University of M'sila
Faculty of Technology
Department of Electrical Engineering
(Date 15/06/2026)

Resit examination

academic year 2025/2026
field: ST
module: vibrations and waves
(duration : 1h30min)

I)Theoretical questions(6pts)

Choose the correct answer(s):

1- The natural angular frequency ω_0 of a Simple Harmonic Oscillator (SHO) (m, k) is:

a-$\sqrt{km}$, b-$\sqrt{\frac{k}{m}}$, c-$\sqrt{\frac{m}{k}}$.

2- A lightly damped SHO satisfies the condition: a-$\delta < \omega_0$, b-$\delta > \omega_0$, c- $\delta = \omega_0$

3- The logarithmic decrement D of a lightly damped SHO is equal to: a-δT_a, b-$\frac{2\pi\xi}{\sqrt{1-\xi^2}}$, c- $\delta\omega_a$

4- At resonance, the amplitude of an oscillating system is: a- zero, b- maximum, c- minimum

5- A two-degree-of-freedom system has:
a- one generalized coordinate, b- two generalized coordinates, c- three generalized coordinates

6- A two-degree-of-freedom vibrating system has:
a- one natural mode of vibration, b- two natural modes of vibration, c- not available.

Exercise №1(8pts)

A mass $m = 10kg$ whose dimensions are negligible, is mounted on two vertical springs, each having a stiffness $k = 3500N/m$, and a viscous damper with damping coefficient $\alpha = 30kg/s$ (fig.1). At $t = 0$, the mass is displaced by $5cm$ from its equilibrium position and then released without initial velocity. At equilibrium, the two springs are compressed by the same amount Δl (parallel arrangement).

1. Determine the equivalent spring stiffness k_e.
2. Derive the Lagrangian $\mathscr{L}$ of the system and the dissipation function $\mathcal{D}$.
3. Study the damped free motion of the system. Then, determine the displacement $x(t)$ (measured with respect to the equilibrium position) as a function of time.
4. A vertical harmonic force $F(t) = F_0\, cos(\omega t)$ is applied to the mass. Determine the vibration amplitude of m as a function of ω. For which value of F_0 the vibration amplitude at resonance will not exceed $0.2m$?

Note: Assume that at resonance $\omega_r \approx \omega_0$.

Exercise №2(6pts)

Consider the circuit shown in fig.2.

1. Derive the differential equation governing the oscillations in the circuit in terms of the voltage V_C across the capacitor C.
2. Assume that $e(t) = 0$ (free oscillations). The two terminals of the capacitor are connected to the input of an oscilloscope. The waveform shown in fig.3 is observed on the screen. Write the expression of $v_c(t)$. Determine the values of ω_a and δ (the damped angular frequency and the damping coefficient, respectively). So, deduce the values of the inductance L and the resistance R, given that $C = 0.1\mu F$
3. Knowing that $e(t) = E_m \cos\omega_e t$, give the form of the oscillations in the steady-state regime and show that resonance can occur in the circuit by calculating the damping ratio ξ.
4. Calculate the angular frequency ω_R at resonance and the bandwidth $\Delta\omega$.

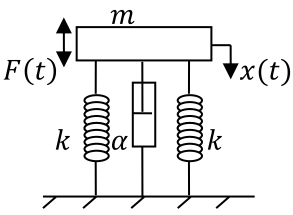


fig.1

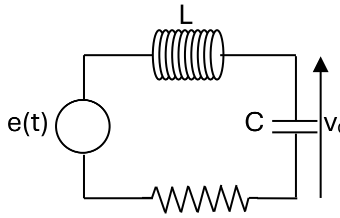


fig.2

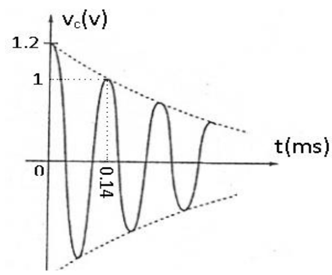


fig.3

Good Luck

جامعة محمد بوضياف بالمسيلة كلية التكنولوجيا قسم الهندسة الكهربائية (التاريخ: 2026/06/15)	**إمتحان الدورة الإستدراكية**	السنة الجامعية 2026/2025 ميدان التكنولوجيا المادة: الاهتزازات والأمواج (المدة: 1سا و30د)

أسئلة نظرية(6ن)

إختر الجواب أو الأجوبة الصحيحة:

1- يعطى النبض الذاتي ω_0 لهزاز توافقي بسيط (m,k) بالعبارة: a- $\sqrt{km}$، b- $\sqrt{\frac{k}{m}}$، c- $\sqrt{\frac{m}{k}}$

2- يحقق الهزاز التوافقي البسيط ذو التخامد الضعيف الشرط التالي: a- $\delta < \omega_0$، b- $\delta > \omega_0$، c- $\delta = \omega_0$

3- يعطى التناقص اللوغرتمي D بالعبارة: a- δT_a، b- $\frac{2\pi\xi}{\sqrt{1-\xi^2}}$، c- $\delta\omega_a$

4- تأخذ سعة الاهتزاز لنظام مهتز عند الرنين قيمة: a- معدومة، b- أعظمية، c- أصغرية

5- يقبل نظام مهتز ذو درجتين من الحرية: a- إحداثي معمم واحد، b- إحداثيين معممين، c- ثلاث إحداثيات معممة

6- يقبل نظام مهتز ذو درجتين من الحرية: a- نمط أساسي واحد من الاهتزاز، b- نمطين أساسيين من الاهتزاز، c- لا أحد من الحالتين

التمرين الأول(6ن)

تستند كتلة $m = 10kg$ ذات أبعاد مهملة على نابضين شاقوليين ثابت مرونة كل منهما $k = 3500N/m$ وعلى مخمد معامل تخامده اللزج $\alpha = 30kg/s$ (الشكل1). في اللحظة $t = 0$ نزيح الكتلة عن وضع التوازن بمقدار $5cm$ ثم نحررها دون سرعة ابتدائية. علما أن النابضين يتقلصان عند التوازن بنفس المقدارΔl (النابضان على التوازي)

1) جد ثابت المرونة k_e للنابض المكافىء.

2) جد دالة لاغرونج $\mathscr{L}$ ودالة التبدد $\mathcal{D}$.

3) أدرس الحركة الحرة المخمدة. جد إذن الإزاحة $x(t)$ (بالنسبة لوضع التوازن) بدلالة الزمن t.

4) نطبق على الكتلة قوة عمودية $F(t) = F_0\, cos(\omega t)$. أحسب سعة الاهتزاز للكتلة m بدلالة ω. ما هي قيمة F_0 التي من أجلها لا تتعدى سعة الاهتزاز عند الرنين $0.2m$.

ملاحظة: نقبل أن الرنين يحدث من أجل $\omega_r \approx \omega_0$.

التمرين الثاني(8ن)

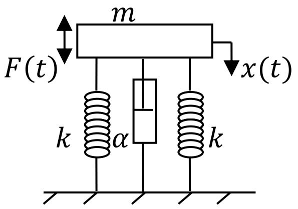
لتكن الدارة RLC المبينة في الشكل2.

1- جد المعادلة التفاضلية للاهتزازات في الدارة بدلالة الفرق في الجهد V_C بين طرفي المكثفة C

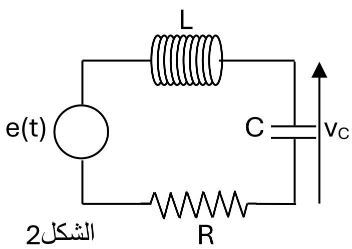
2- نعتبر $e(t) = 0$ (اهتزاز حر) . نربط طرفي المكثفة بمدخل راسم الاهتزاز فنلاحظ على شاشته الاهتزاز الممثل في الشكل3. أكتب عبارة $v_c(t)$. حدد قيمة كل من δ و ω_a (معامل التخامد وشبه نبض الاهتزاز على التوالي) ثم استنتج قيمة الذاتية L و المقاومة R. يعطى $(C = 0.1\mu F)$

3- نعتبر $e(t) = E_m \cos\omega_e t$. أعط شكل الحل الدائم وبين إمكانية حدوث الرنين في الدارة (بحساب نسبة التخامد ξ)

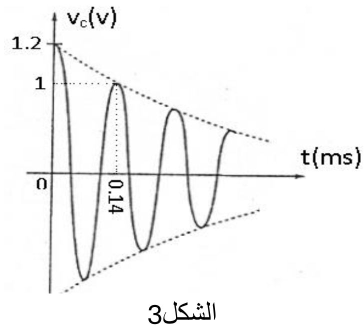
4- أحسب النبض ω_R عند الرنين والحزمة العابرة $\Delta\omega$.



الشكل1



الشكل2



الشكل3

بالتوفيق

Model Answer of Resit Exam in Vibrations and Waves

Date: 15/06/2026

Theoretical Questions (6 pts)

1- b,	2- a,	3- a and b,	4- b,	5- b,	6- b
1.00	1.00	1.00	1.00	1.00	1.00

Exercise №1 (8 pts)

1. The two springs are connected in parallel so:

$k_e = k + k = 3500 \times 2 = 7000\, N/m.$ 0.50

The equivalent system is then represented in the fig opposite:

0.50

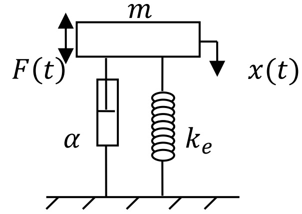


fig.1

2. Expressions of $\mathcal{L}(x, \dot{x})$ and $\mathcal{D}$:

2.a. $\mathcal{L}(x, \dot{x}) = T - U = 1/2\, m\, \dot{x}^2 - 1/2\, ke\, x^2$ 0.50

2.b. $\mathcal{D} = 1/2\, \alpha\, \dot{x}^2$ 0.50

3. For a damped free vibration, the Lagrange's equation is:

$$d/dt(\partial\mathcal{L}/\partial\dot{x}) - \partial\mathcal{L}/\partial x + \partial\mathcal{D}/\partial\dot{x} = 0$$ 0.50

So, after deriving $\mathcal{L}$ and $\mathcal{D}$ partially with respect to x and $\dot{x}$ we get :

$$\ddot{x} + 2\delta\dot{x} + {\omega_0}^2 x = 0$$ 0.50

with:

$2\delta = \frac{\alpha}{m} = 30/10 = 3 \Rightarrow \delta = 1.5\, s^{-1}$ 0.50

$\omega_0^2 = \frac{k_e}{m} = 700 \Rightarrow \omega_0 = \sqrt{700} \approx 26.5\, rad/s$ 0.50

Since $\delta < \omega_0$, then the system undergoes lightly damped oscillations:

Therefore :

$x(t) = Ae^{-\delta t}\cos(\omega_a t + \varphi)$ with $\omega_a = \sqrt{\omega_0^2 - \delta^2} = 26{,}4 rad.s^{-1}$ 1.00

Knowing that at $t = 0$:

$$x(0) = 5\, cm \Rightarrow x(0) = A\cos\varphi = 5cm$$
$$\dot{x}(0) = 0 \Rightarrow -A\delta\cos\varphi - A\omega_a\sin\varphi = 0$$

It's a set of two equations of two unknowns A and φ

One obtains:

1.00

$$\varphi = -0.06\, rad$$
$$A = 5.01\, cm$$

Finally:

$$x(t) = 5.01e^{-1.5t}\cos(26.4\, t - 0.06)$$

4. For a forced vibration the system admits the Lagrange's equation of the form:

$$d/dt(\partial\mathcal{L}/\partial\dot{x}) - \partial\mathcal{L}/\partial x + \partial\mathcal{D}/\partial\dot{x} = F_0 \cos(\omega t)$$

Therefore:

$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = (F_0/m)\cos(\omega t)$ $\boxed{0.50}$

so $x(t)$ is of the form: $\boxed{0.50}$

$x(t) = x_0 \cos(\omega t + \theta)$

Using the complex method one obtains the amplitude x_0 as follows:

Amplitude:

$x_0 = (F_0/m) / \sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}$ $\boxed{0.50}$

assuming that $\omega_r \approx \omega_0$ then at resonance, the amplitude is maximum and equal to :

$$(x_0)_{max} = {F_0/m}\Big/{2\delta\omega_0}$$

if we suppose that $(x_0)_{max} \lesssim 0.2m$ $\boxed{0.50}$

one obtains:

$$(x_0)_{max} = {F_0/m}\Big/{2\delta\omega_0} \lesssim 0.2 \;\Rightarrow\; F_0 \lesssim 158.7N$$

Exercise №2 (6pts)

1- Kirchhoff's voltage law gives:

$$V_L + V_R + V_C = e(t) \quad \boxed{0.50}$$

where: $V_R = Ri = RC\dot{V}_C$ and $V_L = L\frac{di}{dt} = LC\ddot{V}_C$

so, we get:

$$\ddot{V}_C + 2\delta\dot{V}_C + \omega_0^2 V_C = \omega_0^2 e(t) \quad \boxed{0.50}$$

With : $2\delta = {R}/{L}$ and $\omega_0^2 = {1}/{LC}$ $\boxed{0.50}$

2- If $e(t) = 0 \;\Rightarrow\; \ddot{V}_C + 2\delta\dot{V}_C + \omega_0^2 V_C = 0$ $\boxed{0.50}$

the graph observed on the screen of the oscilloscope shows that the system undergoes lightly damped oscillations and therefore the solution of the above differential equation is of the form:

$$V_C(t) = V_0 e^{-\delta t}\cos(\omega_a t + \varphi) \quad \boxed{0.50}$$

From the graph we get:

$$V_0 = 1.2V$$

$$T_a = 0.14ms \;\Rightarrow\; \omega_a = {2\pi}/{T_a} = 44880 rad.s^{-1} \quad \boxed{0.50}$$

Algorithmic decrement D is given by:

$$D = \delta T_a = \ln\left({1.2}/{1}\right) \;\Rightarrow\; \delta = 1302.3 s^{-1} \quad \boxed{0.50}$$

The relationship between ω_a and ω_0 is: $\omega_a = \sqrt{\omega_0^2 - \delta^2} \;\Rightarrow\; \omega_0^2 = \omega_a^2 + \delta^2 \;\Rightarrow \omega_0 = 44899 rad.s^{-1}$

on the other hand, we have: $\omega_0^2 = {1}/{LC} \;\Rightarrow L = {1}/{C\omega_0^2} = 5mH$ $\boxed{0.50}$

and: $2\delta = {R}/{L} \;\Rightarrow\; R = 2\delta L = 13.02\Omega$

3- In presence of $e(t)$ (e.m.f) the circuit undergoes forced oscillations, so the angular frequency is the same as of the external excitation $e(t)$, then in steady state regime the form of $V_C(t)$ is:

$$V_C(t) = V_0\cos(\omega_e t + \theta) \quad \boxed{0.50}$$

The condition of resonance in the circuit is:

$$\xi < {1}/{\sqrt{2}} \quad \boxed{0.50}$$

however

$$\xi = {\delta}/{\omega_0} = 0.03 < {1}/{\sqrt{2}} \quad \boxed{0.50}$$

Therefore, the resonance is possible

The angular frequency at resonance is then: $\omega_R = \omega_0\sqrt{1 - 2\xi^2} = 44823;5 rad.s^{-1}$ $\boxed{0.50}$

While the bandwidth is: $\Delta\omega = 2\delta = 2604.6 rad.s^{-1}$

End