Ministry of Higher Education and Scientific Research
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Module: Control Systems
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LAB WORK No. 2

Time Domain Analysis of First and Second Order Continuous Linear Systems

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I. OBJECTIVE

The aim of this lab work is to study and identify the transient and steady-state regimes in the step response of single-input single-output (SISO) continuous linear systems of 1st and 2nd order. We will study both open-loop and closed-loop control cases.

II. THEORETICAL BACKGROUND

II.1 First Order System

A first-order system is governed by a first-order differential equation of the form:
$T \cdot dy(t)/dt + y(t) = k \cdot x(t)$
where x(t) is the system input and y(t) its output.

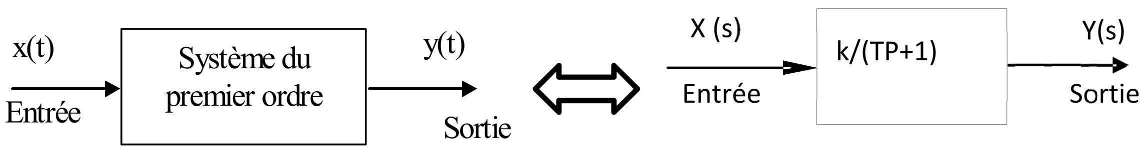


Figure 1.2. Block Diagram

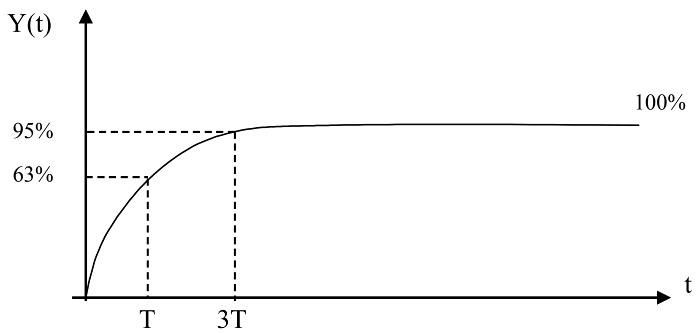


Figure 1.3 Time Response (Step Response)

With K: The static gain (steady state)
T: The system time constant

The transfer function obtained by Laplace Transform:
$G(s) = Y(s)/X(s) = k/(T \cdot s + 1)$

The response to a step input of amplitude E for such a system will be given in the time domain by the solution: $y(t) = kE(1 - e^{-\frac{t}{T}})$ where the final value of the steady state is equal to k times the input. At t=T, the system response reaches 63% of its final value and 95% after tr=3T, the latter being called the 5% settling time.

II.2 Second Order System

The normalized equation:
$d^2y(t)/dt^2 + 2\xi\omega_0 \cdot dy(t)/dt + \omega_0^2 \cdot y(t) = k \cdot \omega_0^2 \cdot x(t)$

Transfer function:
$G(s) = Y(s)/X(s) = k \cdot \omega_0^2/(s^2 + 2\xi\omega_0 \cdot s + \omega_0^2)$ Where:

- **ω_0** = undamped natural frequency
- **ξ** = damping ratio
- **ω_d** = damped natural frequency $\omega_d = \omega_0\sqrt{1-\xi^2}$

which is obtained when $|\xi| < 1$. The roots of the denominator are therefore given by:

$$s_{1,2} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

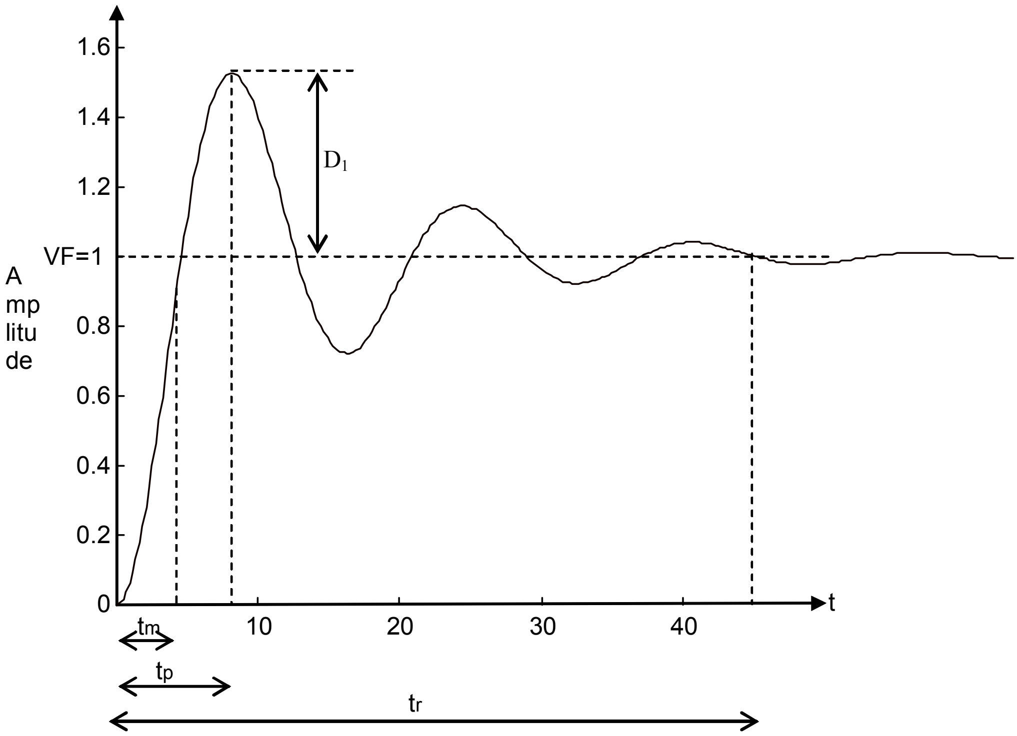
In this case, the system response to a unit step input with $k=1$ is given by the relation: $y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin(\omega_0\sqrt{1-\xi^2}\,t + \phi)$

where $\phi = \cos^{-1}\xi$

The system response is characterized by the following parameters:

- **tm**: **Rise Time** - characterizes the time taken for the response to increase from 0% to 100% of its final value (in the case of an underdamped oscillatory response).
- **V_F**: **Final Value** - the steady-state output value obtained as t → ∞.
- **D_1**: **First Maximum Overshoot** - the first peak value exceeding the final steady-state value.

- **tp**: **Peak Time** - defined as the time required for the output to reach its first overshoot peak.
- **t_r**: **Settling Time** - defined as the time required for the output to reach and remain within a specified tolerance band around the final value (±10%, ±5%, or ±2%).



III. PREPARATORY WORK

- **Preparatory Work**

For the first-order system controlled in closed-loop with unit feedback and a gain k in the forward path:

1. Provide the closed-loop transfer function of the system
2. Express the new time constant T' as a function of T and k. Deduce the response time t'r
3. Express the static gain k' and the steady-state error E'(∞) as a function of k

For the second-order system placed in a unit feedback control loop (closed-loop control):

1. Provide the closed-loop transfer function of the system
2. Put the obtained transfer function into standard form
3. Express the new time constant T' as a function of T and k. Deduce the system response time t'r (CL)
4. Express the static gain k' and the steady-state error E'(∞) as a function of k

IV. SIMULATION

1. CASE OF A FIRST-ORDER SYSTEM

a) Open-loop study (O.L)

1. Implement the corresponding setup using SIMULINK
2. Apply a step signal to the system input
3. Plot the open-loop system response by completing Tables 1 and 2 for:

a) Open-Loop Study

Table 1: k=1

τ	**0.5**	**1.0**	**1.5**
Settling Time (~5%)			

*Table 2: τ=1

k Value	**1**	**3**	**10**
Steady-State Error			

b) Closed-Loop Study

Let us now consider the previous case controlled in closed-loop with unit feedback.
Repeat the same steps mentioned in (a) by completing the tables below for:

Table 3: k=1

τ **Value**	**0.5**	**1.0**	**1.5**
Time Constant T'			
Settling Time (~5%)			

Table 4: τ=1

k Value	**1**	**3**	**10**
Gain K' (CL)			
Steady-State Error E'(∞)			

Work Required:

- Interpret the step responses obtained in open-loop and closed-loop. Deduce the advantage of closed-loop control.
- Discuss the benefit of proportional action in both steady-state and dynamic performance.

2. SECOND ORDER SYSTEM

a) Open-Loop Study

Implement the corresponding simulation diagram using Simulink.

• **1st Case:** Set ω_0 to 1 [rad/s] with k=1

- Plot the system's step response for: ξ = 0; 0.1; 0.5; 0.7; 1; 1.5; 2.
- Record the maximum value of each response and complete the table below:

Table 5: ω=1 rad/s, k=1

ξ Value	**0**	**0.1**	**0.5**	**0.7**	**1.0**	**1.5**	**2.0**
y_max							
D_1(%)							

• **Second Case:** Taking ω = 1 [rad/s], ξ = 0.25, and k = 1

- Determine the characteristic times of the system: t_m , t_p , and t_r
- Determine graphically the value of ω_8 (pseudo-oscillation frequency)

• **Third Case:** With k = 1 and ξ = 0.5
Plot the step response for: ω = 1, 3, and 10 [rad/s]
What do you observe?

b) Closed-Loop Study:

- Implement the simulation diagram in Simulink for the system in closed-loop with unit feedback and a gain k in the forward path. Take ω= 1 [rad/s], ξ = 0.3
- Excite the system with a unit step and observe the output while varying the gain k: k = 1, 5, 10, 100
- For each response, record the overshoot (D_1%), peak time (t□), and the corresponding steady-state error E(tf).

c) Case of an Unstable Second-Order System
Plot the step response for the following two cases:
• ξ = -0.1; ω_0 = 1 rad/sec
• ξ = -0.1; ω_0 = 10 rad/sec
What can be concluded in this case?

WORK REQUIRED

1. Compare open-loop vs. closed-loop responses
2. Discuss proportional action effects
3. Analyze system performance in both regimes
4. Draw conclusions about control system behavior