SOLUTION OF EXERCISE 1 (10 pts)

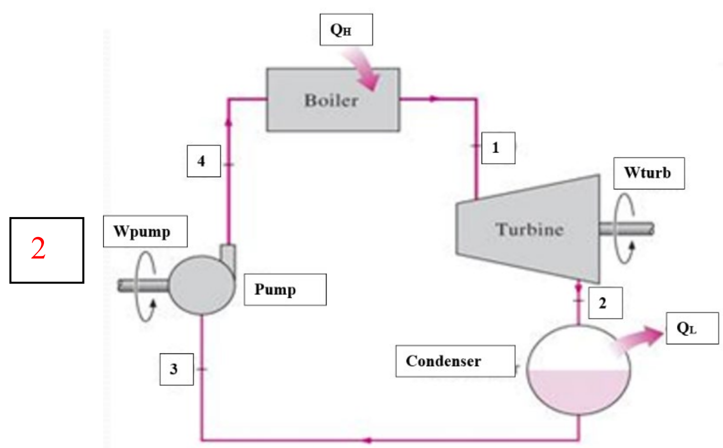
The Hirn cycle is a Rankine cycle with superheating. It includes the following transformations:
- isentropic expansion from pressure P_1 to pressure P_2, in a turbine. The steam is dry at the inlet of the turbine, represented by point 1 on the entropic diagram
- condensation in the condenser at constant P_2 pressure and T2 temperature, up to the saturated liquid state.
- isentropic compression of liquid water from P_2 to P_1 in a pump;
- heating at constant pressure P_1, then vaporization at the same pressure in the boiler.

At the outlet of the boiler, the fluid is located at point 1 and its temperature is $T_1 = 350$ °C.
Data: $P_1 = 3$ MPa and $P_2 = 0.075$ MPa.

1. Represent the schematic diagram of this steam engine installation, indicating the direction of travel of the fluid and numbering the position of the fluid at the inlets and outlets of the various elements (boiler, condenser, pump, turbine).
2. Using the data from the thermodynamic tables of water, determine the mass enthalpy of the fluid at the turbine outlet.
3. Calculate the amount of heat q exchanged in the boiler between the outside environment and 1 kg of water.
4. Calculate the quantity of heat exchanged in the condenser between the external environment and 1 kg of water.
5. Calculate the thermal efficiency of the cycle.

Given Data:

- $P_1 = 3$ MPa,
- $P_2 = 0.075$ MPa,
- $T_1 = 350$°C.



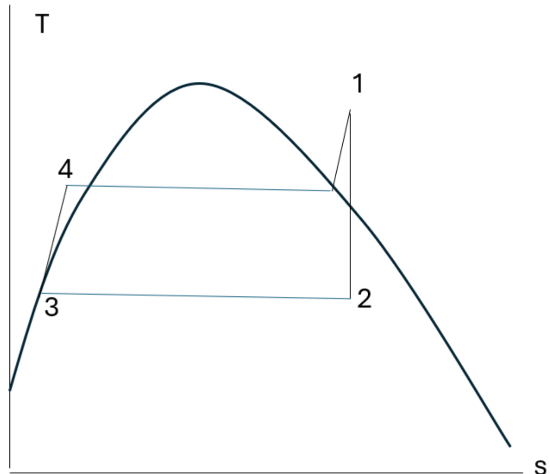
1. Schematic Diagram of the Steam Engine Installation

The cycle consists of four main components:
- **Boiler**: Heats and vaporizes water from point 4 to point 1.
- **Turbine**: Isentropic expansion from point 1 to point 2.
- **Condenser**: Condenses steam from point 2 to point 3 at constant pressure P_2.
- **Pump**: Isentropically compresses liquid water from point 3 to point 4.

Flow direction: 1 → Turbine → 2 → Condenser → 3 → Pump → 4 → Boiler → 1.

Shape of the Cycle on the T-s Diagram

- **1 → 2**: Isentropic expansion (vertical line downward).
- **2 → 3**: Condensation at constant pressure P_2 (horizontal line to the left).
- **3 → 4**: Isentropic compression (vertical line upward).
- **4 → 1**: Heating and vaporization at constant pressure P_1 (horizontal line to the right).



2. Mass Enthalpy at the Turbine Outlet (Point 2)

Using **steam tables**:

	P = 2.50 MPa (223.95°C)				P = 3.00 MPa (233.85°C)				P = 3.50 MPa (242.56°C)			
Sat.	0.07995	2602.1	2801.9	6.2558	0.06667	2603.2	2803.2	6.1856	0.05706	2603.0	2802.7	6.1244
225	0.08026	2604.8	2805.5	6.2629								
250	0.08705	2663.3	2880.9	6.4107	0.07063	2644.7	2856.5	6.2893	0.05876	2624.0	2829.7	6.1764
300	0.09894	2762.2	3009.6	6.6459	0.08118	2750.8	2994.3	6.5412	0.06845	2738.8	2978.4	6.4484
350	0.10979	2852.5	3127.0	6.8424	0.09056	2844.4	3116.1	6.7450	0.07680	2836.0	3104.9	6.6601
400	0.12012	2939.8	3240.1	7.0170	0.09938	2933.6	3231.7	6.9235	0.08456	2927.2	3223.2	6.8428

- At $P_1 = 3$ MPa and $T_1 = 350$°C:
  $\boldsymbol{h_1 \approx 3116.1}$ **kJ/kg**, 2
  $s_1 \approx 6.7450$ kJ/kg·K.

  At $P_2 = 0.075$ MPa,

h

h_f h_g s_f

s_g

75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545

h_f=384.44kJ/kg , h_g=2662.4kJ/kg , s_f=1.2132kJ/kg.K , s_g=7.4558kJ/kg.K

$$x = \frac{s_1 - s_f}{s_g - s_f} = \frac{h_1 - h_f}{h_g - h_f} = 0.866 \rightarrow h_2 = 2403 kJ/kg$$ 2

3. Heat Exchanged in the Boiler (Q_H)

$$Q_H = h_1 - h_3$$

Therefore, $Q_H = 3116.1 - 384.44 = 2731.66$ kJ/kg. 2

4. Heat Exchanged in the Condenser (Q_L)

$$Q_L = h_2 - h_3$$

- $h_3 \approx 384.4$ kJ/kg (saturated liquid at P_2).
- Therefore, $Q_L = 2403 - 384.44 = 2018.56$ kJ/kg. 2

5. Thermal Efficiency of the Cycle (η)

$$\eta = \frac{W_{\text{net}}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = \frac{2731.66 - 2018.56}{2731.66} =$$

Therefore, $\eta = 0.26 = 26\%$ 2

SOLUTION OF EXERCISE 2 (10 pts)

The air-fuel mixture of a four-stroke engine is admitted at a pressure $p_1 = 100$ kPa and a temperature $T_1 = 288$ K.
The gas constant r of the mixture is 287 J.kg^{-1}.K^{-1}.
The ratio of specific heats c_p and c_v is 1.4.
The engine displacement is 500 cm^3.
Its volumetric compression ratio is 7.5.
It runs at a speed of 3600 rpm, and its mechanical efficiency is 85%.
During isochoric combustion, the temperature rises by 1800 K.

1. Calculate the volumes V_1 (at bottom dead center) and V_2 (at top dead center).
2. Determine the different states (1-2-3-4) of the thermodynamic cycle.
3. Sketch the shape of this cycle.
4. Calculate the work done per cycle.
5. Calculate the mean effective pressure of the cycle.
6. Calculate the thermodynamic power of this engine.
7. Calculate its thermodynamic efficiency.
8. Calculate the calorific power it absorbs.
9. Calculate its actual mechanical power.
10. Calculate its overall efficiency.

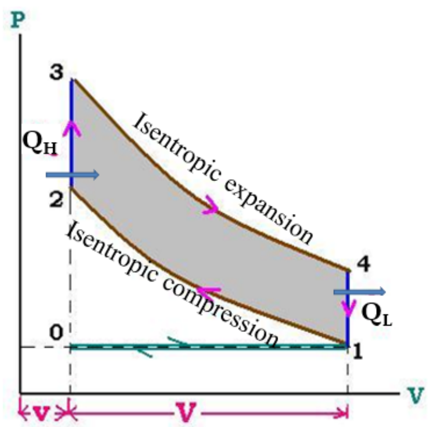
يُعمَد خليط الهواء-الوقود لمحرك رباعي الأشواط عند ضغط $p_1 = 100$ kPa ودرجة حرارة $T_1 = 288$ K.
ثابت الغاز r للخليط يساوي 287 J.kg^{-1}.K^{-1}.
نسبة السعات الحرارية c_p و c_v تساوي 1.4.
سعة المحرك 500 سم^3.
نسبة الانضغاط الحجمي تساوي 7.5.
يدور المحرك بسرعة 3600 دورة/دقيقة، وكفاءته الميكانيكية 85%.
خلال الاحتراق عند حجم ثابت، ترتفع درجة الحرارة بمقدار 1800 كلفن.

1. احسب الحجمين V_1 (عند النقطة الميتة السفلى) و V_2 (عند النقطة الميتة العليا).
2. حدد الحالات المختلفة (1-2-3-4) للدورة الترموديناميك.
3. ارسم شكل هذه الدورة.
4. احسب الشغل المبذول في كل دورة.
5. احسب الضغط المتوسط الفعال للدورة.
6. احسب القدرة الترموديناميكية لهذا المحرك.

7. احسب كفاءته الترموديناميكية.
8. احسب القدرة الحرارية التي يمتصها.
9. احسب قدرته الميكانيكية الحقيقية.
10. احسب كفاءته العامة.

Given data

- Admission pressure, $p_1 = 100\,\text{kPa}$
- Admission temperature, $T_1 = 288\,\text{K}$
- Gas constant, $r = 287\,\text{J/kg·K}$
- Ratio of specific heats, $\gamma = \frac{c_p}{c_v} = 1.4$
- Engine displacement, $V_{\text{displacement}} = 500\,\text{cm}^3 = 0.5 \times 10^{-3}\,\text{m}^3$
- Compression ratio, $\tau = \frac{V_1}{V_2} = 7.5$
- Engine speed, $N = 3600\,\text{rpm}$
- Mechanical efficiency, $\eta_m = 85\% = 0.85$
- Temperature rise during isochoric combustion, $\Delta T = 1800\,\text{K}$



1. Calculate the volumes V_1 and V_2

The compression ratio is given by:

$$\tau = \frac{V_1}{V_2} = 7.5$$

The displacement volume is given by:

$$V_1 - V_2 = 0.5 \times 10^{-3}\,\text{m}^3$$

Using the compression ratio:

$$V_1 = 7.5\,V_2$$

Substitute V_1 in the displacement equation:

$$7.5V_2 - V_2 = 0.5 \times 10^{-3}$$
$$6.5V_2 = 0.5 \times 10^{-3}$$

$$\boldsymbol{V_2} = \frac{0.5 \times 10^{-3}}{6.5} \approx \mathbf{7.69 \times 10^{-5}\,m^3}$$ 0.25

$$\boldsymbol{V_1} = 7.5 \times 7.69 \times 10^{-5} \approx \mathbf{5.77 \times 10^{-4}\,m^3}$$ 0.25

2. Determine the different states (1-2-3-4) of the thermodynamic cycle

The cycle follows the **Otto cycle**:

- **1-2**: Isentropic compression
- **2-3**: Isochoric heat addition (combustion)
- **3-4**: Isentropic expansion
- **4-1**: Isochoric heat rejection

Calculations for each state:

- **State 1**: $p_1 = 100\,\text{kPa}$, $T_1 = 288\,\text{K}$, $V_1 = 5.77 \times 10^{-4}\,\text{m}^3$
- **State 2**: After isentropic compression, use the relation:

$$\boldsymbol{T_2} = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 288 \times (7.5)^{0.4} \approx \mathbf{645\,K}$$ 0.5

$$\boldsymbol{p_2} = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100 \times (7.5)^{1.4} \approx \mathbf{1679\,kPa}$$ 0.5

- **State 3**: After isochoric heat addition, the temperature rises by 1800 K:

$$\boldsymbol{T_3} = T_2 + 1800 = 645 + 1800 = \mathbf{2445\,K}$$ 0.5

$$\boldsymbol{p_3} = p_2 \times \frac{T_3}{T_2} = 1679 \times \frac{2445}{645} \approx \mathbf{6365\,kPa}$$ 0.5

- **State 4**: After isentropic expansion, use the relation:

$$\boldsymbol{T_4} = \boldsymbol{T_3}\left(\frac{V_3}{V_4}\right)^{\gamma-1} = 2445 \times \left(\frac{1}{7.5}\right)^{0.4} \approx \mathbf{1092\,K}$$ 0.5

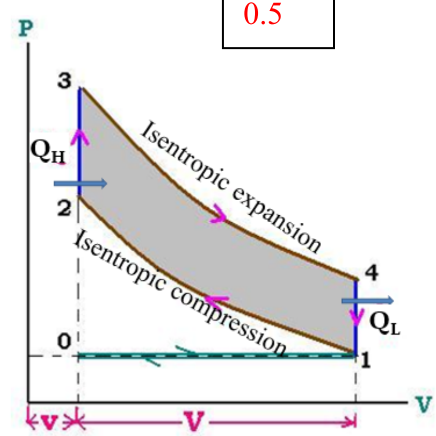
$$\boldsymbol{p_4} = p_3\left(\frac{V_3}{V_4}\right)^{\gamma} = 6365 \times \left(\frac{1}{7.5}\right)^{1.4} \approx \mathbf{379\,kPa}$$ 0.5

3. Sketch the cycle

The cycle is represented on a $p - V$ diagram:

- **1-2**: Isentropic compression curve.
- **2-3**: Vertical line (isochoric) upwards.
- **3-4**: Isentropic expansion curve.
- **4-1**: Vertical line (isochoric) downwards.

1



4. Calculate the work done per cycle

The net work done per cycle is given by:

Isentropic Process: For an isentropic process of an ideal gas, the relationship between pressure and volume is given by: $PV^{\gamma} = \text{constant}$ where γ is the ratio of specific heats ($\gamma = \frac{c_p}{c_v}$).

Work during an Isentropic Process: The work W done by or on the gas during an isentropic process is given by: $W_{1-2} = \int_{V_1}^{V_2} P\ dV$

sentropic Compression (1-2):

Integration: Using the relation $PV^{\gamma} = \text{constant}$, we can express P as a function of V: $P = \frac{\text{constant}}{V^{\gamma}}$

Therefore, $W_{1-2} = \int_{V_1}^{V_2} \frac{\text{constant}}{V^{\gamma}}\, dV$

After integration, we get: $W_{1-2} = \frac{\text{constant}}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$

Since the constant is $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$, we can write: $W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-\gamma}$

During this phase, work is done on the gas (compression). This work is negative because it is provided by the piston to the gas. $W_{1-2} = \int_{V_1}^{V_2} P\ dV$

For an isentropic transformation of an ideal gas, we can use the relation: $W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-\gamma}$ where γ is the ratio of specific heats ($\gamma = \frac{c_p}{c_v}$).

Isochoric Combustion (2-3):

During this phase, there is no work done because the volume remains constant ($dV = 0$). $W_{2-3} = 0$

Isentropic Expansion (3-4):

During this phase, the gas does work as it expands. This work is positive because it is provided by the gas to the piston. $W_{3-4} = \int_{V_3}^{V_4} P\ dV$

For an isentropic transformation, we can use the relation: $W_{3-4} = \frac{P_3V_3 - P_4V_4}{1-\gamma}$

Isochoric Cooling (4-1):

During this phase, there is no work done because the volume remains constant ($dV = 0$). $W_{4-1} = 0$

Net Work (W_{net}):

The net work is the sum of the work done during all phases of the cycle: $W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$

Since $W_{2-3} = 0$ and $W_{4-1} = 0$, we have: $W_{\text{net}} = W_{3-4} + W_{1-2}$

Using the relations for isentropic transformations, we get: $W_{\text{net}} = \frac{P_3V_3 - P_4V_4}{1-\gamma} + \frac{P_2V_2 - P_1V_1}{1-\gamma}$

Simplification:

For an Otto cycle, $V_3 = V_2$ and $V_4 = V_1$, so: $W_{\text{net}} = \frac{P_3V_2 - P_4V_1}{1-\gamma} + \frac{P_2V_2 - P_1V_1}{1-\gamma}$

Using the calculated values:

$$\boldsymbol{W_{net} \approx 450\,\mathrm{J}}$$

1

5. Calculate the mean effective pressure of the cycle

The mean effective pressure is given by:

$$\boldsymbol{p_m} = \frac{W_{\text{net}}}{V_1 - V_2} = \frac{450}{0.5 \times 10^{-3}} = \mathbf{900\,kPa}$$

1

6. Calculate the thermodynamic power of the engine

1

The thermodynamic power is given by:

$$P_{\text{th}} = W_{\text{net}} \times \frac{N}{2 \times 60}$$

$$\boldsymbol{P_{\text{th}}} = 450 \times \frac{3600}{120} = 13500\ \text{W} = \mathbf{13.5\,kW}$$

1

7. Calculate the thermodynamic efficiency

The thermodynamic efficiency is given by:

$$\boldsymbol{\eta_{\text{th}}} = 1 - \frac{1}{\tau^{\gamma-1}} = 1 - \frac{1}{7.5^{0.4}} \approx 0.55 \approx \mathbf{55\%}$$

0.5

8. Calculate the calorific power absorbed

The calorific power absorbed is given by:

$$\boldsymbol{Q_{\text{in}}} = \frac{\boldsymbol{P_{\text{th}}}}{\boldsymbol{\eta_{\text{th}}}} = \frac{13.5}{0.55} \approx \mathbf{24.5\,kW}$$

0.5

9. Calculate the actual mechanical power

The actual mechanical power is given by:

$$\boldsymbol{P_{\text{mec}}} = P_{\text{th}} \times \eta_m = 13.5 \times 0.85 \approx \mathbf{11.475\,kW}$$

0.5

10. Calculate the overall efficiency

The overall efficiency is given by:

$$\boldsymbol{\eta_{\text{global}}} = \eta_{\text{th}} \times \eta_m = 0.55 \times 0.85 \approx 0.48 \approx \mathbf{47\%}$$

0.5