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Module : Probabilities and Statistics

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Solutions to the Directed Works Series n°3

Exercice n°1 :

In some countries, car number plates begin with a letter of the alphabet, followed by five digits. Calculate how many number plates are possible if :

a) The first digit following the letter cannot be 0.

b) The first letter cannot be O or I and the first digit cannot be 0 or 1.

Solution :

Letter

a) Letter choice (26 options : A–Z)
First digit choice (9 options : 1–9)
second digit choice (10 options : 0–9)
third digit choice (10 options : 0–9)
fourth digit choice (10 options : 0–9)
fifth digit choice (10 options : 0–9)
according to the product rule
Total = $26 \times 9 \times 10\times 10\times 10\times 10 = 2{,}340{,}000$

b) Letter choice (24 options : excluding O and I) ↓ First digit choice (8 options : 2–9)
second digit choice (10 options : 0–9)
third digit choice (10 options : 0–9)
fourth digit choice (10 options : 0–9)
fifth digit choice (10 options : 0–9)
according to the product rule
Total = $24 \times 8 \times 10\times 10\times 10\times 10 = 1920000$

Exercice n°2 :

9 people are seated around a round table.

1. How many different ways can they sit ? (only the relative position of the nine people in relation to each other is taken into account). relative to each other)
2. Same question, but people A and B want to be near to each other.

Solution :

1. Nine people around a round table, so we have circular permutations :
   $(n\text{-}1)! = (9\text{-}1)! = 8! = 40320$
2. A and B must sit next to each other
   Treat A and B as a single block.
   Then we have
   Number of circular permutations : $(8-1)! = 7! = 5{,}040$.
   But inside the block, A and B can switch places (2 ways).
   Ways = $7! \times 2 = 5{,}040 \times 2 = 10{,}080$

Exercice n°3 :

Find the number of anagrams of the word MISSISSIPPI. Of these anagrams, how many begin and end with the letter S ? **Solution :**

The word MISSISSIPPI has 11 letters with repeated counts : $M = 1, S = 4, I = 4?P = 2.$

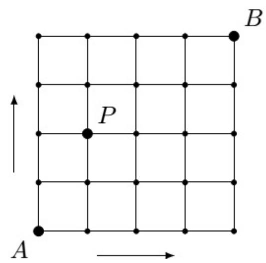
The number of permutations with repetitions is

$$\bar{P}_{11}\left(1,4,4,2\right) = \frac{11!}{1!4!4!2!} = 34650$$

Anagrams that begin and end with S We fix in the first and last positions, leaving 9 interior positions with letters $M = 1, S = 2, I = 4?P = 2.$ The number of distinct arrangements of these 9 letters is

$$\bar{P}_{9}\left(1,4,2,2\right) = \frac{11!}{1!4!2!2!} = 3780$$

Exercice n°4 : On this 4×4 grid, you can only move to the right or upwards.



a) How many paths are there from point A to point B ?
b) How many of these paths pass through point $P\left(1;2\right)$?

Solution :

a) The paths from A to B are words consisting of 4 letters u and letters r. So the number of paths is the number of permutations with repetition.

$$\bar{P}_{8}\left(4,4\right) = \frac{8!}{4!4!} = 3780$$

b) From A (0 , 0) to P (1 , 2) : 3 moves (1 r, 2 u).

$$\bar{P}_{3}\left(1,2\right) = \frac{3!}{1!2!} = 3$$

From P (1 , 2) to B (4 , 4) : 5 moves (3 r, 2 u).

$$\bar{P}_{5}\left(2,3\right) = \frac{5!}{2!3!} = 10$$

According to the Product rule
$3 \times 10 = 30$ paths.

Solution :

Exercice n°5 :

Let $C_r^n = \frac{n!}{r!(n-r)!}$,, where $n, r \in \mathbb{N}$ and $r \leq n$.

1) Show that :

a) $C_0^n = C_n^n) = 1 \quad , C_r^n = C_r^{n-r}$ (symmetry formula)

b) $C_1^n = n$.

c) $C_{r+1}^{n+1} = C_r^n + C_{r+1}^n$ (Pascal's triangle).

d) $\sum\limits_{i=0}^{n} C_i^n = 2^n$

2) Using Newton's Binomial formula $(a+b)^n = \sum\limits_{r=0}^{n} \left(\begin{array}{c} n \\ r \end{array} \right) a^{n-r} b^r$, calculate

$A = \sum\limits_{r=0}^{n} \left(\begin{array}{c} n \\ r \end{array} \right) \quad , \quad B = \sum\limits_{r=0}^{n} \left(\begin{array}{c} n \\ r \end{array} \right) (-1)^r$.

3) What is the coefficient of x^6 in the development of $(x+2)^8$ and $(x^2-5)^7$?

Solution :

For $n, r \in \mathbb{N}$ with $0 \le r \le n$, define

$$C_r^n = \frac{n!}{r!(n-r)!} = \binom{n}{r}.$$

1) Show the identities.

(1) Edge values and symmetry.

$$C_0^n = \frac{n!}{0!\,n!} = 1, \qquad C_n^n = \frac{n!}{n!\,0!} = 1,$$

$$C_r^n = \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,r!} = C_{n-r}^n.$$

(2) Single-choice value.

$$C_1^n = \frac{n!}{1!\,(n-1)!} = n.$$

(3) Pascal's identity.

$$C_{r+1}^{n+1} = C_r^n + C_{r+1}^n.$$

$$\begin{aligned}
C_r^n + C_{r+1}^n &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!} \\
&= \frac{n!}{r!(n-r)(n-r-1)!} + \frac{n!}{(r+1)r!(n-r-1)!} \\
&= \frac{n!}{r!(n-r-1)!}[\frac{1}{(n-r)} + \frac{1}{(r+1)}] \\
&= \frac{n!}{r!(n-r-1)!}[\frac{(n-r)+(r+1)}{(n-r)(r+1)}] \\
&= \frac{n!}{r!(n-r-1)!}[\frac{n+1}{(n-r)(r+1)}] \\
&= \frac{(n+1)!}{(r+1)!(n-r)!} \\
&= C_{r+1}^{n+1}. \\
&=
\end{aligned}$$

2) Using the binomial theorem.

From $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$, $n > 0$:

$$A = \sum_{r=0}^{n} \binom{n}{r} = (1+1)^n = 2^n,$$

$$B = \sum_{r=0}^{n} \binom{n}{r} (-1)^r = (1-1)^n = 0, n \geq 1.$$

3) Coefficient of x^6.
(a) In $(x+2)^8$.

$$(x+2)^8 = \sum_{r=0}^{8} \binom{8}{r} x^{8-r} 2^r.$$

We need $8 - r = 6 \Rightarrow r = 2$, hence the coefficient is

$$\binom{8}{2} \cdot 2^2 = 28 \cdot 4 = 112.$$

(b) In $(x^2-5)^7$.

$$(x^2-5)^7 = \sum_{k=0}^{7} \binom{7}{k} (x^2)^k (-5)^{7-k}.$$

We need $2k = 6 \Rightarrow k = 3$, hence the coefficient is

$$\binom{7}{3} \cdot (-5)^4 = 35 \cdot 625 = 21875.$$

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Exercice n°6 :
An urn contains 12 balls numerated from 1 to 12.
3 balls are drawn simultaneously.

i) Determine the number of different draws.

ii) Same question if these three balls are drawn successively.

iii)* What if, after each draw, the ball is put back into the urn.

Solution :

An urn contains 12 balls numbered from 1 to 12. Three balls are drawn.

i) Simultaneous draw (without order).
We simply choose 3 balls out of 12, so the number of different draws is

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220.$$

ii) Successive draw without replacement (order matters).
Here we are counting permutations of 3 elements out of 12 :

$$P(12,3) = \frac{12!}{(12-3)!} = 12 \cdot 11 \cdot 10 = 1320.$$

iii) Successive draw with replacement.
After each draw the ball is put back, so each of the 3 draws has 12 possible outcomes. Thus :

$$12^3 = 1728.$$

Exercice n°7 :
Consider the set $E = \{1; 2; 3; 4; 5; 6\}$. Using the 6 digits of this set, each taken only once, how many distinct numbers can be formed in each of the following cases :

a) Numbers of 6 digits ?

b) Numbers of 4 digits ?

c) Numbers with 4 digits starting with 3 ?

d) Numbers of 4 digits containing the digit 3 ?

e) Numbers of 4 digits containing the digits 3 and 6 ?

Solution :

Let $E = \{1, 2, 3, 4, 5, 6\}$. Form distinct numbers using these digits, each used at most once.

a) Numbers of 6 digits.
All permutations of the 6 digits :

$$6! = 720.$$

b) Numbers of 4 digits.
Permutations of length 4 from 6 digits :

$$A_4^6 = 6 \cdot 5 \cdot 4 \cdot 3 = 360.$$

c) Numbers of 4 digits starting with 3.
Fix the first digit as 3, then permute 3 digits from the remaining 5 :

$$A_3^5 = 5 \cdot 4 \cdot 3 = 60.$$

d) Numbers of 4 digits containing the digit 3.
Count all 4-digit numbers minus those with no 3 :

$$A_4^6 - A_3^5 = 360 - 120 = 240.$$

e) Numbers of 4 digits containing the digits 3 and 6.

$$4 \times 3 \times A_2^4 = 144,$$

Module manager. Merini.A