GUIDED EXERCISES CHAPTER 1, BACHELOR'S YEAR 3 ENERGY

Course reminders

1. **Compression Ratio (r):**
   - Defined as the ratio of the maximum volume to the minimum volume in the cylinder:
   $$r = \frac{V_1}{V_2}$$
   - Where V_1 is the volume at the start of compression, and V_2 is the volume at the end of compression.
2. **Cut-off Ratio (r_c):**
   - Defined as the ratio of volumes at the end and start of the heat addition process (for Diesel and Sabathé cycles):
   $$r_c = \frac{V_3}{V_2}$$
   - Where V_3 is the volume at the end of heat addition, and V_2 is the volume at the start of heat addition.
3. **Pressure Ratio (r_p):**
   - Defined as the ratio of pressures at the end and start of the constant-volume heat addition process (for Sabathé cycles):
   $$r_p = \frac{P_3}{P_2}$$
   - Where P_3 is the pressure at the end of constant-volume heat addition, and P_2 is the pressure at the start of heat addition.

EXERCISES

1. Carnot Cycle Exercises

Exercise 1.1: Ideal Carnot Cycle

Given:

- Hot reservoir temperature $T_H = 800\,\mathrm{K}$
- Cold reservoir temperature $T_C = 300\,\mathrm{K}$
- Heat input $Q_H = 1000\,\mathrm{kJ}$

Questions:

1. Calculate the thermal efficiency of the cycle.

2. Determine the work output W.
1. Calculate the heat rejected Q_C.

Exercise 1.2: Carnot Engine with Power Output

Given:

- Hot reservoir temperature $T_H = 700\,\mathrm{K}$
- Cold reservoir temperature $T_C = 280\,\mathrm{K}$
- Engine power output $= 5\,\mathrm{kW}$

Questions:

1. Calculate the thermal efficiency.
2. Determine the heat input rate $\dot{Q}_H$ required to produce this power.

Exercise 1.3: Carnot Cycle with Ideal Gas

Given:

- Ideal gas with $R = 0.287\,\mathrm{kJ/kg{\cdot}K}$, $\gamma = 1.4$
- Initial pressure $P_1 = 1\,\mathrm{bar}$, initial temperature $T_1 = 300\,\mathrm{K}$
- $T_2 = 600\,\mathrm{K}$, $T_3 = 1200\,\mathrm{K}$

Questions:

1. Determine P_2, P_3, and P_4.
2. Calculate the net work produced per kg of gas.

2. Otto Cycle Exercises

Exercise 2.1: Ideal Otto Cycle

Given:

- Initial pressure $P_1 = 1\,\mathrm{bar}$, initial temperature $T_1 = 300\,\mathrm{K}$
- **Compression ratio** $r = 8$
- Heat added $Q_{23} = 800\,\mathrm{kJ/kg}$
- $\gamma = 1.4$, $c_v = 0.718\,\mathrm{kJ/kg{\cdot}K}$

Questions:

1. Calculate T_2 and P_2.
2. Determine T_3 and P_3.

3. Calculate the thermal efficiency of the cycle.

Exercise 2.2: Otto Cycle with Mass of Air

Given:

- Mass of air $m = 1\,\text{kg}$
- Initial pressure $P_1 = 1\,\text{bar}$, initial temperature $T_1 = 298\,\text{K}$
- **Compression ratio** $r = 9$
- Heat added $Q_{23} = 1000\,\text{kJ}$

Questions:

1. Calculate the net work output.
2. Determine the mean effective pressure (MEP).

Exercise 2.3: Effect of Compression Ratio

Given:

- Initial temperature $T_1 = 300\,\text{K}$, heat added $Q_{23} = 750\,\text{kJ/kg}$
- $\gamma = 1.4$

Questions:

1. Calculate the thermal efficiency for **compression ratios** $r = 6$**,** $r = 8$**, and** $r = 10$.
2. Discuss the effect of the compression ratio on efficiency.

3. Diesel Cycle Exercises

Exercise 3.1: Ideal Diesel Cycle

Given:

- Initial pressure $P_1 = 1\,\text{bar}$, initial temperature $T_1 = 300\,\text{K}$
- **Compression ratio** $r = 16$
- **Cut-off ratio** $r_c = 2$
- $\gamma = 1.4$, $c_p = 1.005\,\text{kJ/kg·K}$

Questions:

1. Calculate T_2, P_2, T_3, and P_3.
2. Determine the heat added Q_{23}.

3. Calculate the thermal efficiency.

Exercise 3.2: Diesel Cycle with Power Output

Given:

- Engine power output $= 100\,\text{kW}$
- Mass flow rate of air $m = 0.5\,\text{kg/s}$
- **Compression ratio** $r = 14$
- **Cut-off ratio** $r_c = 1.8$
- $\gamma = 1.4$, $c_p = 1.005\,\text{kJ/kg·K}$

Questions:

1. Calculate the heat input rate $\dot{Q}_{23}$.
2. Determine the thermal efficiency.

Exercise 3.3: Comparison with Otto Cycle

Given:

- **Compression ratio** $r = 12$
- **Cut-off ratio** $r_c = 2$
- $\gamma = 1.4$

Questions:

1. Calculate the thermal efficiency for a Diesel cycle.
2. Calculate the thermal efficiency for an Otto cycle with the same compression ratio.
3. Compare the two efficiencies.

4. Sabathé (Dual) Cycle Exercises

Exercise 4.1: Ideal Sabathé Cycle

Given:

- Initial pressure $P_1 = 1\,\text{bar}$, initial temperature $T_1 = 300\,\text{K}$
- **Compression ratio** $r = 10$
- **Pressure ratio** $r_p = 1.5$
- **Cut-off ratio** $r_c = 1.8$

- $\gamma = 1.4$, $c_p = 1.005\,\text{kJ/kg·K}$, $c_v = 0.718\,\text{kJ/kg·K}$

Questions:

1. Calculate P_2, T_2, P_3, T_3, P_4, and T_4.
2. Determine the heat added Q_{23} and Q_{34}.
3. Calculate the thermal efficiency.

Exercise 4.2: Sabathé Cycle with Power Output

Given:

- Engine power output $= 150\,\text{kW}$
- Mass flow rate of air $m = 0.3\,\text{kg/s}$
- **Compression ratio** $r = 11$
- **Pressure ratio** $r_p = 1.6$
- **Cut-off ratio** $r_c = 2$
- $\gamma = 1.4$, $c_p = 1.005\,\text{kJ/kg·K}$

Questions:

1. Calculate the total heat input rate $\dot{Q}_{in}$.
2. Determine the thermal efficiency.

Exercise 4.3: Comparison of Cycles

Given:

- **Compression ratio** $r = 12$
- **Pressure ratio** $r_p = 1.5$
- **Cut-off ratio** $r_c = 2$
- $\gamma = 1.4$

Questions:

1. Calculate the thermal efficiencies for Otto, Diesel, and Sabathé cycles.
2. Discuss the advantages and disadvantages of each cycle.

5. Synthesis Exercises

Exercise 5.1: Comparative Study

Given:

- Initial temperature $T_1 = 300\,\text{K}$, initial pressure $P_1 = 1\,\text{bar}$
- **Compression ratio** $r = 10$
- **Cut-off ratio** $r_c = 2$
- **Pressure ratio** $r_p = 1.5$
- $\gamma = 1.4$

Questions:

1. Sketch the $P - V$ diagrams for Otto, Diesel, and Sabathé cycles.
2. Compare the thermal efficiencies of the three cycles.

Exercise 5.2: Efficiency Optimization

Given:

- Initial temperature $T_1 = 300\,\text{K}$, heat input $Q_{in} = 1000\,\text{kJ/kg}$
- $\gamma = 1.4$

Questions:

1. For a Sabathé cycle, determine the optimal values of r, r_p, and r_c to maximize efficiency.
2. Discuss practical constraints to achieve this optimal efficiency.

Exercise 5.3: Industrial Application

Given:

- An industrial Diesel engine has a measured efficiency of 40% with **compression ratio** $r = 14$ and **cut-off ratio** $r_c = 2$.
- $\gamma = 1.4$

Questions:

1. Compare this efficiency with the theoretical efficiency of an ideal Diesel cycle.
2. Propose improvements to increase the real efficiency.