

Worksheet n°6

(Random variables)

Exercise n°1 :

We toss a fair coin 3 times. Let X be the number of heads obtained.

- 1) Find the sample space Ω of this random experiment.
- 2)) Find the probability distribution of X , its expectation $E(X)$ and its variance $V(X)$.
- 3) Calculate $P(X < 2)$ and $P(X \geq 2)$ and $P(1 < X < 2.5)$.
- 4)) Let $Y = 3X + 1$. deduce its expectation $E(Y)$ and its variance $V(Y)$.

Exercise n°2 :

Let X be a random variable whose probability distribution is given by

X	4	5	6	7	8
P	0.15	0.35	0.10	0.25	0.15

- 1) Find and represent the distribution function of X .
- 2)) Calculate the following probabilities : $P(X \leq 7.5)$; $P(X > 8)$; $P(4 \leq X \leq 6.5)$; $P(5 < X \leq 6)$.
- 3) Determine the expectation and variance of X .
- 4)) Let $Z = \frac{1}{X}$. Find the distribution of Z , and its expectation $E(Z)$ and variance and its variance $V(Z)$.

Exercise n°3 :

We consider a biased cubic die whose faces are numbered from 1 to 6, and let X be the random variable given by the number on the top face. We assume that $\exists a \in \mathbb{R}^+$ such that

$$P(X = k) = ka, \quad k = 1, 2, 3, 4, 5, 6.$$

1. Determine the probability distribution of X , and compute its expectation.
2. Let $Y = \frac{1}{X}$. Determine the distribution of Y , and compute its expectation and variance.

Exercise n°4 :

Let X be a random variable with The probability density function (PDF) given by

$$f(x) = \begin{cases} kx^2, & \text{if } -1 \leq x < 2 \\ 0, & \text{otherwise} \end{cases}$$

1. Find the constant k .
2. Calculate $E(X)$ and $Var(X)$.
3. Determine $F(x)$.
4. Calculate $P_{(0 \leq X \leq 2)}(X \geq 1)$.

Exercise n°5 :

The cumulative distribution function of a continuous r.v. Y is given by

$$F(y) = \begin{cases} 1 - \frac{9}{y^2}, & \text{if } 3 < y \\ 0, & \text{otherwise} \end{cases}$$

1. Find $P(Y \leq 5)$. $P(Y > 8)$.
2. The pdf of Y
3. Calculate $E(X)$ and $Var(X)$.