C_1^2 - C_2^2}{2}$

Now the enthalpy h: $dh = \int \frac{dP}{\rho} + \int TdS$

3-2-Principles and Energy Equation for Fixed Channels (Turbine): (Fig3-3)

-adiabatic flow: Qe=0 and without technical work exchange WT=0

► (1er P) : $0 = \Delta h + \frac{\Delta C^2}{2} \Rightarrow \boldsymbol{h_{01} = h_{02}} \;\; \boldsymbol{ou} \;\; \boldsymbol{H_1 = H_2}$

⇒The total enthalpy of the fluid remains constant.

►**Speed slowdown coefficient**: $\boldsymbol{\varphi = \frac{C_2}{C_{2th}}}$;

With: C_{2th} : theoretical speed without losses

►**Calculation of adiabatic losses:** $\pi_s = h_2 - h_{2s} = \frac{C_{2th}^2}{2}(1 - \varphi^2)$

Siq_1=h_1-h_2s: available static drop (charge)⇒ $\boldsymbol{\pi_s = \left(q_1 + \frac{C_1^2}{2}\right)(1 - \varphi^2)}$

►**Coefficient of pressure loss in the stator:** $\boldsymbol{\zeta_s = 1 - \frac{P_{02}}{P_{01}}}$

3-3- Principles and Energy Equation in mobile channels (Turbine): (Fig3-4)

-in mobile channels the transformations are treated as adiabatic Qe=0 but with exchange of technical work therefore WT≠0

▸By combining (PFD) and (1st P): $\Rightarrow h_2 + \frac{W_2^2}{2} - \frac{U_2^2}{2} = h_3 + \frac{W_3^2}{2} - \frac{U_3^2}{2} \Rightarrow \boldsymbol{h_{u2} = h_{u3}} =$: **Rothalpie**

⇒So rothalpy is preserved

▸ Case of axial turbomachine: $U_1 = U_2 \Rightarrow h_2 + \frac{W_2^2}{2} = h_3 + \frac{W_3^2}{2} \Rightarrow \boldsymbol{h_{R2} = h_{R3}}$: Relative total enthalpy

⇒So in the case of an axial turbomachine the relative total enthalpy is conserved

► Speed slowdown coefficient: We set: $\psi = \frac{W_3}{W_{3th}}$

with: C_1th: theoretical speed without losses and ψ: Speed slowdown coefficient

►Calculation of adiabatic losses: $\pi_r = h_3 - h_{3s} = \frac{W_{3th}^2}{2}(1-\psi^2)$

Siq_2=h_2-h_3s: static drop available in the mobile⇒ $\boldsymbol{\pi_r = \left(q_2 + \frac{W_2^2}{2}\right)(1-\psi^2)}$

►Coefficient of pressure loss in the Rotor: $\boldsymbol{\zeta_r = 1 - \frac{P_{R3}}{P_{R3s}}}$ **(ζ: zêta)**

3-4- Losses per remaining speed: (Fig3-5)

Several cases should be considered:

—during complete recovery of the output kinetic energy, the kinetic energy recovery coefficient m= 1.

—during a complete loss of output kinetic energy, we have m= 0.

—in intermediate cases, we do not completely lose the kinetic energy $C_3^2/2$; we therefore recover m $m\,C_3^2/2$.

⇒The loss is then: $\boldsymbol{\pi_v = (1-m)\frac{c_3^2}{2}}$

3-5-Isentropic and polytropic yield of a transformation:

▸For a turbine, the isothermal efficiency $\boldsymbol{\eta_{iso}}$, isentropic $\boldsymbol{\eta_s}$ and polytropic $\boldsymbol{\eta_{pol}}$ are:

$$\boldsymbol{\eta_{iso}} = \frac{W_{rD}}{W_{isoD}}, \qquad \eta_s = \frac{W_{rD}}{W_{sD}} \qquad \text{et} \qquad \eta_{pol} = \frac{W_{rD}}{W_{pD}} == \frac{(k-1)/k}{(\gamma-1)/\gamma}$$

So that the relationship between the two yields: $\boldsymbol{\eta_s} = \frac{W_{rD}}{W_{sD}} = \frac{\left[1-\left(\frac{P_{02}}{P_{01}}\right)^{\boldsymbol{\eta_{pol}}\,\frac{(\gamma-1)}{\gamma}}\right]}{\left[1-\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

▸On the other hand for a compressor:

$$\boldsymbol{\eta_{iso}} = \frac{W_{isoC}}{W_{rC}}, \qquad \eta_s = \frac{W_{sC}}{W_{rC}} \qquad \text{et} \qquad \eta_{pol} = \frac{W_{pC}}{W_{rC}} = \frac{\frac{\gamma-1}{\gamma}}{\frac{k-1}{k}} = \frac{(\gamma-1)/\gamma}{(k-1)/k}$$

Noticed:

►The polytropic yield η_{pol} considers that expansion or compression takes place following a series of infinitesimal steps. It is therefore a mathematical abstraction. This efficiency is independent of the pressure ratio between the inlet and outlet.

►Four different forms of isentropic efficiency (yields) can be identified:

1. Total to Total: This applies when the kinetic energies at the input and output are large.

2. Total to Static: When kinetic energy is available at the input and output is not useful;

3. Static to total: applies when there is no kinetic energy at the input, and the output is useful.

4. Static to static: applies when the kinetic energies at the input and output are insignificant.

3-6-Definition of Turbine stage aerodynamic efficiency: (Fig3-6).

Having neglected disk friction losses and leakage losses, we will use:

▬ Total to total yield of the stage; $\boldsymbol{\eta_{tt} = \frac{h_{01}-h_{03}}{h_{01}-h_{03s}}}$

it is the one which best represents the intrinsic quality of the turbine (m=1).

▬ Total static efficiency of the stage: $\boldsymbol{\eta_{ts} = \frac{h_{01}-h_{03}}{h_{01}-h_{3ss}}}$

When taking into account the loss per remaining speed (m=0).

● As we can find a relationship between the two yields: $\boldsymbol{\frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2W_T}}$

3-7- Aerodynamic efficiency as a function of losses in a turbine stage: (Fig3-6).

The yield is written: $\eta = \frac{h_{01}-h_{03}}{C_d^2/2} = \frac{U_2^2-U_3^2}{C_d^2} + \frac{C_2^2-C_3^2}{C_d^2} - \frac{W_2^2-W_3^2}{C_d^2}$

► Where $(\overline{\Delta h})$: the enthalpy drop available for the stage: $\overline{\Delta h} = \frac{C_d^2}{2} = h_{01} - h_{3ss} - m\frac{C_3^2}{2}$

With: C_d Being a fictitious speed, representing the available fall.

If we put: $\pi_s = h_2 - h_{2s} \approx h_{3s} - h_{3ss}$

► We can therefore write the yield as a function of losses in the form:

$$\eta = 1 - \frac{\pi_s}{C_d^2/2} - \frac{\pi_r}{C_d^2/2} - \frac{\pi_v}{C_d^2/2} = 1 - \xi^2\left[\frac{1-\varphi^2}{\varphi^2}\left(\frac{C_1}{U}\right)^2 + \frac{1-\psi^2}{\psi^2}\left(\frac{W_2}{U}\right)^2 + m\left(\frac{C_2}{U}\right)^2\right] \text{with: } \xi = \frac{U}{C_d}$$

3-8-Definition of the degree of reaction (Turbine):

We can propose two definitions of the degree of reaction R either by using:

—the available static drops: $\boldsymbol{R} = \frac{\boldsymbol{q_2}}{\boldsymbol{q_1}+\boldsymbol{q_2}}$ With: $q_2 = h_2 - h_{3s}$ and $q_1 = h_1 - h_{2s}$

—the real static drops: $\boldsymbol{R}^* = \frac{\boldsymbol{q_2^*}}{\boldsymbol{q_1^*}+\boldsymbol{q_2^*}}$ With: $q_2^* = h_2 - h_3$ and $q_1^* = h_1 - h_2$

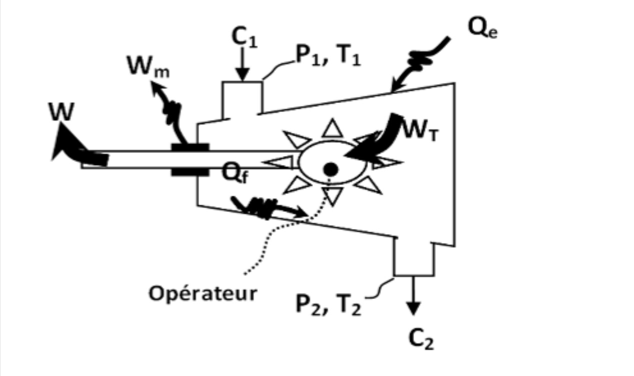

Fig-1-Détente

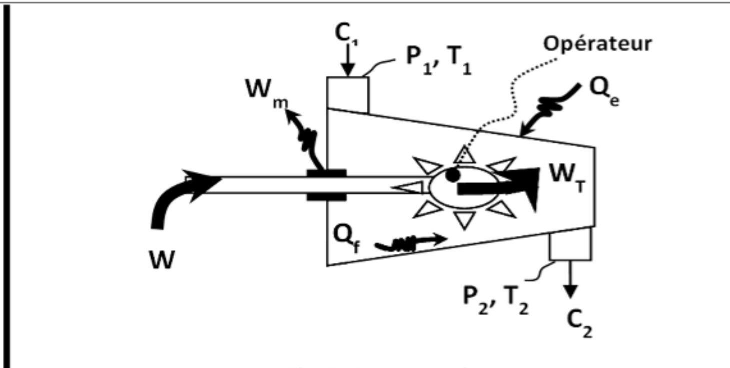

Fig-2-Compression

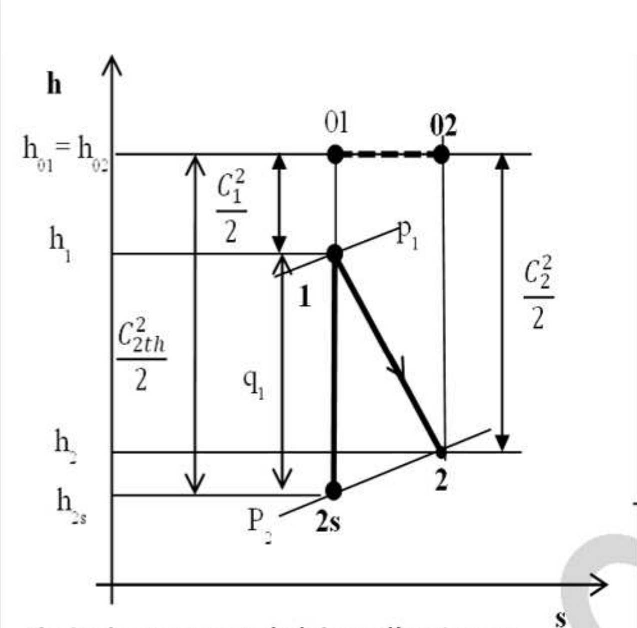

Fig-3-Diagramme enthalpique d'un Stator

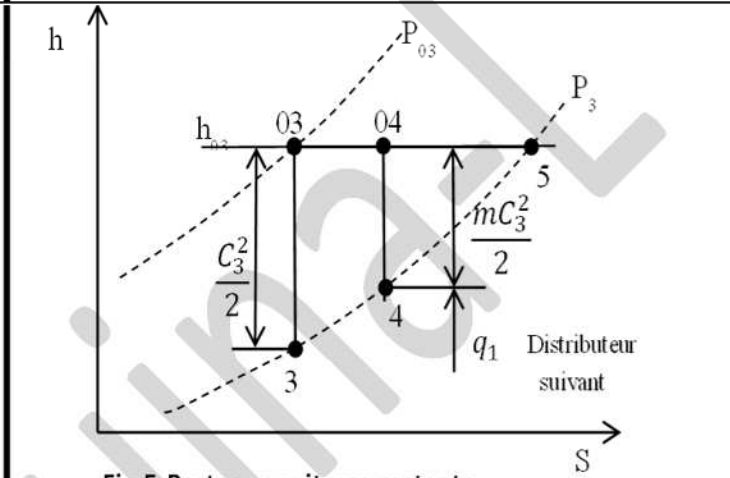

Fig-5-Pertes par vitesse restante

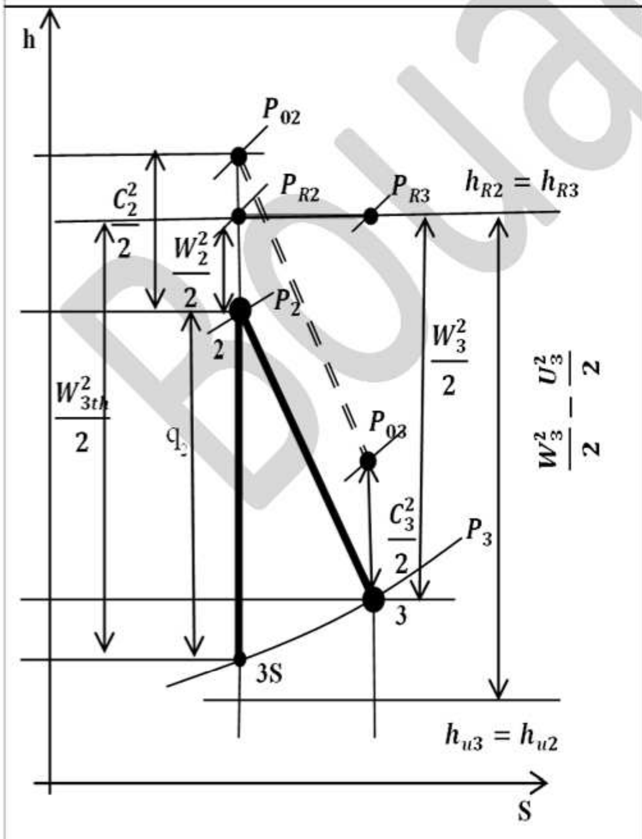

Fig-4-Diagramme enthalpique d'un Rotor

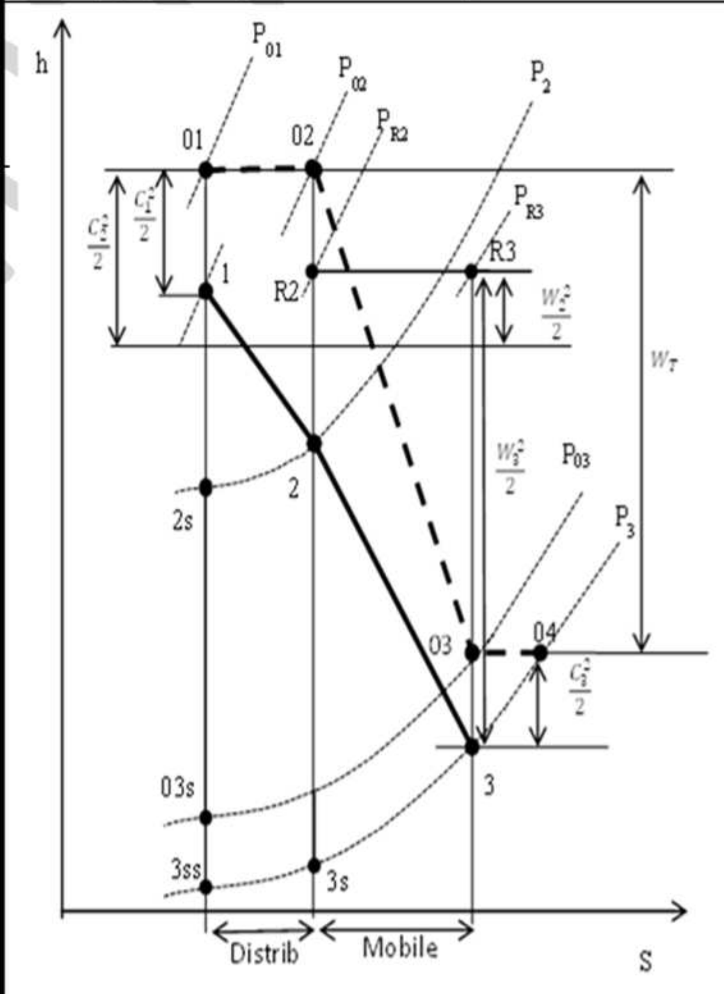

Fig-6-Diagramme enthalpique d'une Turbine Axiale