Mohamed Boudiaf University - M'sila **University Year : 2024/2025**
Faculty of Technology
Electrical Engineering Department
Module : Probability and Statistics

Correcting the final exam

Exercice n°1 : (9 marks)

A car manufacturer wants to study the relationship between a car's fuel consumption (Y) and its speed (X). The 10 measurements are given in the table below :

x_i	50	60	80	90	95	100	110	120	130	140
y_i	3	4	5	5.5	6	6.5	7.5	8.5	10	11.5

a) Construct a scatterplot for the data .

b) What can we say about the relationship between X and Y ?

c) Let $Z = LnY$. Complete the table

x_i	50	60	80	90	95	100	110	120	130	140
Z_i										

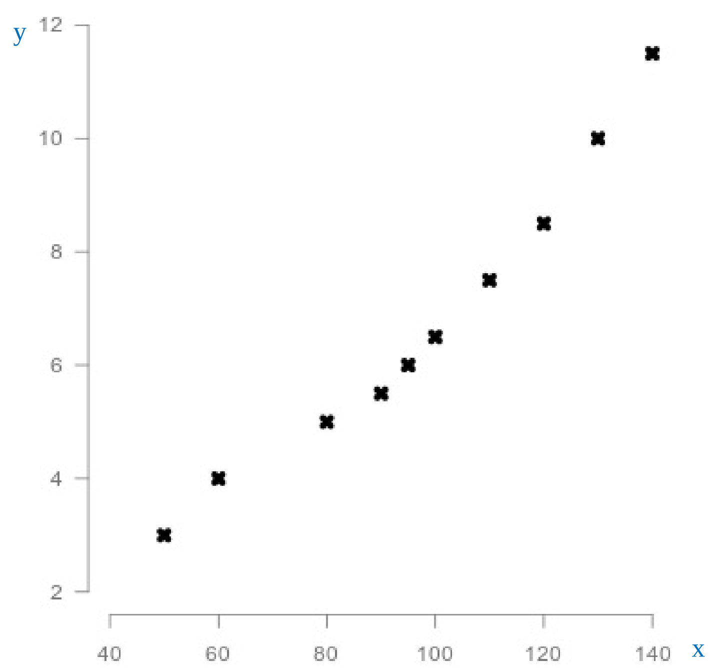
d) Determine the equation of the regression line from z to x using the method of least squares .

e) Deduce the equation of y in terms of x.

f) Estimate the fuel consumption of a vehicle travelling at 150 km/h ?

Solution :

a) Scatterplot for the data : (1 mark)



The relationship between a car's fuel consumption (Y)and its speed (X)

b)X and Y do not have a linear relationship (or X and Y have a weak linear relationship). (0.5 mark)

c) Let $Z = LnY$. Complete the table

x_i	50	60	80	90	95	100	110	120	130	140
Z_i	1.10	1,39	1.61	1.70	1.79	1.87	2.01	2.14	2.30	2.44

(2.5 mark)

d) Equation of the regression line $z = ax + b$ using the method of least squares :

$\bar{x} = \dfrac{1}{10}\sum_{i=1}^{10} x_i = 97.5$ (0.5 mark)

$\bar{z} = \dfrac{1}{10}\sum_{i=1}^{10} z_i = 1.84$ (0.5 mark)

$V(x) = \dfrac{1}{10}\sum_{i=1}^{10} x_i^2 - \bar{x}^2 = 756.25$ (0.5 mark)

$Cov\,(x, z) = \dfrac{1}{10}\sum_{i=1}^{10} x_i z_i - \bar{x}\bar{z} = 11.12$ (0.5 mark)

then $a = \dfrac{11.12}{756.25} = 0.014$ and $B = \bar{z} - A\bar{x} = 1.84 - 0.014 \times 97.5 = 0.48$

So , $(D) : z = 0.48 + 0.014x$ (1 mark)

e) We have $z = lny = 0.48 + 0.0.14x$, so :$y = e^{0.0.14x} \times e^{0.48} = 0.61e^{0.0.14x}$ (1 mark)

f) For $x = 150km/h$ then $y = 1.61e^{0.0.14\times150} = 13.2\ litre$ (1 mark)

<u>Exercice n°2</u> (7 marks)

A box contains three balls numbered 2, 3 and 5. Two balls are drawn in successively with the drawn ball being returned to the box. Let X be the sum of the numbers obtained.

a) Find the sample spac Ω of this random experiment.

b) Find the probability distribution of X, its expectation $E(X)$ and its variance $V(X)$.

c) Let $Y = 3X + 10$. Deduce its expectation $E(Y)$ and its variance $V(Y)$.

<u>Solution :</u>

a) The sample space :

$$\Omega = \{(2;2); (2;3); (2;5); (3;2); (3;3); (3;5); (5;2); (5;3); (5;5)\}$$

(1 mark)

b) We have

$$X : \Omega \longrightarrow \mathbb{R}$$
$$(i, j) \mapsto X(i, j) = i + j$$

Therefore, $X(\Omega) = \{4; 5; 6; 7; 8; 10\}$. (1 mark)

The probability distribution of X :

X	4	5	6	7	8	10
$P(X = x_i)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(2 mark)

The expectation :

$$
\begin{aligned}
E(X) &= \Sigma_{i=1}^{6} P(X = x_i) \times x_i \quad (0.25\ mark)\\
&= \frac{1}{9} \times 4 + \frac{2}{9} \times 5 + \frac{1}{9} \times 6 + \frac{2}{9} \times 7 + \frac{2}{9} \times 8 + \frac{1}{9} \times 10 \quad (0.25\ mark)\\
&= \frac{20}{3}\\
&\simeq 6.67 \quad (0.25\ mark)
\end{aligned}
$$

The variance :

$$
\begin{aligned}
V(X) &= \Sigma_{i=1}^{6} P(X = x_i) x_i^2 - E(X)^2 \quad (0.25\ mark)\\
&= \frac{1}{9} \times 4^2 + \frac{2}{9} \times 5^2 + \frac{1}{9} \times 6^2 + \frac{2}{9} \times 7^2 + \frac{2}{9} \times 8^2 + \frac{1}{9} \times 10^2 - (\frac{20}{3})^2 \quad (0.25\ mark)\\
&= \frac{28}{9}\\
&\simeq 3.11 \quad (0.25\ mark)
\end{aligned}
$$

c) Let $Y = 3X + 10$.
The expectation

$$
\begin{aligned}
E(Y) &= = E(3X + 10) \quad (0.25\ mark)\\
&= 3E(X) + 10\\
&= 3 \times \frac{20}{3} + 10 \quad (0.25\ mark)\\
&= 30 \quad (0.25\ mark)
\end{aligned}
$$

Thr variance

$$
\begin{aligned}
V(Y) &= V(3X + 10) \quad (0.25\ mark)\\
&= 3^2 V(X) \quad (0.25\ mark)\\
&= 9 \times \frac{28}{9}\\
&= 28 \quad (0.25\ mark)
\end{aligned}
$$

Exercice n°3 (4 marks)
Two machines A and B respectively produce 65% and 35% of the total number of parts manufactured in a factory. The percentages of defective parts produced by these machines are 3% and 2% respectively. If an part selected at random, find the probability :

a) that a part is defective.

b) that a defective part comes from A.

c) that a non-defective part comes from B.

Solution :
Let be the following events A : "the part was produced by machine A", so $P(A) = 0.65$.
B : "the part was produced by machine B", so $P(B) = 0.35$.

d = defective part. $\bar{d}$ = non-defective part. then $P(d/A) = 0.03$ and $P(d/B) = 0.02$

a) we're looking for :

$$\begin{aligned} P(d) &= P(d\cap A)+P(d\cap B) \quad (0.5\ mark)\\ &= P(A)P(d/A)+P(B)P(d/B) \quad (0.5\ mark)\\ &= 0.65\times 0.03+0.35\times 0.02 \quad (0.25\ mark)\\ &= 0,0265 \quad (0.25\ mark)\end{aligned}$$

b) we're looking for :

$$\begin{aligned} P(A/d) &= \frac{P(A\cap d)}{P(d)} \quad (0.25\ mark)\\ &= \frac{P(A)P(d/A)}{P(d)} \quad (0.25\ mark)\\ &= \frac{0.65\times 0.03}{0.0265} \quad (0.25\ mark)\\ &\simeq 0.736 \quad (0.25\ mark)\end{aligned}$$

.

c) we're looking for :

$$\begin{aligned} P(B/\bar{d}) &= \frac{P(B\cap \bar{d})}{P(\bar{d})} \quad (0.5\ mark)\\ &= \frac{P(B)P(\bar{d}/B)}{1-P(d)} \quad (0.5\ mark)\\ &= \frac{0.35\times 0.98}{1-0.0265} \quad (0.25\ mark)\\ &\simeq 0,35 \quad (0.25\ mark)\end{aligned}$$

Module manager. Merini.A
Good luck