T.D Module maths 3

Exercice01 : Show that the following series is convergent and give its sum :

$a)\ \sum_{n\geq1}\frac{1}{n^2-4}$,b) $\sum_{n\geq1}\frac{1}{n^2+5n+4}$

Exercice02 Study the nature of the following series:

1)$\sum_{n\geq2}\frac{n^2+5n-1}{n(n+4)^{\frac{3}{2}}}$; $\sum_{n\geq1}\sqrt{\frac{n-\ln n}{n+4n^3}}$; $\sum_{n\geq0}\frac{(3n)!3^n}{n^{2n}2^{2n}}$; $\sum_{n\geq0}\frac{2^n+n^2}{3^n}$

Exercice03 Study the nature of the following series:

1)$a)\ \sum_{n\geq3}\frac{(-1)^n}{n^2+1}$; $b)\ \sum_{n\in N^*}\frac{(-1)^n}{n^n}$; $\sum_{n\geq1}\sin(\frac{(-1)^n}{n^2+1})$; $\sum_{n\geq0}\ln\left(1+\frac{(-1)^n}{n}\right)$

Exercice04

Find the following fourier series ;

$$\begin{cases} 1 & t\in[0.1[\\ t & t\in[-1.0[\end{cases} \quad \text{of period T=2}$$

Exercice05

Find the following fourier series ;$f(t)=|t|, |t|<2,$ of period 4

1) Graph this function over at least two periods.

2) Studying convergence .

3) Deduce the value of $\sum_{n=1}^{\infty}\frac{1}{(2n+1)^2}$, $\sum_{n=1}^{\infty}\frac{1}{(2n+1)^4}$

Exercice06

Find the following sinus fourier series ;$f(t)=t(\pi-t)$,$t\in(0,\pi)$

1) Studying convergence .

2) Deduce the value of $\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)^3}$, $\sum_{n=1}^{\infty}\frac{1}{(2n+1)^6}$

Exercice07

Find the fourier series

$$\begin{cases} \cos(t) & 0<t<\pi \\ -\cos(t) & -\pi\leq t<0 \end{cases}$$

Deduce the value of $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2n-1)}{(4n-1)(4n-3)}=\frac{\pi}{8\sqrt{2}}$

Exercice08

Find the fourier tronsform :

$$\begin{cases} 1 & |t| < 1 \\ \frac{1}{2} & |t| = 1 \\ 0 & |t| > 1 \end{cases}$$

Deduce the value of $\int_0^{\infty} \frac{sin\alpha cos\alpha x}{\alpha} d\alpha$

Exercice09

Find the image by the Laplace transform of:

a)f(t)= 1+cost-e^t , b) f(t)=$2t + e^{3t}$cos3t , c)$t^2 e^{-t}$, c) $\frac{sin2t}{t}$, d) $\frac{1-e^{-t}}{t}$

Calculate the value of: $\int_0^t \frac{sint}{t}$dt et $\frac{e^{-at}+e^{-bt}}{t}$, and a,b>0

b) Find the inverse of the “origin” by the Laplace transform

$F(p) = \frac{p^2+p+9}{p(p^2+9)}$, F(p)=$\frac{1}{(p-4)^3}$, $F(p) = \frac{p}{p^2-p-2}$, $F(p) = \frac{e^{-p}}{p^2+p+2}$, $F(p) = log\left(\frac{p+3}{p+2}\right)$

c) Use the Laplace transform to solve the following equation

$\begin{cases} y'(t) + 2y(t) = e^{-t} \\ y(0) = 2 \end{cases}$, $\begin{cases} y'(t) - 2y(t) = sint \\ y(0) = 1 \end{cases}$