Chapter 05

Correlation of signals

5.1 Introduction

In signal processing, it is often necessary to compare two signals, this is done in several ways. The most commonly used method is to measure their similarity in shape and position by translating one of the signals relative to the other mathematically. This operation is a scalar product.

Correlation functions (autocorrelation and cross-correlation) are used in the spectral analysis of finite energy or finite average power signals. Physically, the correlation function is obtained by shifting one of the signals by multiplying the shifted signal by the other signal and then integrating the product obtained. The cross-correlation function also allows us to measure the degree of resemblance between two signals.

5.2 Definitions

It is used for the purpose of signal interpretation (information extraction).

What is correlation?

- Comparison of two signals with each other.
- This function acts mainly in the time domain.
- We are interested in signals of finite energy (finite time support), or in signals with finite average power.

5.3 Autocorrelation function of finite energy signals

It is given by:

$$R_s(\tau) = \int_{-\infty}^{+\infty} s^*(t)\, s(t+\tau)dt$$

For a real signal:

$$R_s(\tau) = \int_{-\infty}^{+\infty} s(t)\, s(t+\tau)dt$$

5.4 Properties of correlation

1) $R_s(0) = \int_{-\infty}^{+\infty} s^*(t)\, s(t)dt = E_{s(t)}$ → (Energy of a signal).

2) $R_s(\tau) = R_s(\tau)$ → (Even function).

3)

$$TF\{R_s(\tau)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s(t).s(t+\tau)dt.e^{-j2\pi f\tau}d\tau$$

$$= \int_{-\infty}^{+\infty} s(t)\left[\int_{-\infty}^{+\infty} s(t+\tau)e^{-j2\pi f\tau}d\tau\right].dt$$

We have:

$$S(f)e^{-j2\pi ft} = \left[\int_{-\infty}^{+\infty} s(t+\tau)e^{-j2\pi f\tau}d\tau\right]$$

So:

$$TF\{R_s(\tau)\} = S(f)\int_{-\infty}^{+\infty} s(t)e^{j2\pi ft}dt$$

We have:

$$S^*(f) = \int_{-\infty}^{+\infty} s(t)e^{j2\pi ft}dt$$

Eventually:

$$TF\{R_s(\tau)\} = S(f).S^*(f)$$

$TF\{R_s(\tau)\} = \|S(f)\|^2 = \varphi_s(f)$ → (Energy Spectral Density (ESD)).

4)

$$s(t) = e(t) * h(t) \Rightarrow S(f) = E(f).H(f)$$

$$\|S(f)\|^2 = \|E(f).H(f)\|^2$$

$$= \|E(f)\|^2.\|H(f)\|^2$$

$$\varphi_s(f) = \|E(f)\|^2.\|H(f)\|^2$$

$$\varphi_s(f) = \varphi_e(f).\|H(f)\|^2$$

Example 5.1:

Consider the following signal: $s(t) = e^{-a.t}.u(t) \quad with \quad a > 0$

1- Graphical representation of this signal.

2- Calculate the autocorrelation function.

3- Graphical representation of the autocorrelation function.

4- Calculate the energy spectral density.

5.5 Cross-correlation function for finite energy signals

It is given by:

$$R_{12}(\tau) = \int_{-\infty}^{+\infty} s_1(t).s_2(t+\tau)dt$$

$$\varphi_{12}(f) = TF\{R_{12}(\tau)\} \qquad \rightarrow \text{(Inter-spectral energy density).}$$

5.6 Autocorrelation function for signals with finite mean power

It is given by:

$$R_p(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{(T)} s(t).s(t+\tau)dt\,[W] \qquad \rightarrow \text{(Power).}$$

$$TF\{R_p(\tau)\} = \int_{-\infty}^{+\infty} R_p(\tau)e^{-j2\pi f\tau}d\tau$$

$$TF\{R_p(\tau)\} = \phi(f) \qquad \rightarrow \text{(Power spectral density).}$$

$$R_p(\tau) = \int_{-\infty}^{+\infty} \phi(f)e^{j2\pi f\tau}d\tau$$

$$R_p(0) = p = \int_{-\infty}^{+\infty} \phi(f)df$$

5.7 Cross-correlation function for signals with finite mean power

It is given by:

$$R_{12p}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{(T)} s_1(t).s_2(t+\tau)dt$$

$$TF\{R_{12p}(\tau)\} = \phi_{12p}(f) = \int_{-\infty}^{+\infty} R_{12p}(\tau)e^{-j2\pi f\tau}d\tau \qquad \rightarrow \text{(Inter-spectral power density).}$$

Properties:

$$\phi_s(f) = \phi_e(f).\|H(f)\|^2$$

$$p_s = \int_{-\infty}^{+\infty} \phi_s(f)df$$