VI.1 Shafts

VI.1.1 Definition of a Shaft

A shaft is a solid, generally rotational body that undergoes rotary motion around its axis and typically supports transmission elements such as gears, pulleys, flywheels, cranks, and sprockets for chains, among others. The shaft is a commonly used component in machines. Its role is multifaceted: generally, it serves to transmit power from one part of the machine to another, but it can also be used to ensure the positioning of one element relative to another. Due to its geometry and functions, a shaft can have different names, such as:

- **Transmission Shaft**: It transmits torque from a motor to a machine or machine element.
- **Return Shaft**: It supports machine elements (gears, pulleys, etc.) and transmits torque between each element.
- **Axle**: A stationary or rotating shaft that does not transmit torque but serves for positioning.

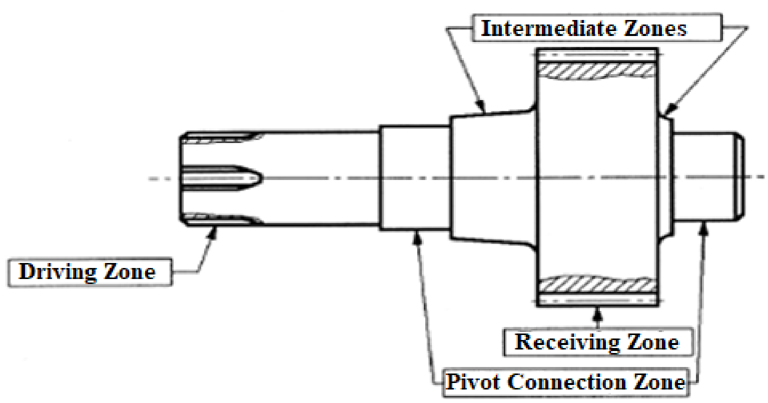


Figure IV.1 Characteristic Zones of a Shaft

Based on their geometric shapes, shafts are classified as straight, bent (crankshaft), or flexible. Axes can only be straight. Straight shafts can be: smooth, stepped (in most cases), flanged, and gear shafts.

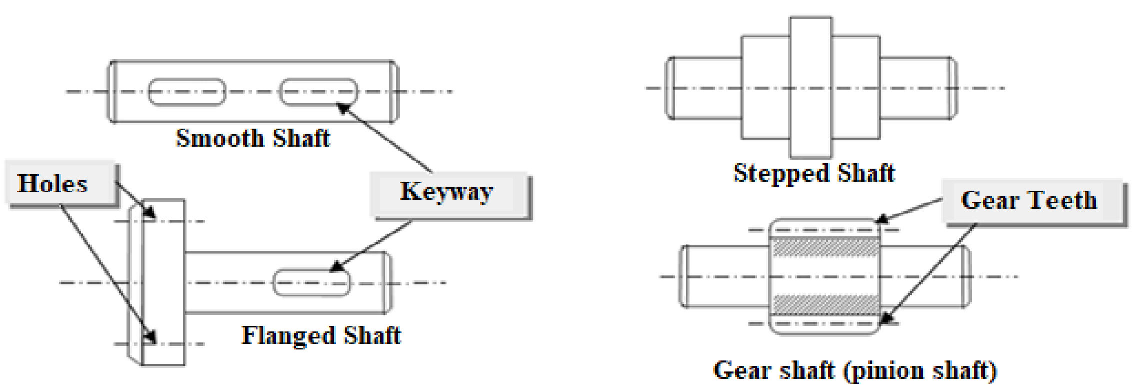


Figure IV.2 Types of Shafts

According to the geometric shape of the cross-section, shafts and axles can be categorized as: solid, hollow, splined, etc.

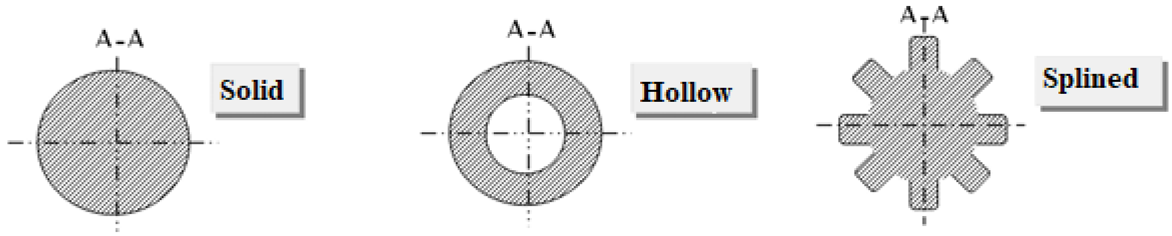


Figure IV.3 Straight Section of Shafts

The supporting parts of shafts and axles are called journals. The journals located at the ends of the shafts and subject to radial reactions are referred to as ends. The intermediate journals are called collars. The journals subjected to axial reactions are known as pivots.

Different types of ends include cylindrical, conical, and spherical.

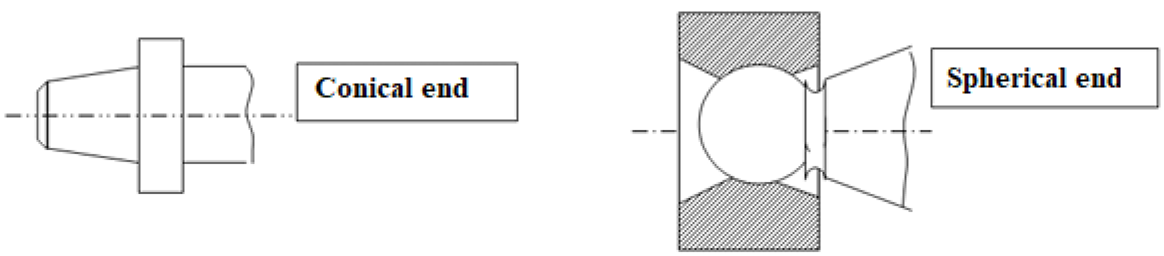


Figure IV.4 Supports of Shafts and Axles

Depending on its designated role, a shaft is subjected to bending, torsion, or a complex loading involving torsion, bending, and axial loads of tension or compression. The preliminary design calculations for a shaft are carried out:

- Using classical strength of materials (RDM) calculations:
  - ✓ For strength;
  - ✓ For deformation.
- Utilizing fatigue theory.
- Through vibration analysis (critical speed).

The chosen criterion depends on the geometry and the specifications imposed by the potential function of the shaft.

IV.2 Bearings

IV.2.1 Introduction

Sliding with friction in the case of bushings has been replaced by bearings in order to reduce friction.

Figure IV.5 illustrates a ball bearing with its various components. The outer ring is positioned in the housing of the casing and is fitted to its bore, while the inner ring is mounted on the shaft. The cage maintains a consistent spacing between the rolling elements, which can be either balls, rollers, or needles.

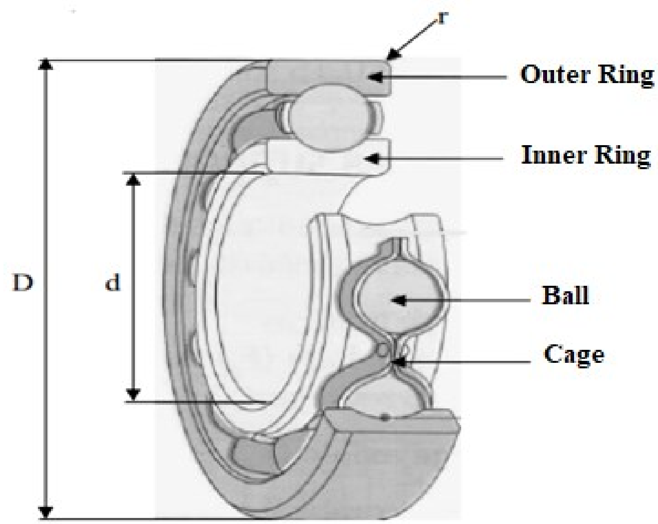


Figure IV.5 Components of a Ball Bearing

The inner ring rotates at a speed N, while the ball rotates at a significantly lower speed of N (rpm). The friction of the ball against the inner ring is minimal due to the small contact area; consequently, the power dissipated due to friction P_{pr} will be low and is given by:

$$\boldsymbol{P_{pr} = F_r . V_{bille}} \qquad \text{(IV. 1)}$$

Where $\boldsymbol{Fr}$ is the friction force and $\boldsymbol{V_{ball}}$ is the speed of the ball.

Bearings are manufactured from either alloy or non-alloy steel with a high yield strength (elastic limit) (e.g., 100C6, C60, etc.).

IV.2.2 Types of Bearings

Table IV.1 presents the different types of bearings and the loads they can support. Indeed, bearings are designed to accommodate either a radial load, an axial load, or a combined load (both radial and axial).

Bearings	Type of Bearing	Load Capacity	Characteristics
	Single Row Radial Ball Bearing	Supports relatively large axial and radial loads.	Requires good coaxiality of the shaft and hole.
	Double Row Radial Ball Bearing	Supports significant radial loads and moderate axial loads.	Requires good coaxiality of the shaft and hole.
	Single Row Angular Contact Ball Bearing	Supports relatively high axial loads (in one direction) and moderate radial loads.	Recommended for high rotational speeds; requires good coaxiality; does not tolerate alignment defects.
	Double Row Angular Contact Ball Bearing	Supports fairly large radial loads and alternating axial loads (in both directions).	Acceptable rotational speeds are lower than those for single row angular contact ball bearings.
	Tapered Roller Bearing	Supports relatively large radial and axial loads (in one direction).	Used in pairs and mounted oppositely; allows for high rotational speeds and requires good coaxiality; adjustable operational play is possible.
	Single Row Cylindrical Roller Bearing	Supports only high radial loads with no axial or combined loads.	Contact is made along a line, thus the radial load capacity is high; allows for high rotational speeds and requires good coaxiality.

Bearings	Type of Bearing	Load Capacity	Characteristics
	Spherical Roller Bearing (Double Row Cylindrical Rollers)	Supports high radial loads and medium alternating axial loads.	High friction necessitates oil lubrication; operates at medium rotational speeds.
	Ball Thrust Bearing	Supports only relatively large axial loads.	Rotational speed is low to reduce the centrifugal force applied to the balls; used for vertical shafts that are heavily loaded axially and rotate slowly.
	Needle Thrust Bearing	Supports only relatively large axial loads.	Allows both sliding and rolling; sensitive to shock; features a compact design.
	Needle Roller Bearing (with or without inner ring)	Supports only high radial loads with no axial or combined loads.	Allows slight axial displacement (axial play) of the shaft relative to the hole.

Table IV.1: The Different Types of Bearings

IV.2.3 Bearing Symbols

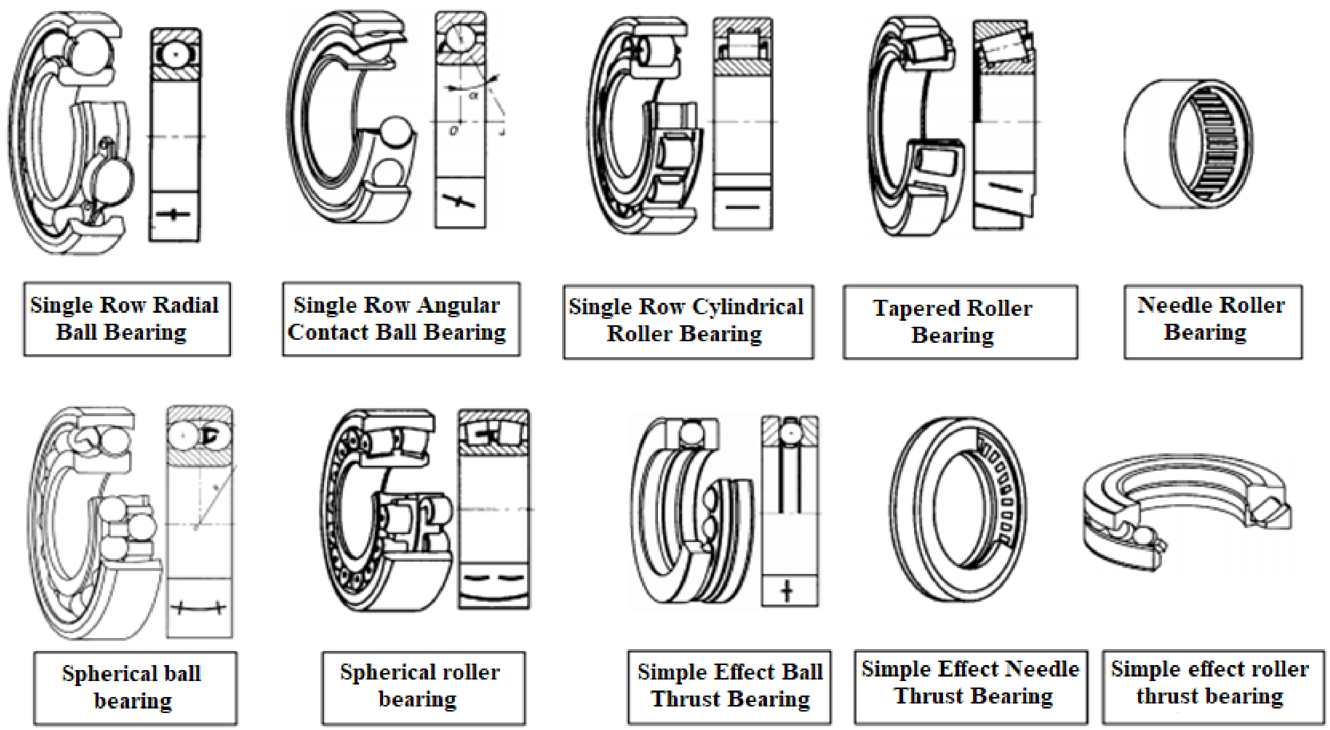


Figure IV.6 Bearing Symbols

IV.2.3.1 Real and Schematic Representations

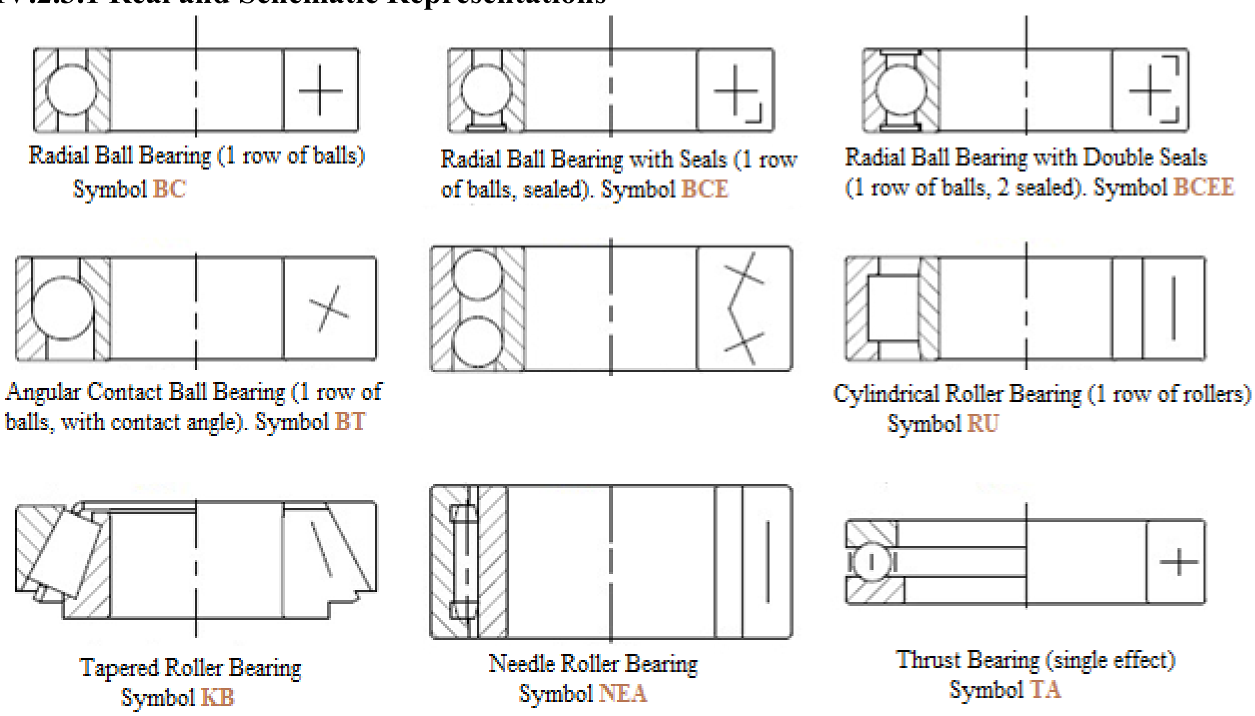
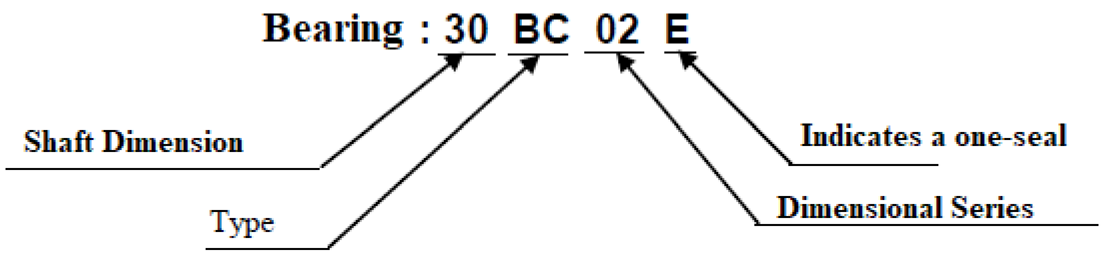


Figure IV.7 Real and Schematic Representations

IV.2.3.2 Designation of a Bearing



IV.2.4 Loads Supported by Bearings

The mechanical contact forces exerted by the rolling elements on either ring are generally represented by forces or loads (Figure IV.8).
Three cases are observed: Axial, Radial, and Combined.

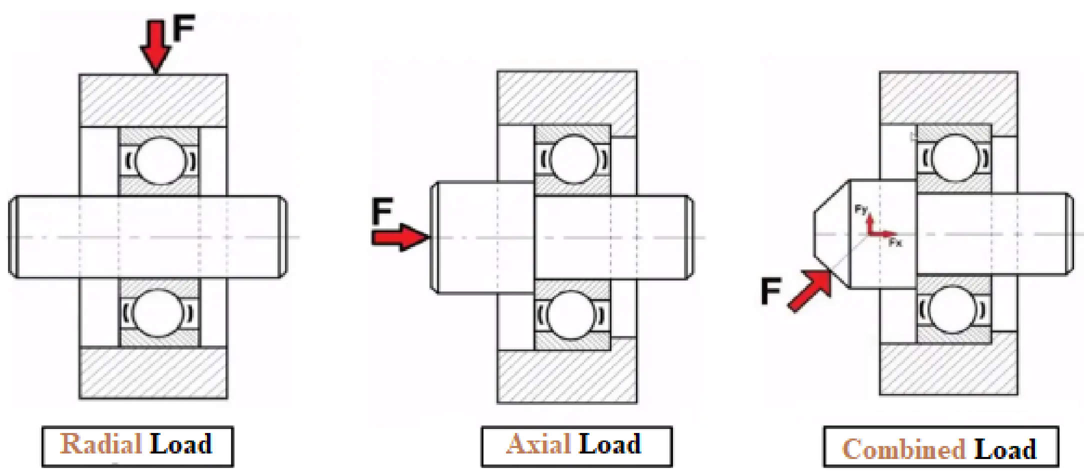


Figure IV.8 Loads Supported by a Bearing

IV.2.5 Selection of Bearings

The choice of the type of bearings to be used depends on the specific technical requirements of each case (required lifespan, significance of loads applied to the bearing, available space, rotational speed, operating temperatures, clearances, precision, lubrication) and can only be made with a thorough understanding of the technical characteristics of each type.

IV.2.6 Bearing Calculation and Sizing

IV.2.6.1 Bearing Lifetime

The lifetime of a bearing is defined by the number of revolutions or hours required, at a given constant speed, before the bearing shows the first signs of material fatigue.

$$L_h = \left(\frac{C}{P}\right)^n \cdot \frac{10^6}{60.N} \qquad (\text{IV}.2)$$

Where:

- L_h: Nominal lifetime in hours of operation.
- *C*: Basic dynamic load rating, in Newtons (N).
- *P*: Equivalent dynamic load, in Newtons (N).
- *n*: Exponent
  - n=3: for ball bearings
  - n=$\frac{10}{3}$: for roller bearings
- N: Revolutions per minute (RPM).

Let's consider two groups of bearings subjected to different load conditions F_1 and F_2. Their respective lifetimes L_1 and L_2 can be calculated using the following equation:

$$\frac{L_1}{L_2} = \left(\frac{F_2}{F_1}\right)^n \qquad (\text{IV}.3)$$

a) Relation Between Lifetime L_{10} and Basic Dynamic Load C

The basic dynamic load C is the constant load that a bearing can support to achieve a nominal lifetime of 1 million revolutions of the inner ring while the outer ring is fixed. For radial bearings, the nominal load is a radial load applied at the center of the bearing, with a constant magnitude and direction. For thrust bearings, the nominal load is a constant thrust load, parallel to the axis of the bearings. The load rating C for NSK bearings has been calculated according to ISO 281.

The relationship between the basic dynamic load of the bearing, the applied load, and the lifetime is given by the following formula:

$$L_{10} = \left(\frac{C}{P}\right)^n \qquad (\text{IV}.4)$$

Where:

- L_{10}: Nominal lifetime in millions of revolutions.
- P: Applied load.

If the load P on the bearing and the speed N are known, the appropriate dynamic factor for the machine's lifetime can be determined, and then the basic dynamic load can be calculated using the following formula:

$$C = \frac{f_h . P}{f_n} \qquad \text{(IV. 5)}$$

Where:

- f_h: Lifetime factor.
- f_n: Speed factor.

A bearing that meets this value of C should be selected from the catalogs.

b) Equivalent Dynamic Loads

When a bearing is subjected simultaneously to a radial force and an axial force, the equivalent dynamic load is given by:

$$P = X . F_r + Y . F_a \qquad \text{(IV. 6)}$$

Where:

- Fr: Radial component of the load, in Newtons (N).
- Fa: Axial component of the load, in Newtons (N).
- X: Radial load factor.
- Y: Axial load factor.

The values of X and Y can be found in the dimensional tables of our general catalog. For a single-row ball bearing, the axial load is not considered as long as the ratio Fa/Fr is below a certain value ***e*** (see table). For a double-row ball bearing, even a small axial load will influence the equivalent load.

To calculate a given bearing, it is preferable to use the values provided by the manufacturer, as they have been established based on geometry and the ratio Fa/Fr.

From the dimensions of the bearing and the manufacturer's catalog, C_0 is determined, then Fa/C_0 is calculated, and subsequently, the corresponding values of ***e*** and Y are determined using Table IV.2.

$\frac{F_a}{C_0}$	0,014	0,028	0,056	0,084	0,110	0,170	0,283	0,42	0,5
X	0,56	0,56	0,56	0,56	0,56	0,56	0,56	0,56	0,56
Y	2,3	1,99	1,71	1,55	1,45	1,31	1,15	1,04	1,00
e	0,19	0,22	0,26	0,28	0,30	0,34	0,38	0,42	0,44

Table IV.2: Factors X and Y for a Radial Contact Ball Bearing

The equivalent dynamic load **P** will be calculated based on the following conditions:

- If $\frac{F_a}{F_r} \leq e, then\ P = F_r (where\ X = 1, Y = 0)$
- If $\frac{F_a}{F_r} > e, then\ P = 0{,}56. F_r + Y. F_a$

b) Equivalent Static Load

For bearings that are stationary or rotating at very low speeds (n < 20 rpm), the basic static load C_0 is considered, and the equivalent static load is determined as follows:

$$P_0=X_0.F_{0r}+Y_0.F_{0a} \qquad \text{(IV.7)}$$

Where:

- F_{0r}: Radial static load, in Newtons (N).
- F_{0a}: Axial static load, in Newtons (N).
- X_0: Radial static load factor.
- Y_0: Axial static load factor.

The values of factors X_0 and Y_0 are provided in dimensional tables.

This is valid for all bearings except for thrust roller bearings subjected to both radial and axial loads, where:

$$P_0 = F_{0a} + 2{,}7\ F_{r0} \qquad \text{(IV. 8)}$$

The required basic static load is:

$$C_0 = P_0. f_s \qquad \text{(IV. 9)}$$

Where:

- P_0: Equivalent static load (N).
- *fs*: Correction factor.
- *fs* ≥2: For bearings subjected to vibrations or shocks and also meeting high precision requirements.
- *fs* =1: For normal operation.
- *fs* =0.5… 1: For operation without vibration.

IV.2.6.2 Reliability of a Bearing

In many application cases, it is desirable to select a bearing with a reliability other than 90%. Statistical studies on bearing failures have established a relationship between lifetime and reliability. The reliability **R** can be calculated using the following equation:

$$R = exp\left[-\left(\frac{L_{hR}}{m.L_{10}}\right)^b\right] \qquad \text{(IV. 10)}$$

Where:

- L_{hR}: Desired lifetime, expressed in hours.

- L_{10}: Lifetime in hours corresponding to a reliability of 90%.
- m, b: Constants.

IV.2.7 Bearing Installation

If the inner ring of a bearing (see Figure IV.9) is mounted with radial clearance on the rotating shaft, the balls and the shaft behave like roller elements in a mill, and the inner ring risks being rolled, which leads to progressive deterioration of the bearing. To avoid this phenomenon, the inner ring must be mounted with a tight fit; if the housing is rotating, then the outer ring should also be mounted with a tight fit.

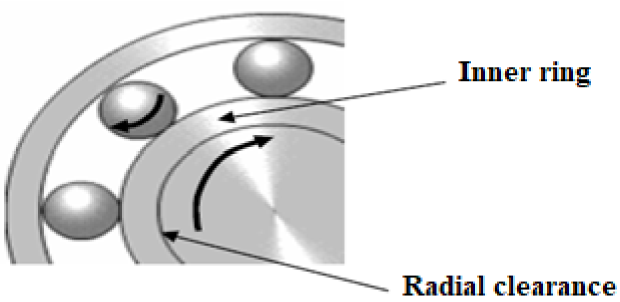


Figure IV.9: Phenomenon of Ring Rolling

IV.2.7.1 Rolling

Rolling refers to the wear of the shaft or hub due to the friction of the ring connected to the moving part (hub or shaft). There are two solutions to avoid this problem:

a) Tight fit of the ring with the moving part.

b) Economical solution but difficult to achieve by gluing.

IV.2.7.2 Thermal Expansion

Even slight friction within the bearing generates heat. As a result, due to the shape of the shaft, it expands much more than the hub. If the bearings are mounted tightly on both rings, this expansion cannot occur, leading to rapid wear of the bearings.

a) **Installation of Single-Row Ball Bearings**

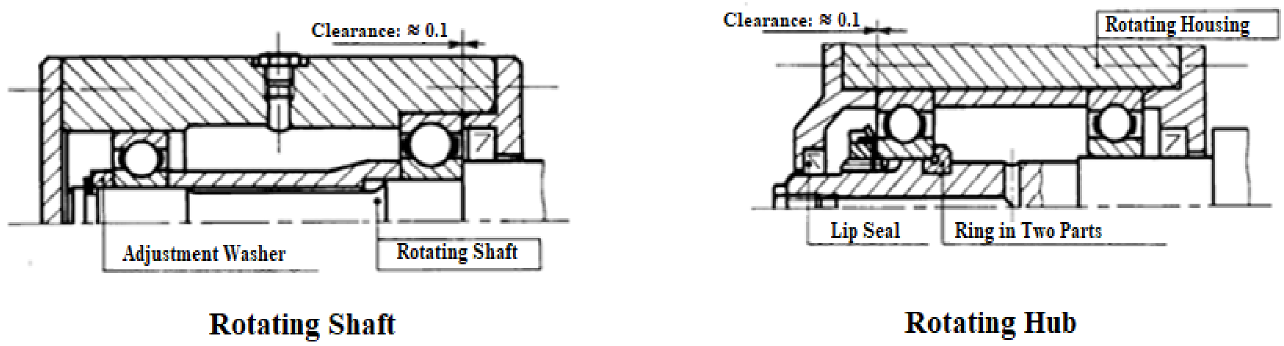
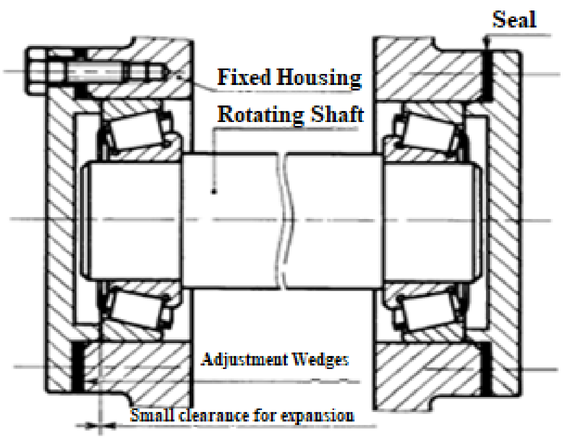
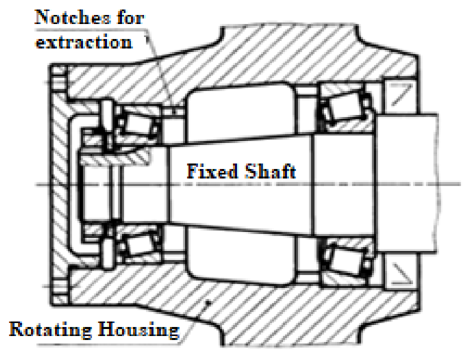


Figure IV.10: Single Row Ball Bearing Assemblies

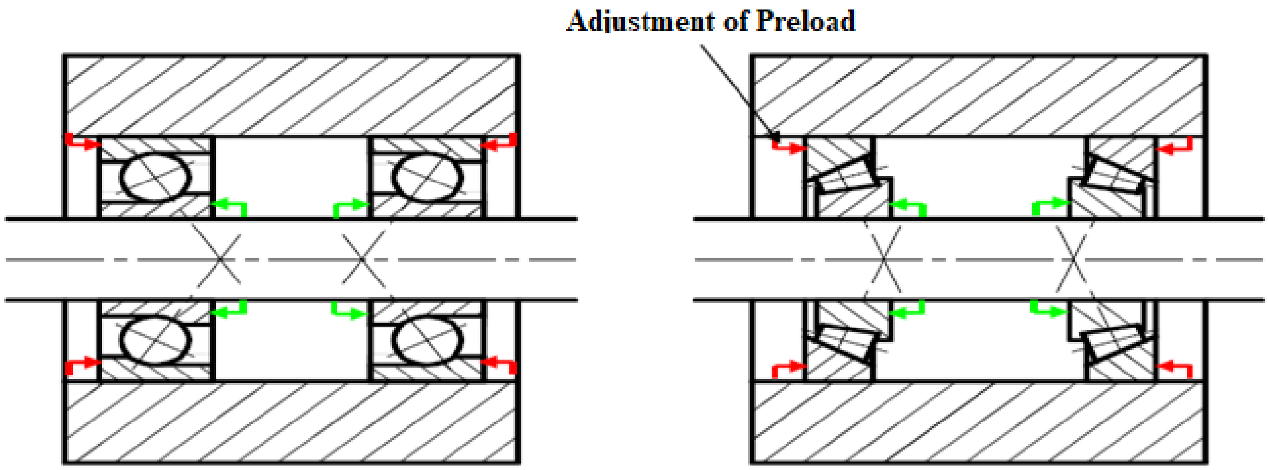
b) Assemblies of Tapered Roller Bearings or Angular Contact Bearings



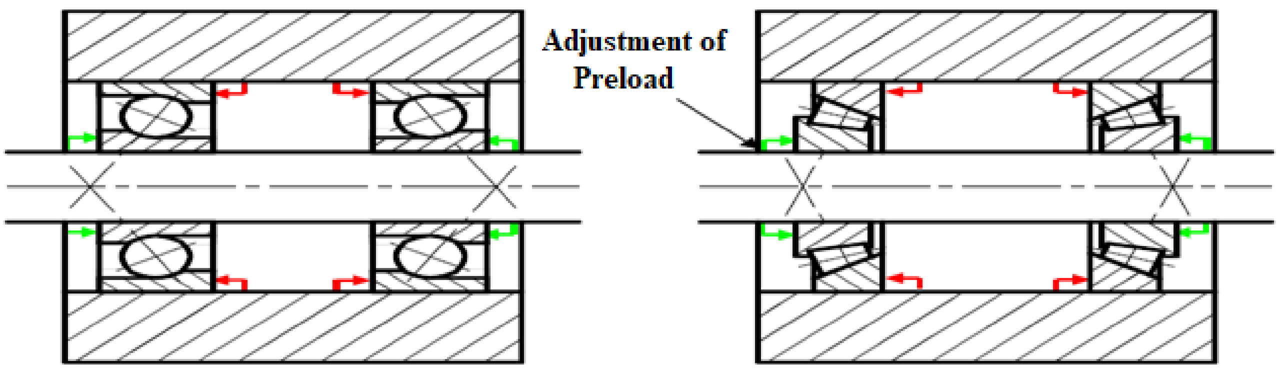
Rotating Shaft



Rotating Hub



(a) Assembly X - Rotating Shaft



(b) Assembly O - Rotating Hole

Figure IV.11: Assembly of Roller Bearings or Angular Contact Bearings

IV.2.8 Lubrication of Bearings

The purpose of lubrication in bearings is to protect against rust pitting, corrosion caused by air and water, prevent dust ingress, dissipate heat, cool the bearing components, reduce friction on the raceways, and facilitate movement. At low temperatures, a semi-fluid oil is used; for high temperatures, a slightly less fluid lubricant (grease) is utilized. There are also bearings available that are lubricated for life.

IV.2.9 Sealing

A bearing must be protected against atmospheric dust, metal particles, abrasives, water, steam, etc. It is essential to prevent lubricant loss; thus, bushings should be sealed as effectively as possible using sealing elements. The choice of sealing device depends on the nature of the lubricant (which must not escape) and the surrounding environment (which must not enter), as well as the rotation speed.

IV.11 Advantages and Disadvantages of Bearings

The advantages of using bearings include:

- Reduced friction between the shaft and the bore due to the balls.
- Capability to support radial, axial, or combined loads and high rotation speeds.
- Wear is transferred to the bearing, protecting the bore and shaft.
- Extended lifespan.
- Moderate lubrication requirements, unlike plain bearings, which require a significant amount of grease or oil to ensure proper functioning.
- Simplified assembly and maintenance compared to plain bearings.
- Provides good guidance (no radial play, axial play, or angular misalignment).

The main disadvantage of using bearings is that they can sometimes be noisy, and their cost is relatively high.