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Chapter 2.Static and total thermodynamic quantities in turbomachines

2-1-Types of thermodynamic transformations:

In turbomachines there are generally two types of transformations:

a)-compressing

b)-relaxing

These two types can be done in several ways:

Adiabatically, isentropically, isenthalpically, polytropically,…….

2-2-Visualization of a thermodynamic transformation:

1°)--for visualizations we generally use:

a)-the entropy diagram (T-S)

b)-the enthalpy diagram (h-S)

2°)--for the following interests:

-either for compression or relaxation, they directly visualize energy transfers

a)-quantitatively on the ordinate: (example: for an ideal gas we have: $\Delta h = C_p \Delta T$

b)-qualitatively on the abscissa: (for an ideal gas we have: $\Delta S_{12} = C_p ln\left(\frac{T_2}{T_1}\right) - rln\left(\frac{P_2}{P_1}\right)$)

2-3-Calculation of the parameters of a transformation:

1°)-isentropic transformation: (dS=0) of an ideal gas (PV=RT et $\frac{C_p}{C_v} = \gamma$):

if γ=const → **a)** $PV^{\gamma} = const$ **b)** $\frac{T}{P^{\frac{\gamma-1}{\gamma}}} = const$ **c)** $\frac{T}{V^{\gamma-1}} = const$

2°) polytropic transformation (real):

the real evolutions 1-2 are often described by the equation of a polytrope:

$P_1V_1^k = P_2V_2^k = PV^k = const \Rightarrow \frac{dP}{P} + k\frac{dV}{V} = 0$, with n: polytropic coefficient;

if k=const ⟹ **a)** $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^k$ **b)** $\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}}$ **c)** $\frac{T_1}{T_2} = \left(\frac{V_1}{V_2}\right)^{k-1}$

Noticed :

► if we know the initial and final parameters of a transformation, we can calculate the polytropic coefficient k: $k = \frac{lnP_1 - lnP_2}{lnV_1 - lnV_2}$

► The two types of transformations can be carried out in several ways depending on k: (Figure 2-0)

k = γ; isentropic compression or expansion (**reversible adiabatic**)

k > γ (**theoretical case**); compression with heat input or cooled expansion

k < γ; cooled compression or expansion with heat input

k = 1; isothermal compression or expansion

2-4-Definition of an absolute stagnation variable:

► The stopping conditions are the conditions that would be obtained by a fictitious transformation bringing the fluid back to zero speed isentropically (reversibly and without heat exchange) and without exchange of useful work.

-For the enthalpy at point 2 includes the static enthalpy, the kinetic energy and the potential energy at 1 hence its name total enthalpy:

$$H = h_* = h_0 = h + \frac{C^2}{2} + gz$$

For an ideal gas: : $h_0 = C_p T_0$

- The total temperature: $T_0 = T + \frac{C^2}{2C_P} + \frac{gz}{C_P}$

- Total pressure: $P_0 = P + \rho\frac{C^2}{2} + \rho gz$

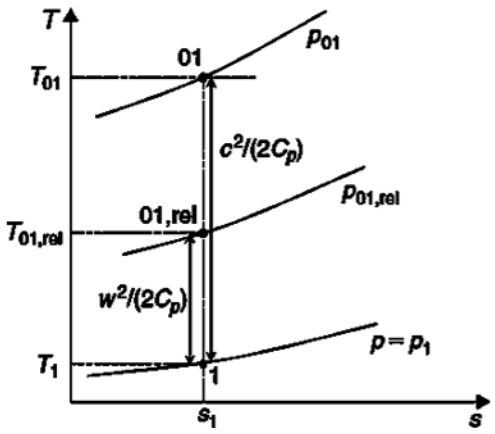

Fig.A : Relation entre la variable statique avec la variable d'arret absolue et relative sur un diagramme T-S

► **Generally neglects the potential energy which gives:**

-For the stagnation enthalpy: $H = h_* = h_0 = h + \frac{C^2}{2}$

-For an ideal gas: $h_0 = C_p T_0$

-For the stagnation temperature: : $T_0 = T + \frac{C^2}{2C_P}$

- Stagnation pressure and density: $P_0 = P + \rho\frac{C^2}{2}$

► **For an isentropic transformation:**

- Stagnation pressure and density: $P_0 = P\left(1 + \frac{C^2}{2C_P T}\right)^{\frac{\gamma}{\gamma-1}}$

- Stagnation density and $\rho_0 = \rho\left(1 + \frac{C^2}{2C_P T}\right)^{\frac{1}{\gamma-1}}$

► **By introducing the Mach number:** $M = \frac{C}{a} = \frac{C}{\sqrt{\gamma r T}} \Rightarrow \frac{C^2}{2C_P T} = \frac{\gamma-1}{2}M^2$

$$\Rightarrow T_0 = T\left[1 + \frac{\gamma-1}{2}M^2\right] \; ; \; P_0 = P\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}, \qquad \rho_0 = \rho\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$$

Or: $\frac{T_0}{T} = \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho_0}{\rho}\right)^{\gamma-1} = 1 + \frac{\gamma-1}{2}M^2$ (**Generator state or Barré Saint V equation**)

- The fluid speed for adiabatic flow: $\boldsymbol{C} = \left[\frac{2\gamma r}{\gamma-1}(T_0 - T)\right]^{1/2}$

- The flow reaches the maximum speed when T=0 : $\boldsymbol{C_{max}} = \left[\frac{2\gamma r T_0}{\gamma-1}\right]^{1/2}$

► **Critical state:** we speak of critical or sonic quantities when M=1:

$\frac{T_0}{T^*} = \left(\frac{P_0}{P^*}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho_0}{\rho^*}\right)^{\gamma-1} = \frac{\gamma+1}{2}$ and the speed: : $\boldsymbol{C^*} = a = \sqrt{\gamma r T^*} = \left[\frac{2\gamma r T_0}{\gamma+1}\right]^{1/2}$

- the case of entropy: $S = C_V log\left(\frac{P}{\rho r}\right) = S_0$: It does not depend on the benchmark considered.

2-5-Definition of a relative stagnation variable:

- The relative stagnation enthalpy: $H_R = h_R = h_R = h + \frac{W^2}{2}$

-Relative stagnation temperature: $T_R = T + \frac{W^2}{2C_P}$

2-6-Calculation of static enthalpy gain and drop:

If γ=Cst, R= Cst, k= Cst, we can calculate either the gain or drop in enthalpy in the case of an isentropic or real transformation:(+: compression (gains), -: expansion (drop))

1°)-for an isentropic transformation:

$$\pm(h_{2s}-h_1)=\pm C_p(T_{2s}-T_1)=\pm RT_1\frac{\gamma}{\gamma-1}\left[\Pi^{\frac{\gamma-1}{\gamma}}-1\right]$$

2°)-for a real transformation: Real (effective) work:

$$\pm(h_2-h_1)=\pm C_p(T_2-T_1)=\pm C_PT_1\left(\frac{T_2}{T_1}-1\right)=\pm RT_1\frac{\gamma}{\gamma-1}\left[\Pi^{\frac{\gamma-1}{\gamma}}-1\right]$$

With: * $C_P=R\frac{\gamma}{\gamma-1}$, *$\Pi=\Pi_C=\frac{P_2}{P_1}>1$: compression rate * $\Pi=\Pi_D=\frac{P_2}{P_1}<1$: expansion rate.

3°)-for a polytropic transformation:

The process can be described by the equation $PV^k=Cst \implies \frac{T_{2s}}{T_1}=\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$$\int_1^2(\pm dh)=\int_1^2(\pm VdP)=\pm\frac{k}{k-1}\frac{P_1}{\rho_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}-1\right]$$

2-7-Calculation of absolute total enthalpy gain and drop:

1°)-for an isentropic transformation: With: * $C_P=R\frac{\gamma}{\gamma-1}$,

The process can be described by the equation $PV^\gamma=Cst \implies \frac{T_{02s}}{T_{01}}=\frac{T_{02}}{T_{01}}=\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$

$$W_s=\pm(h_{02s}-h_{01})=\pm C_p(T_{02s}-T_{01})=\pm\frac{\gamma}{\gamma-1}\frac{P_{01}}{\rho_{01}}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

2°)-for a real transformation: Real (effective) work:

The process can be described by the equation $PV^k=Cst \implies \frac{T_{02s}}{T_{01}}=\frac{T_{02}}{T_{01}}=\left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}}$

$$W_r=\pm(h_{02}-h_{01})=\pm C_p(T_{02}-T_{01})=\pm\frac{\gamma}{\gamma-1}\frac{P_{01}}{\rho_{01}}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}}-1\right]$$

3°)-for a polytropic transformation: $W_p=\sum(\pm dh_s)>\pm(h_{2s}-h_1)$

The process can be described by the equation $PV^k=Cst \implies \frac{T_{02s}}{T_{01}}=\frac{T_{02}}{T_{01}}=\left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}}$

$$W_p=\int_{01}^{02}(\pm dh_s)=\int_{01}^{02}(\pm VdP)=\pm\frac{k}{k-1}V_{01}P_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{k-1}-1\right]=\pm\frac{k}{k-1}\frac{P_{01}}{\rho_{01}}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{k-1}{k}}-1\right]$$

Note1: for an isothermal transformation: (with: $\Delta h=0$ and $\rho_0=\frac{p_0}{rT}$)

The process can be described by the equation PV=Cst

The work on the tree is: $W_{st}=-T\Delta S=rTln\left(\frac{P_{02}}{P_{01}}\right)$ ***or*** $\partial W_{st}=\frac{\partial p_t}{\rho_t}=rT\frac{\partial p_0}{p_0} \implies W_{st}=rTln\left(\frac{p_{02}}{p_{01}}\right)$

Note 2: - for compression the technical work: $W_{st}<(Wp)_k<\gamma<\mathbf{Ws<(Wp)_{k>\gamma}}$

- for relaxation the work is in order: $|Wst|>\mathbf{|(Wp)_k<\gamma|>|Ws|}>|(Wp)_{k>\gamma}|$

Thus, we will always seek to cool the gas during compression (never to heat it), we will often seek to heat a gas during expansion (never to cool it, except for possible questions of resistance of the materials to the temperature).

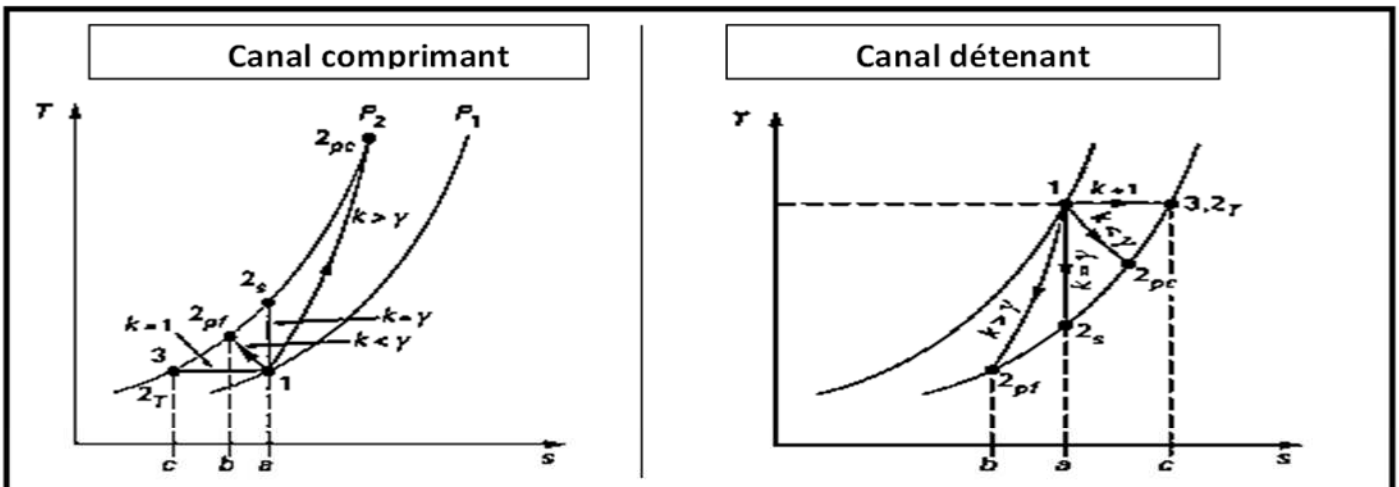

Fig-2-0- :- Visualizing isentropic and polytropic transformations on an entropy diagram **(T-s)**

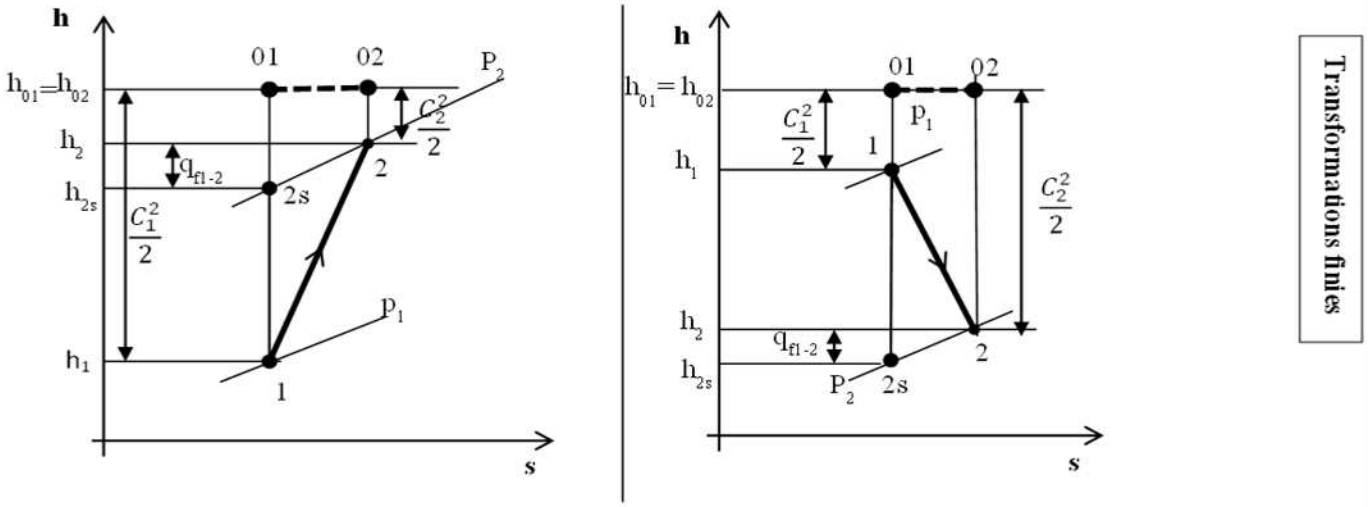

Fig-2-1- :-Transformation adiabatiques du fluide dans les canaux fixes

Transformations finies

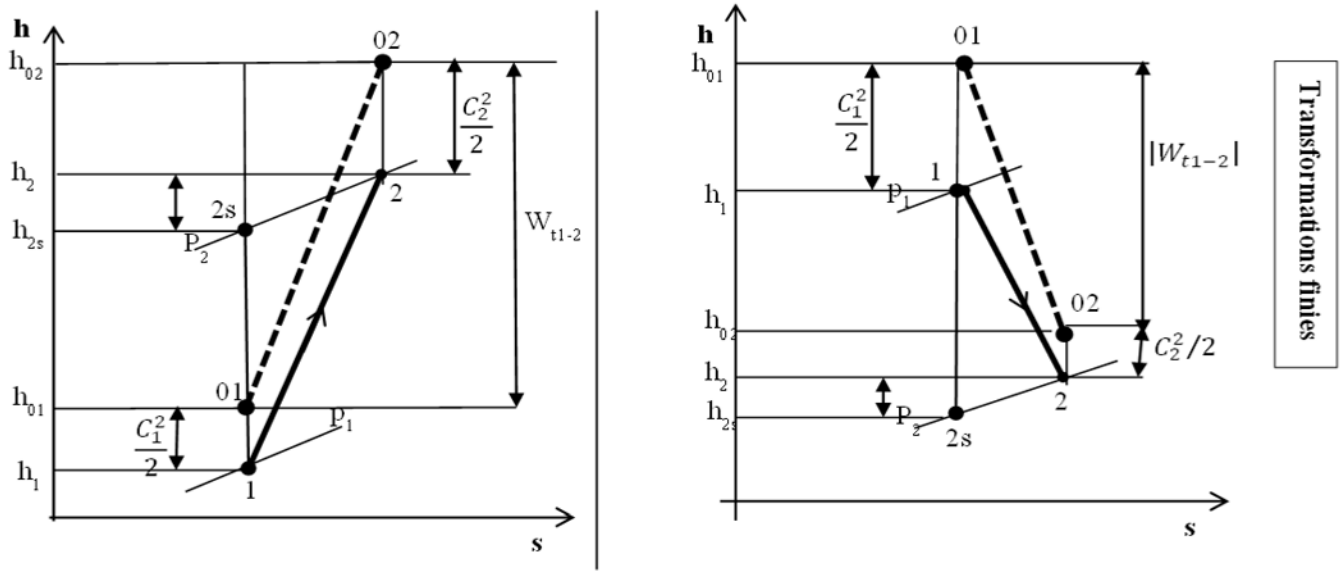

Fig-2-2- :-Transformation adiabatiques du fluide dans les canaux mobiles

Transformations finies

Etat thermodynamique du gaz

Energie totale du gaz