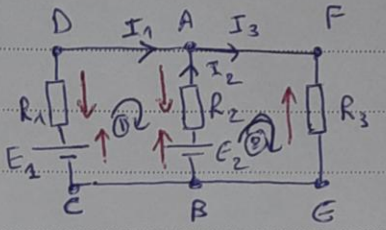
Solution of First series of exercises

Exercise 1:

Exercise 1 :

1] * Number of nodes : two nodes (A and B)



* Number of branches : three branches (AB, AFEB and ADCB)
* Number of meshes : Three meshes (ABCDA, ABEFA and DCBEFAD)

2] The application of Kirchhoff's laws

* Kirchhoff's current law (KCL)

$$\boxed{I_1 + I_2 = I_3} \quad \text{——} \quad (1)$$

* Kirchhoff's voltage Law (KVL)

- From mesh 1 : $E_1 - R_1 I_1 + R_2 I_2 - E_2 = 0$ —— (2)
- From mesh 2 : $E_2 - R_2 I_2 - R_3 I_3 = 0$ —— (3)

3] Determine I_1, I_2, I_3 with : $R_1 = 2\Omega$, $R_2 = 5\Omega$, $R_3 = 10\Omega$, $E_1 = 20V$, $E_2 = 70V$

(1) and (2) becomes

$$\begin{cases} 20 - 2I_1 + 5I_2 - 70 = 0 & \Rightarrow I_1 = -25 + 2{,}5\, I_2 \quad \text{—— (4)} \\ 70 - 5I_2 - 10I_3 = 0 & \xRightarrow{\text{implies that}} I_3 = 7 - 0{,}5\, I_2 \quad \text{—— (5)} \end{cases}$$

Remplacing (4) and (5) in (1) $\Rightarrow \boxed{I_2 = 8A}$

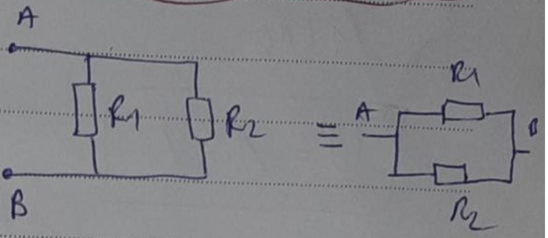
Remplacing the value of I_2 in (4) and (5)

$\Rightarrow \boxed{I_1 = -5A}$ and $\boxed{I_3 = 3A}$

Exercise 2:

1)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \; ; \; R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



AN $\boxed{R_{eq} = \frac{14}{9} = 1{,}55\,\Omega}$

2] $V_g = R_{eq} \cdot i \Rightarrow i = \frac{V_g}{V_{eq}} \Rightarrow \boxed{i = 11{,}57\,A}$ ✓

3] We use the current divider.

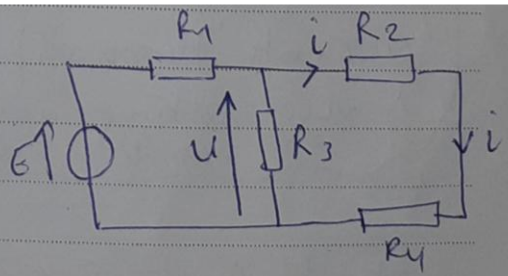
$$I_j = \frac{1/R_j}{\sum_{k=1}^{n} 1/R_k} \cdot I$$

$$I_1 = \frac{1/R_1}{1/R_1 + 1/R_2}\, I = \frac{R_2}{R_1 + R_2}\, I \Rightarrow \boxed{i_1 = 2{,}57\,A}$$

$$I_2 = \frac{1/R_2}{1/R_2 + 1/R_1}\, I = \frac{R_1}{R_1 + R_2}\, I \Rightarrow \boxed{i_2 = 9\,A}$$

Exercise 3:

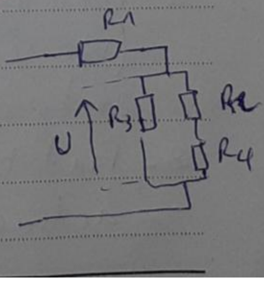
By applying the voltage divider, determine the current i and the voltage u.



$E = 6\,V$, $R_1 = 100\,\Omega$, $R_2 = R_3 = R_4 = 50\,\Omega$

Solu

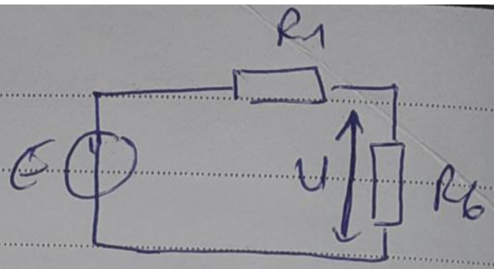
* $R_5 = R_2 + R_4 \Rightarrow \boxed{R_5 = 100\,\Omega}$



$R_6 = R_3 \,//\, R_5 \Rightarrow \boxed{R_6 = 33{,}3\,\Omega}$

$= \frac{R_3 \cdot R_5}{R_1 + R_5}$

$$U = \frac{R_6}{R_1 + R_6} \cdot E$$



$$\boxed{U = 1{,}5\ V}$$

$$i = \frac{U}{R_5}$$

$$\boxed{i = \frac{1{,}5}{100} = 15\ mA}$$

Exercise 4:

Exercise 4:

1) Voltage divider

$$V_1 = \frac{R_1}{R_1 + R_2} E = \frac{1}{1+2} \cdot 3 = 1\ V$$

$$V_2 = \frac{R_2}{R_1 + R_2} E = \frac{2}{2+1} \cdot 3 = 2\ V$$

2) Current divider

$$I_1 = \frac{1/R_1}{(1/R_1) + (1/(R_2 + (R_3 // R_4)))} I = \frac{R_2 + (R_3 // R_4)}{R_1 + R_2 + (R_3 // R_4)} I = \frac{R_2 + \frac{R_3 R_4}{R_3 + R_4}}{R_1 + R_2 + \frac{R_3 R_4}{R_3 + R_4}} I$$

$$\boxed{I_1 = 3{,}63\ A}$$

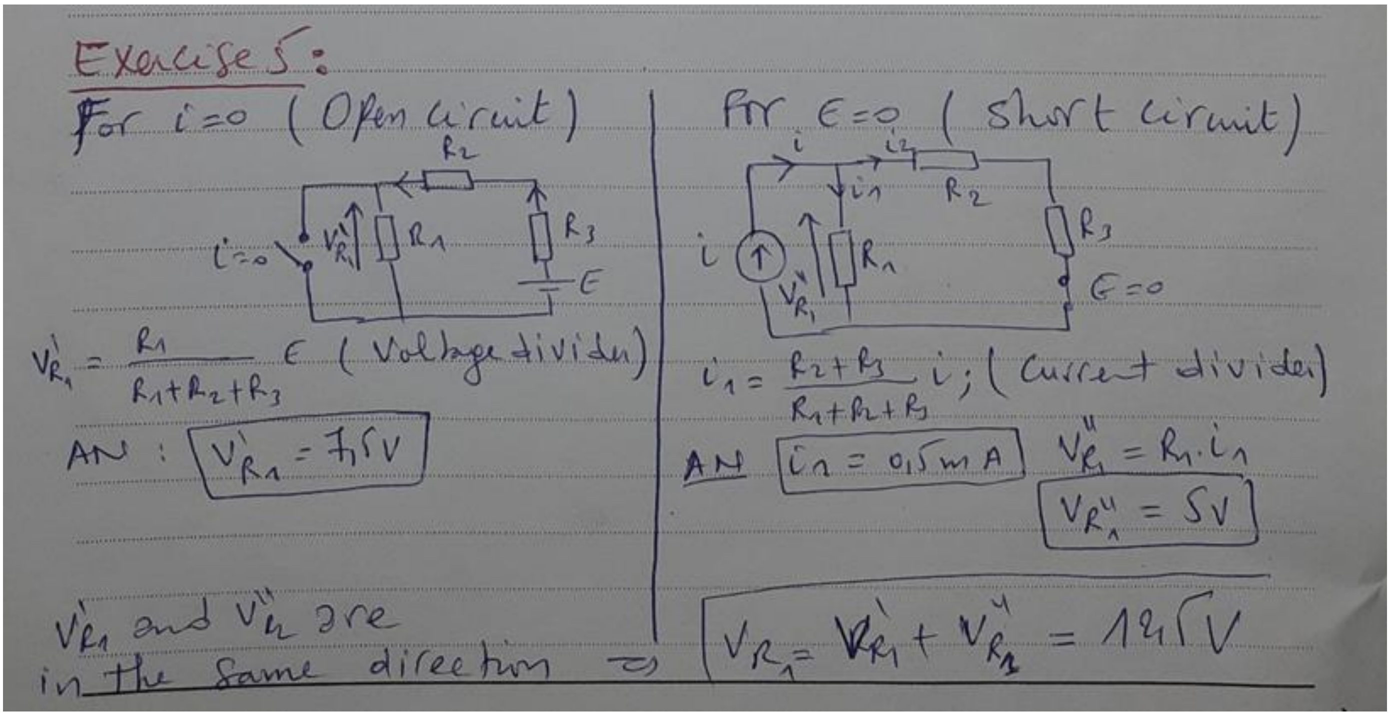
$$I_2 = \frac{1/(R_2 + (R_3 // R_4))}{(1/R_1) + (1/(R_2 + (R_3 // R_4)))} I = \frac{R_1}{R_1 + R_2 + (R_3 // R_4)} I = \frac{R_1}{R_1 + R_2 + \frac{R_3 \cdot R_4}{R_3 + R_4}}$$

$$\boxed{I_2 = 1{,}36\ A}$$

$$I_3 = \frac{R_4}{R_3 + R_4} I_2 = 0{,}9\ A$$

$$I_4 = \frac{R_3}{R_3 + R_4} I_2 = 0{,}45\ A$$

Exercise 5:



Exercise 5:

For $i=0$ (Open circuit)

$$V'_{R_1} = \frac{R_1}{R_1+R_2+R_3}\,E \quad \text{(Voltage divider)}$$

AN : $\boxed{V'_{R_1} = 7,5V}$

For $E=0$ (short circuit)

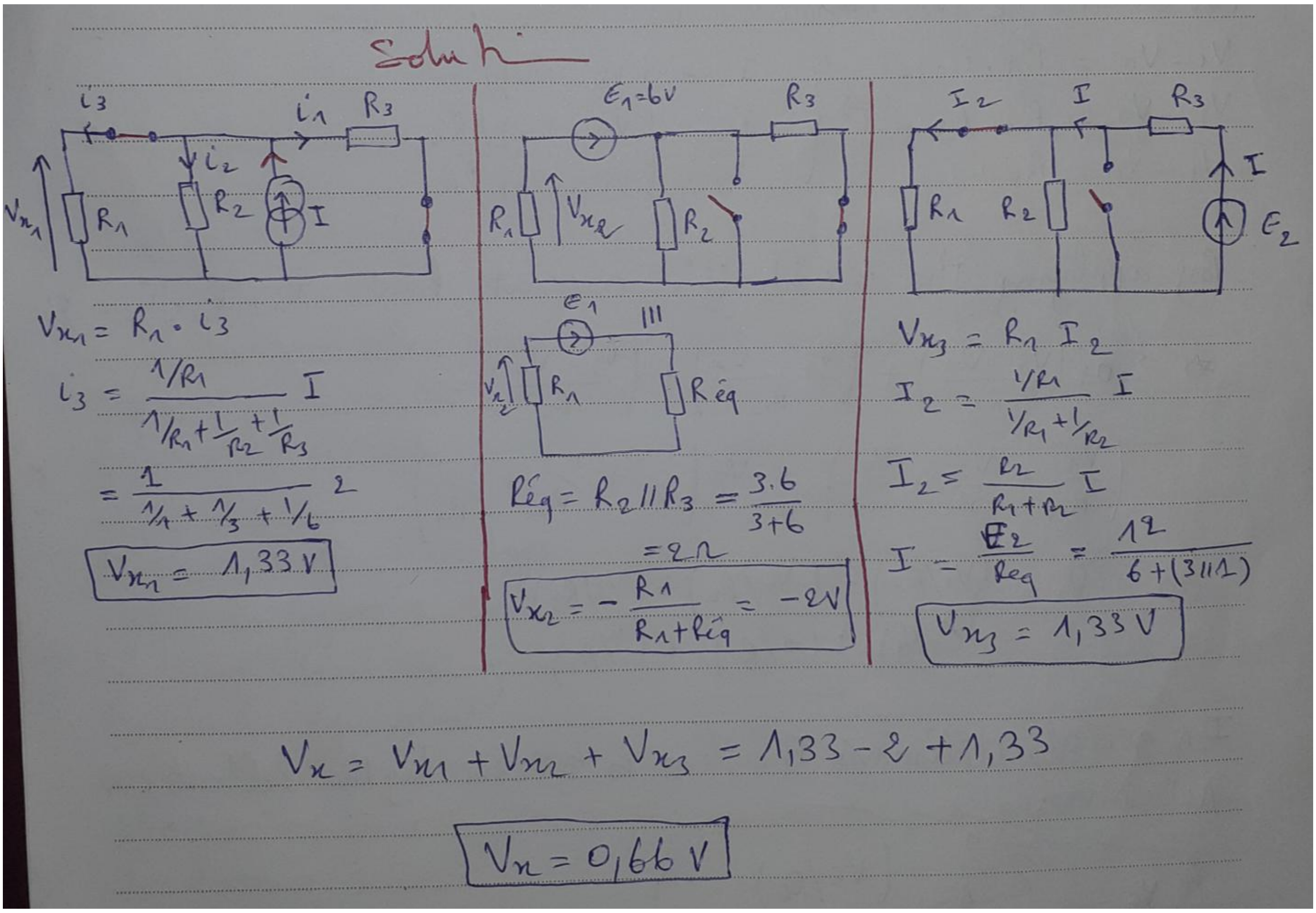
$$i_1 = \frac{R_2+R_3}{R_1+R_2+R_3}\,i ; \quad \text{(Current divider)}$$

AN $\boxed{i_1 = 0,5\,mA}$ $V''_{R_1} = R_1 . i_1$

$\boxed{V''_{R_1} = 5V}$

V'_{R_1} and V''_{R_1} are in the same direction $\Rightarrow$ $\boxed{V_{R_1} = V'_{R_1} + V''_{R_1} = 12,5V}$

Exercise 6:



Solution

$V_{x_1} = R_1 \cdot i_3$

$$i_3 = \frac{1/R_1}{1/R_1 + \frac{1}{R_2} + \frac{1}{R_3}}\,I = \frac{1}{1/4 + 1/3 + 1/6}\,2$$

$\boxed{V_{x_1} = 1,33\,V}$

$E_1 = 6V$

$$R_{eq} = R_2 // R_3 = \frac{3.6}{3+6} = 2\Omega$$

$$\boxed{V_{x_2} = -\frac{R_1}{R_1+R_{eq}} = -2V}$$

$V_{x_3} = R_1\, I_2$

$$I_2 = \frac{1/R_1}{1/R_1 + 1/R_2}\,I$$

$$I_2 = \frac{R_2}{R_1+R_2}\,I$$

$$I = \frac{E_2}{R_{eq}} = \frac{12}{6+(3//1)}$$

$\boxed{V_{x_3} = 1,33\,V}$

$$V_x = V_{x_1} + V_{x_2} + V_{x_3} = 1,33 - 2 + 1,33$$

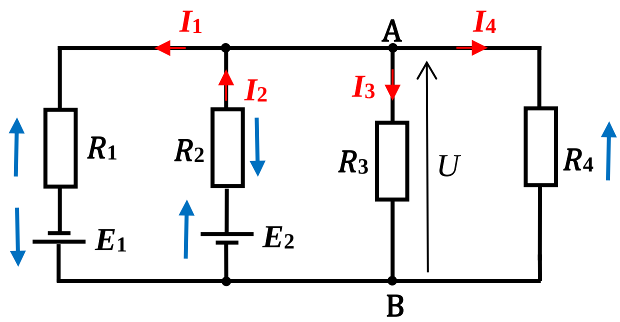
$$\boxed{V_x = 0,66\,V}$$

Exercise 7:

1. **Calculate the potential difference U between points A and B.**

$$\boxed{U=\frac{\frac{-E_1}{R_1}+\frac{E_2}{R_2}+\frac{0}{R_3}+\frac{0}{R_4}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}}=\frac{\frac{-3}{1}+\frac{12}{2}}{\frac{1}{1}+\frac{1}{2}+\frac{1}{2}+\frac{1}{1}}=1V}$$

2. **Calculate and indicate the direction of the current in each resistance.**



$$U+E_1=R_1I_1\Rightarrow I_1=\frac{U+E_1}{R_1}=\frac{1+3}{1\times10^3}=4mA \qquad \boxed{I_1=4mA}$$

$$U=E_2-R_2I_2\Rightarrow I_2=\frac{E_2-U}{R_2}=\frac{12-1}{2\times10^3}=5.5mA \qquad \boxed{I_2=5.5mA}$$

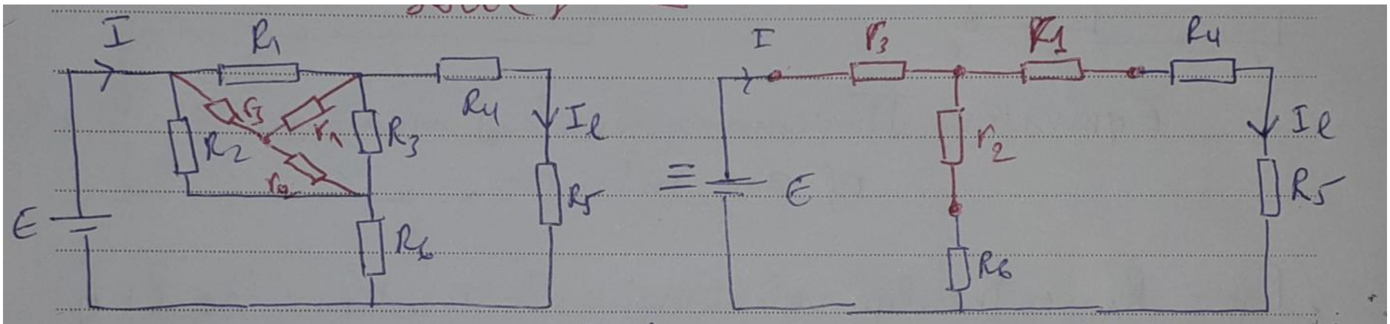
$$U=R_3I_3\Rightarrow I_3=\frac{U}{R_3}=\frac{1}{2\times10^3}=0.5mA \qquad \boxed{I_3=0.5mA}$$

$$U=R_4I_4\Rightarrow I_4=\frac{U}{R_4}=\frac{1}{1\times10^3}=1mA \qquad \boxed{I_4=1mA}$$

3. **Check the law of nodes (Kirchhoff's current law).**

$$\boxed{I_2-I_1-I_3-I_4=5.5-4-0.5-1=0}$$

Exercise 8:



$$r_1 = \frac{R_1 \cdot R_3}{R_1 + R_2 + R_3} \; ; \; r_2 = \frac{R_2 \cdot R_3}{R_1 + R_2 + R_3} \; ; \; r_3 = \frac{R_1 \cdot R_2}{R_1 + R_2 + R_3}$$

$$r_1 = r_2 = r_3 = \frac{100 \cdot 100}{100 + 100 + 100} = 33{,}33\ \Omega$$

$$I = \frac{E}{R_{eq}} \; ; \; R_{éq} = \left[(r_1 + R_4 + R_5) \,//\, (r_2 + R_6) + r_3 \right]$$

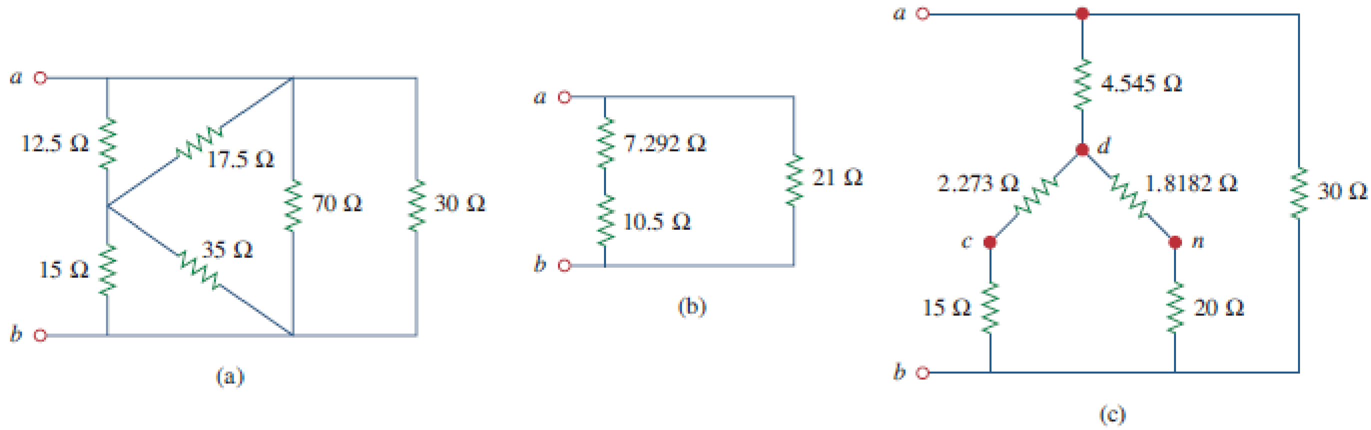
$$R_{eq} = 118{,}17\ \Omega \; ; \; \boxed{I = 84{,}6\,mA}$$

We use the current divider

$$I_l = \frac{\frac{1}{r_1 + R_4 + R_5}}{\frac{1}{r_1 + R_4 + R_5} + \frac{1}{r_2 + R_6}} I = \frac{r_2 + R_6}{r_1 + R_4 + R_5 + r_2 + R_6} I$$

$$I_l = 53{,}83\,mA$$

Exercise 9:



In this circuit, there are two Y networks and three networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5Ω, 10Ω and 20Ω resistors, we may select

$R_1 = 10\ \Omega, \quad R_2 = 20\ \Omega, \quad R_3 = 5\ \Omega$

We have:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$
$$= \frac{350}{10} = 35\ \Omega$$
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Figure. (a). Combining the three pairs of resistors in parallel, we obtain

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$
$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$
$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

So that the equivalent circuit is shown in Figure. (b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

Exercise 10:

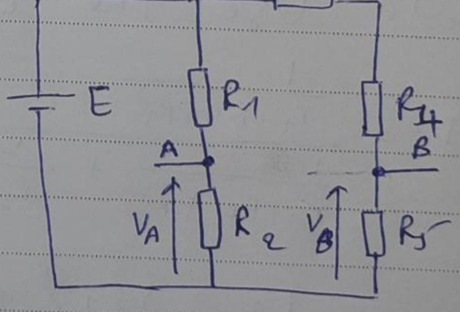
Exercise 10

* 1] Find the Thevenin equivalent of the circuit :

$V_{Th} = V_{AB} = V_A - V_B$

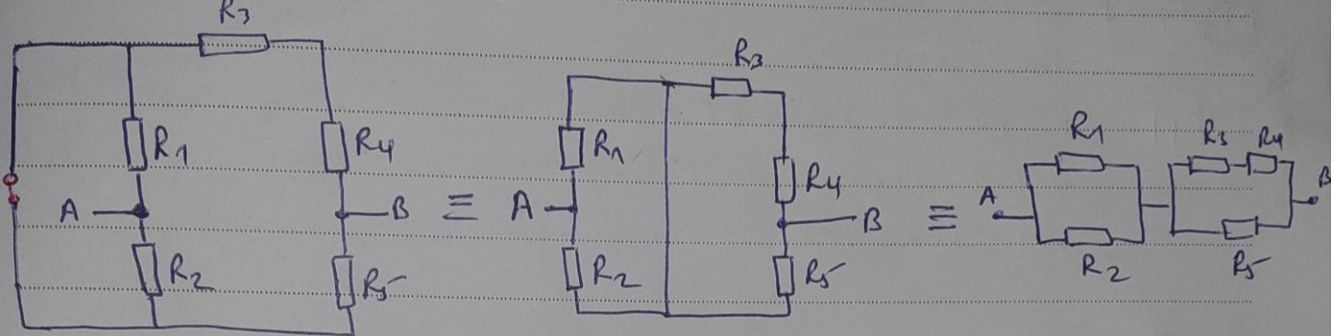
$V_A = \frac{R_2}{R_1+R_2} \cdot E = 5V$

$V_B = \frac{R_5}{R_1+R_4+R_5} \cdot E = 2,5V$

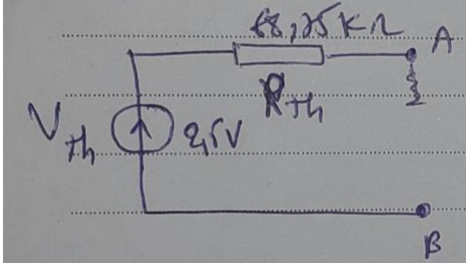


$\boxed{V_{Th} = V_{AB} = V_A - V_B = 5 - 2,5 = 2,5V}$

Calculate $R_{Th} = R_{AB}$

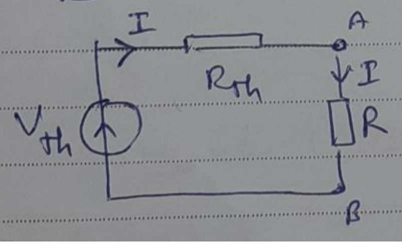


$R_{Th} = R_{AB} = (R_1 // R_2) + [(R_3+R_4)//(R_5)]$ $\boxed{R_{Th} = 68,75\ K\Omega}$



Thevenin equivalent circuit

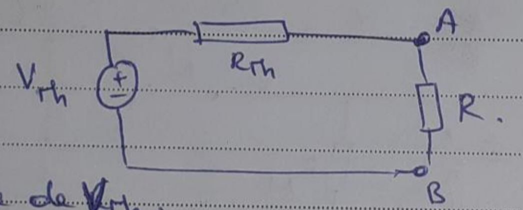
2] Determine the current I folowing in the resistance R



$$I = \frac{V_{Th}}{R_{Th}+R} = 14,81\ \mu A$$

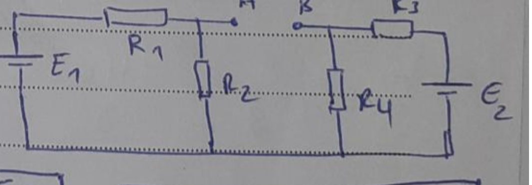
Exercise 11:

Donner le générateur de Thévenin. c-à-d
Mettre le circuit sous la forme suivante



I] Calcule de V_{Th}:

$V_{Th} = V_{AB} = V_A - V_B$

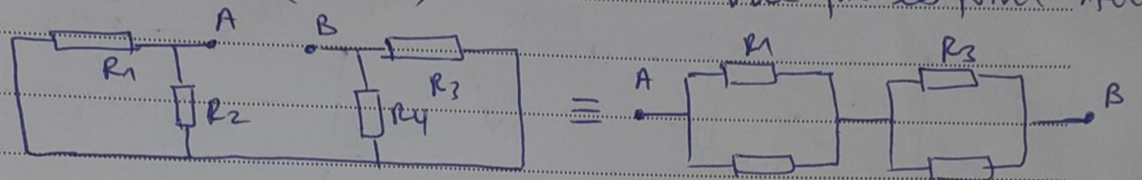


$V_A = \frac{R_2}{R_2+R_1} E_1 = \frac{100}{100+100} \cdot 12$ $\boxed{V_A = 6\ V}$

$V_B = \frac{R_4}{R_3+R_4} \cdot E_2 = \frac{100}{100+100} \cdot 5$ $\boxed{V_B = 2,5\ V}$

$$\boxed{V_{Th} = V_{AB} = 6 - 2,5 = 3,5\ V}$$

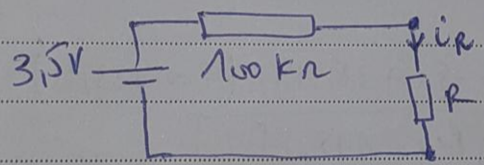
2] Calcul de R_{Th} = ($= R_{AB}$; la résistance vue par les point A et B)



$$R_{Th} = R_{AB} = (R_1 // R_2) + (R_3 // R_4) = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}$$

$$= \frac{100 . 100}{100 + 100} + \frac{100 . 100}{100 + 100}$$

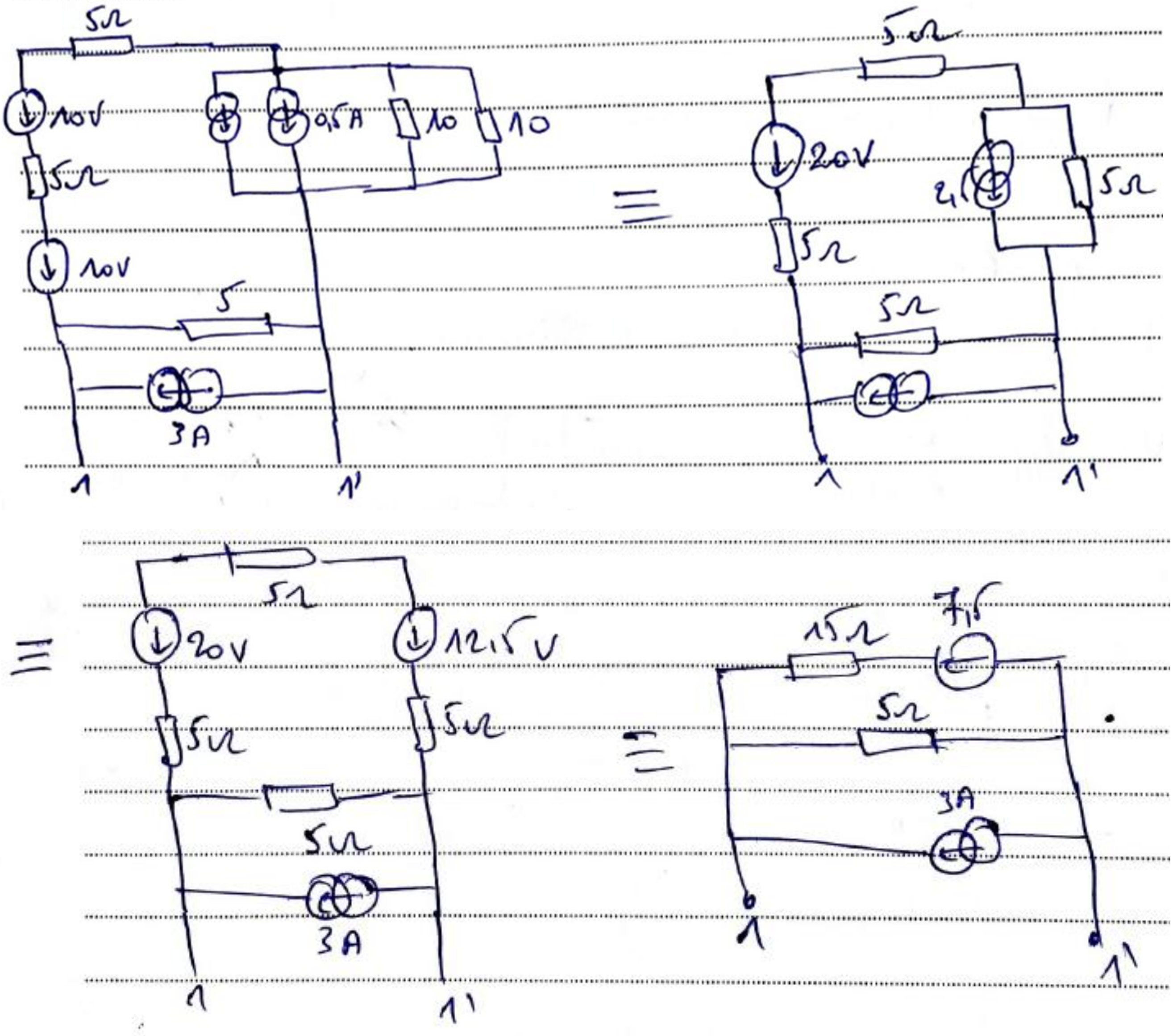
$$\boxed{R_{Th} = 100\,k\Omega}$$

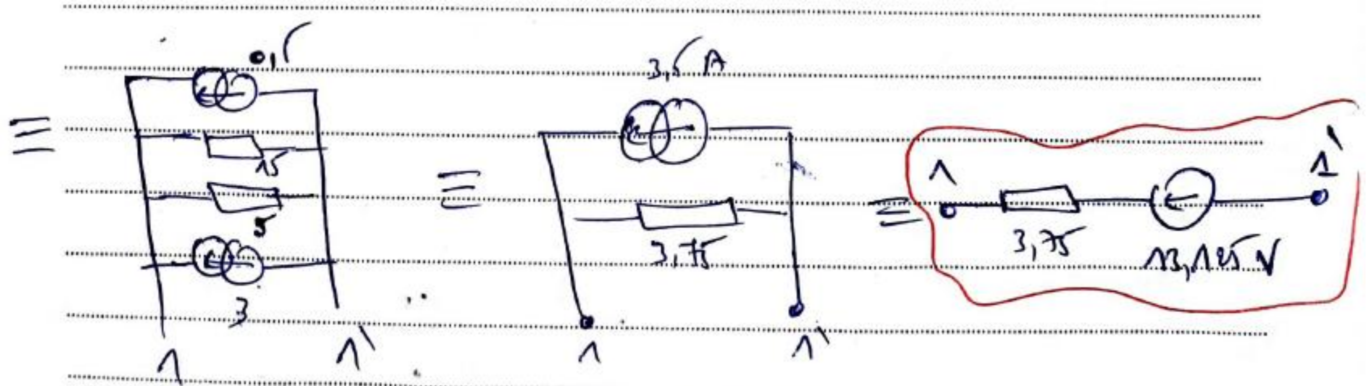


générateur de Thèvenin.

$$i_R = \frac{V_{Th}}{R_{Th} + R} \quad , \quad V_{AB} = V_R = \frac{R}{R + R_{Th}} \cdot E_{Th}$$

Exercise 12:

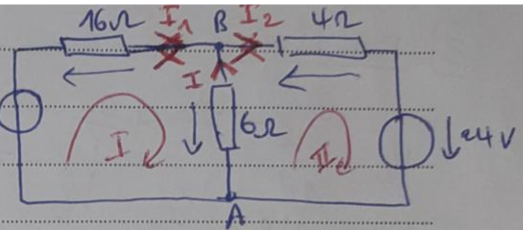




$\boxed{V_{éq} = 13{,}125\ V}$ $\boxed{R_{éq} = 3{,}75\ \Omega}$

Exercise 13:

1) Kirchhoff



* La loi des noeuds donne :

$I_1 + I = I_2$ —— ①

* La loi des mailles. ~~donne~~.

La maille I donne : $4 - 16 I_1 + 6I = 0$ —— ②

La maille II donne : $24 - 6I - 4I_2 = 0$ → ③

Calcule le courant I

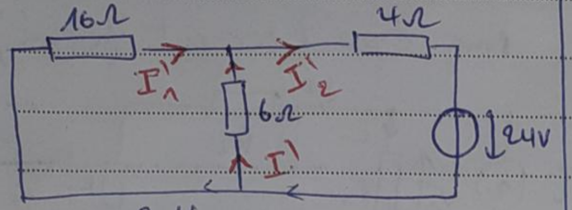
de ② ⇒ $I_1 = 1/4 + \frac{3}{8} I$ → ④

de ③ ⇒ $I_2 = 6 - 3/2\, I$ → ⑤

remplaçant ④ et ⑤ dans ① ⇒ $\boxed{I = 2A}$

2) Superposition

circuit -1-



$$I'_2 = \frac{24}{(6 // 16) + 4}$$

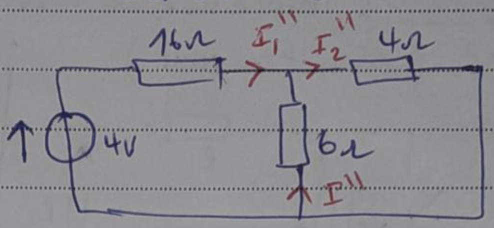
$$\Rightarrow \boxed{I'_2 = +2{,}870\,A}$$

On utilise le diviseur de courant :

$$I' = \frac{16}{16+6} \cdot 2{,}870$$

$$\boxed{I' = 2{,}087\,A}$$

circuit -2-



$$I''_1 = \frac{4}{(6 // 4) + 16}$$

$$\boxed{I''_1 = +0{,}217\,A}$$

On utilise le diviseur de courant

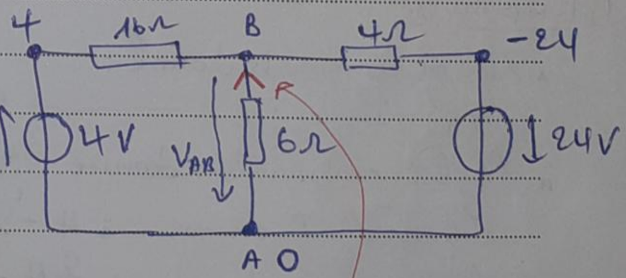
$$I'' = -\frac{4}{4+6} \cdot 0{,}217$$

$$\boxed{I'' = -0{,}087\,A}$$

$$\boxed{I = I' + I'' = 2\,A}$$

3] Millman

$$V_B = \frac{4/16 - \frac{24}{4} + \frac{0}{6}}{1/16 + 1/4 + 1/6} = -12\,V$$



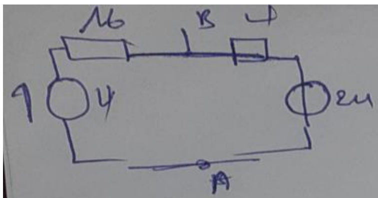
$$V_{AB} = V_A - V_B$$

$$= 0 - (-12)$$

$$\boxed{V_{AB} = 12\,V}$$

$$i' = \frac{V_{AB}}{6} = \frac{12}{6} = 2A$$

$$\boxed{i' = 2A}$$



Thevenin

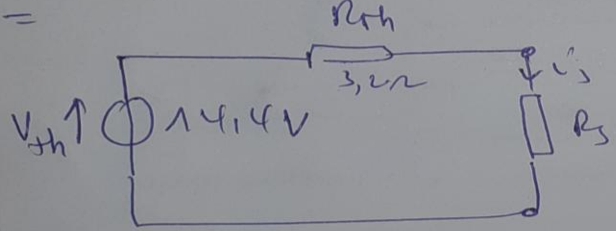
$$i = \frac{24+4}{16+4} = \frac{28}{20} = \frac{14}{10} = 1,4\,A$$

$$V_B = 4 - 16\cdot 1,4 = 18,4\,V$$
$$= 24 - 4\cdot 1,4$$

$$V_{th} = 14,4$$

$$R_{th} = \frac{16\cdot 4}{20} = \frac{64}{20} = \frac{32}{10} = 3,2\,\Omega$$

=



$$i_3 = \frac{18,4}{3,2+6} = \frac{18,4}{9,2} = 2\,A.$$