Chapter II. Dynamics of Gears

II.1. Introduction

In designing a gear, it is important to analyze the magnitude and direction of the forces acting upon the gear teeth, shaft, bearings, etc. In analyzing these forces, an idealized assumption is made that the tooth forces are acting upon the central part of the tooth flank. Table II-1

Table II-1 Forces Acting Upon a Gear

Types of Gears		Tangential Force, F_u	Axial Force, F_a	Radial Force, F_r
Spur Gear		$F_u = \frac{2000\,T}{d}$	———	$F_u \tan\alpha$
Helical Gear			$F_u \tan\beta$	$F_u \frac{\tan\alpha_n}{\cos\beta}$
Straight Bevel Gear		$F_u = \frac{2000\,T}{d_m}$ d_m is the central pitch diameter $d_m = d - b\sin\delta$	$F_u \tan\alpha \sin\delta$	$F_u \tan\alpha \cos\delta$
Spiral Bevel Gear			When convex surface is working:	
			$\frac{F_u}{\cos\beta_m}(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta)$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n \cos\delta + \sin\beta_m \sin\delta)$
			When concave surface is working:	
			$\frac{F_u}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta)$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta)$
Worm Drive	Worm (Driver)	$F_u = \frac{2000\,T_1}{d_1}$	$F_u \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	$F_u \frac{\sin\alpha_n}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$
	Wheel (Driven)	$F_u \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	F_u	
Screw Gear $\left(\begin{matrix}\Sigma = 90^\circ \\ \beta = 45^\circ\end{matrix}\right)$	Driver Gear	$F_u = \frac{2000\,T_1}{d_1}$	$F_u \frac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	$F_u \frac{\sin\alpha_n}{\cos\alpha_n \cos\beta + \mu \sin\beta}$
	Driven Gear	$F_u \frac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	F_u	

II.2. Forces In A Spur Gear Mesh

The spur gear's transmission force *Fn*, which is normal to the tooth surface, as in Figure II–1, can be resolved into a tangential component, *Fu*, and a radial component, *Fr*.

$$F_u = F_n \cos\alpha_b, \qquad F_r = F_n \sin\alpha_b$$

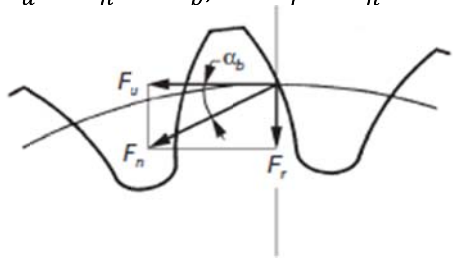

Fig. II–1 Forces Acting on a Spur Gear Mesh

The directions of the forces acting on the gears are shown in Figure II–2. The tangential component of the drive gear, Fu_1, is equal to, the driven gear's tangential component, *Fu*2, but the directions are opposite. Similarly, the same is true of the radial components.

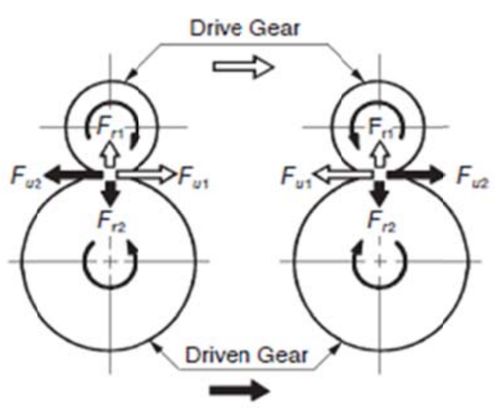

Fig. II–2 Directions of Forces Acting on a Spur Gear Mesh

II.3. Forces In A Helical Gear Mesh

The helical gear's transmission force, *Fn*, which is normal to the tooth surface, can be resolved into a tangential component, *F1*, and a radial component, *Fr*, as shown in Figure II–3.

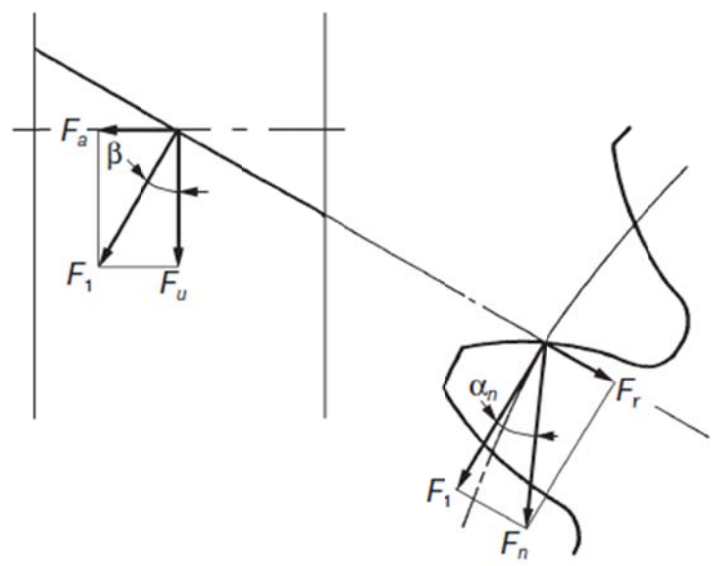

Fig. II–3 Forces Acting on a Helical Gear Mesh

$$F_1 = F_n \cos\alpha_n \qquad F_r = F_n \sin\alpha_n$$

The tangential component, F_1, can be further resolved into circular subcomponent, F_u, and axial thrust subcomponent, F_a.

$$F_u = F_1 \cos\beta \qquad F_a = F_1 \sin\beta$$

Substituting and manipulating the above equations result in:

$$F_a = F_u \tan\beta \qquad F_r = F_u \frac{\tan\alpha_n}{\cos\beta}$$

The directions of forces acting on a helical gear mesh are shown in Figure II–4. The axial thrust sub-component from drive gear, F_{a1}, equals the driven gear's, F_{a2}, but their directions are opposite. Again, this case is the same as tangential components F_{u1}, F_{u2} and radial components F_{r1}, F_{r2}.

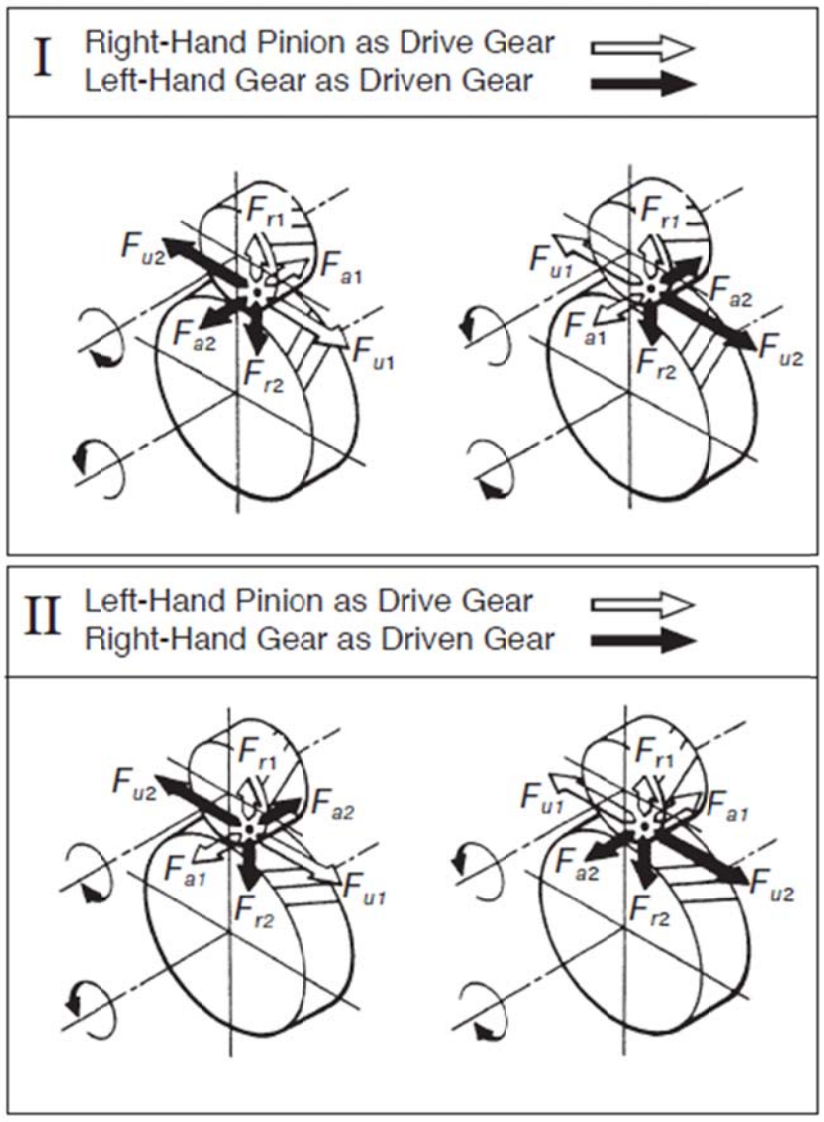

Fig. II–4 Directions of Forces Acting on a Helical Gear Mesh

II.4. Forces On A Straight Bevel Gear Mesh

The forces acting on a straight bevel gear are shown in Figure II–5. The force which is normal to the central part of the tooth face, *Fn*, can be split into tangential component, *Fu*, and radial component, *F*1, in the normal plane of the tooth.

$$F_u = F_n \cos\alpha \qquad F_1 = F_n \sin\alpha$$

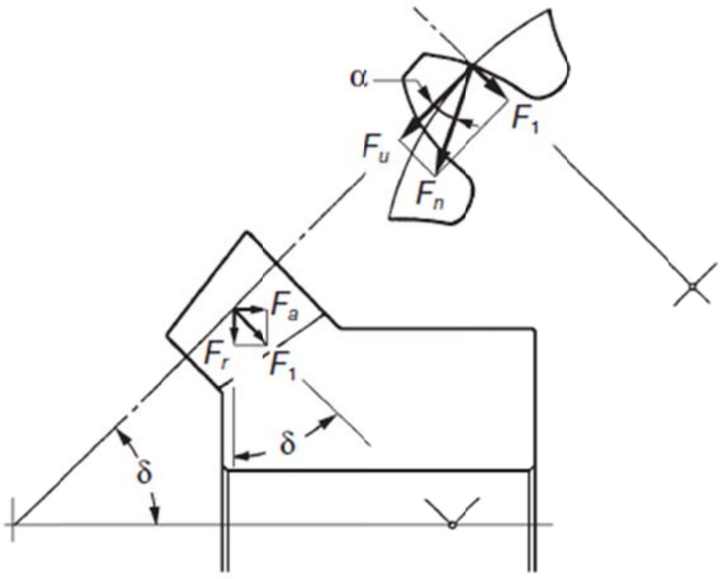

Fig. II–5 Forces Acting on a Straight Bevel Gear Mesh

Again, the radial component, F_1, can be divided into an axial force, F_a, and a radial force, F_r , perpendicular to the axis.

$$F_a = F_1 \sin\delta \qquad F_r = F_1 \cos\delta$$

And the following can be derived:

$$F_a = F_u \tan\alpha_n \sin\delta \qquad F_r = F_u \tan\alpha_n \cos\delta$$

Let a pair of straight bevel gears with a shaft angle Σ = 90°, a pressure angle α_n = 20° and tangential force, F_u, to the central part of tooth face be 100. Axial force, F_a, and radial force, F_r, will be as presented in Table II-2.

Table II-2 Values of Axial Force, F_a, and Radial Force, F_r

(1) Pinion

Forces on the Gear Tooth	Ratio of Numbers of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial Force	25.7	20.2	16.3	13.5	11.5	8.8	7.1
Radial Force	25.7	30.3	32.6	33.8	34.5	35.3	35.7

(2) Gear

Forces on the Gear Tooth	Ratio of Numbers of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial Force	25.7	30.3	32.6	33.8	34.5	35.3	35.7
Radial Force	25.7	20.2	16.3	13.5	11.5	8.8	7.1

Figure II–6 contains the directions of forces acting on a straight bevel gear mesh. In the meshing of a pair of straight bevel gears with shaft angle Σ = 90°, all the forces have relations as

$$F_{u1} = F_{u2}$$
$$F_{r1} = F_{a2}$$
$$F_{a1} = F_{r2}$$

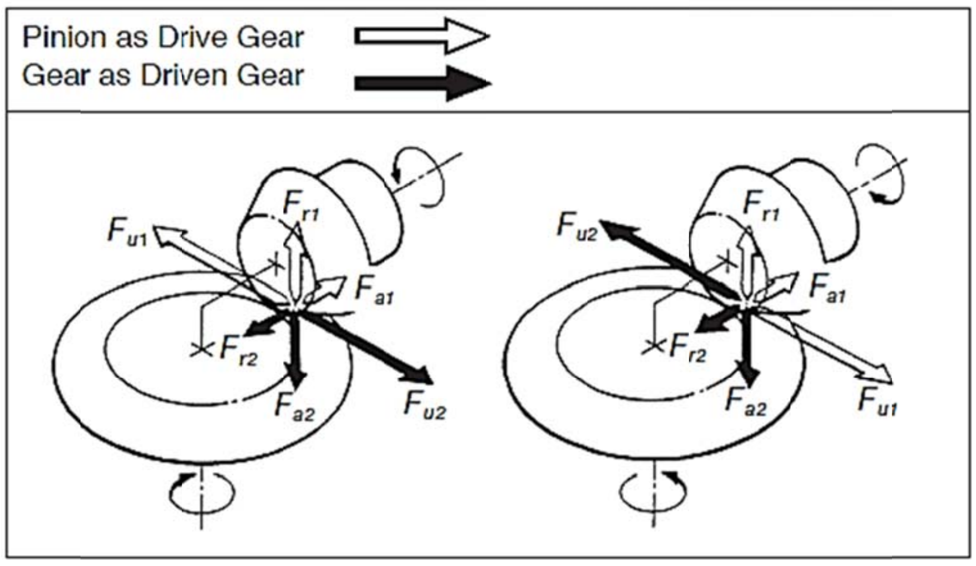

Fig. II–6 Directions of Forces Acting on a Straight Bevel Gear Mesh

II.5. Forces In A Spiral Bevel Gear Mesh

Spiral gear teeth have convex and concave sides. Depending on which surface the force is acting on, the direction and magnitude changes. They differ depending upon which is the driver and which is the driven. Figure II–7 presents the profile orientations of right- and left-hand spiral teeth. If the profile of the driving gear is convex, then the profile of the driven gear must be concave. Table II-3 presents the concave/convex relationships.

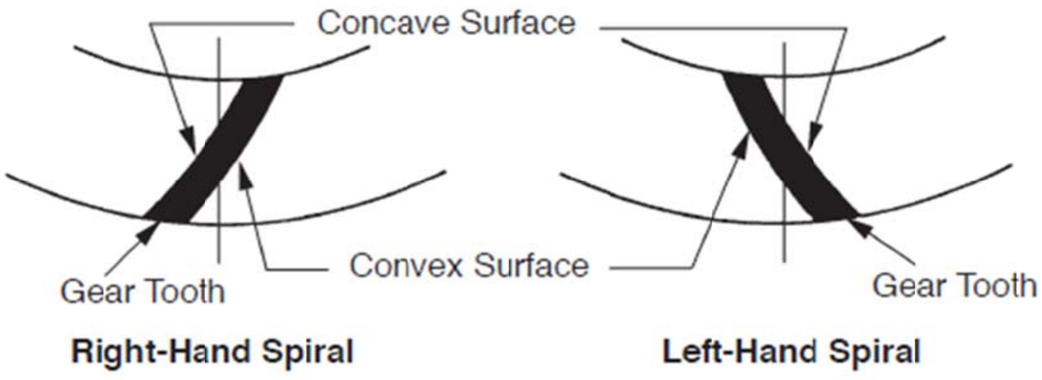

Fig. II–7 Convex Surface and Concave Surface of a Spiral Bevel Gear

Table II-3 Concave and Convex Sides of a Spiral Bevel Gear Mesh

Right-Hand Gear as Drive Gear

Rotational Direction of Drive Gear	Meshing Tooth Face	
	Right-Hand Drive Gear	**Left-Hand Driven Gear**
Clockwise	Convex	Concave
Counterclockwise	Concave	Convex

Left-Hand Gear as Drive Gear

Rotational Direction of Drive Gear	Meshing Tooth Face	
	Left-Hand Drive Gear	**Right-Hand Driven Gear**
Clockwise	Concave	Convex
Counterclockwise	Convex	Concave

NOTE: The rotational direction of a bevel gear is defined as the direction one sees viewed along the axis from the back cone to the apex.

II.5.1 Tooth Forces On A Convex Side Profile

The transmission force, F_n, can be resolved into components F_1 and F_t as (see Figure II–8):

$$F_1 = F_n \cos\alpha_n \qquad\qquad F_t = F_n \sin\alpha_n$$

Then F_1 can be resolved into components F_u and Fs:

$$F_u = F_1 \cos\beta_m \qquad\qquad F_s = F_1 \sin\beta_m$$

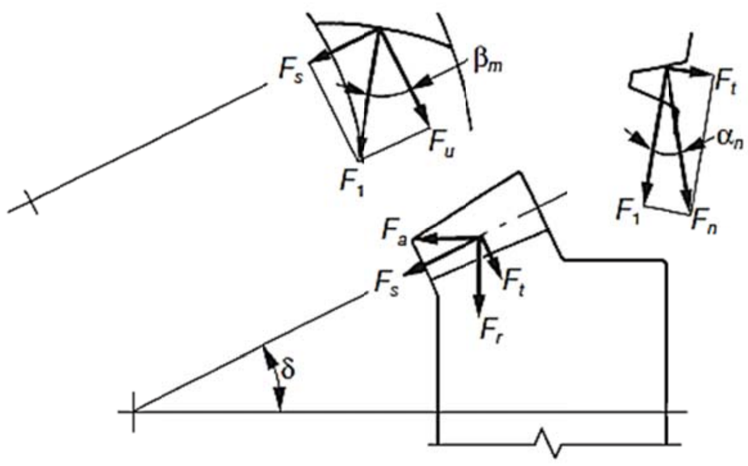

Fig. II–8 When Meshing on the Convex Side of Tooth Face

On the axial surface, F_t and Fs can be resolved into axial and radial subcomponents.

$$F_a = F_t sin\delta - F_s \cos\delta$$
$$F_a = F_t sin\delta + F_s \cos\delta$$

By substitution and manipulation, we obtain:

$$F_a = \frac{F_u}{\cos\beta_m}(\tan\alpha_n sin\delta - \sin\beta_m \cos\delta)$$

$$F_a = \frac{F_u}{\cos\beta_m}(\tan\alpha_n sin\delta + \sin\beta_m\cos\delta)$$

II.5.2 Tooth Forces On A Concave Side Profile

On the surface which is normal to the tooth profile at the central portion of the tooth, the transmission force, *Fn*, can be split into *F*1 and *Ft* as (see Figure II–9):

$$F_1 = F_n\cos\alpha_n \qquad F_t = F_n\sin\alpha_n$$

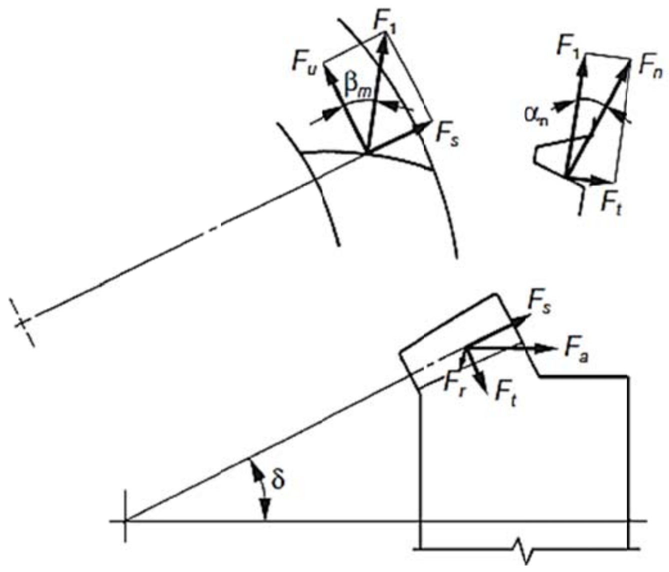

Fig. II–9 When Meshing on the Concave Side of Tooth Face

And F_1 can be separated into components F_u and *Fs* on the pitch surface:

$$F_u = F_1\cos\beta_m \qquad F_s = F_1\sin\beta_m$$

So far, the equations are identical to the convex case. However, differences exist in the signs for equation terms. On the axial surface, *Ft* and *Fs* can be resolved into axial and radial subcomponents. Note the sign differences.

$$F_a = F_t sin\delta + F_s\cos\delta$$
$$F_a = F_t sin\delta - F_s\cos\delta$$

The above can be manipulated to yield:

$$F_a = \frac{F_u}{\cos\beta_m}(\tan\alpha_n sin\delta + \sin\beta_m\cos\delta)$$
$$F_a = \frac{F_u}{\cos\beta_m}(\tan\alpha_n sin\delta - \sin\beta_m\cos\delta)$$

Let a pair of spiral bevel gears have a shaft angle Σ =90°, a pressure angle α_n = 20°, and a spiral angle β_m = 35°. If the tangential force, F_u, to the central portion of the tooth face is 100, the axial thrust force, F_a, and radial force, F_r, have the relationship shown in Table II-4.

The value of axial force, F_a, of a spiral bevel gear, from Table II-4, could become negative. At that point, there are forces tending to push the two gears together. If there is any axial play in the bearing, it may lead to the undesirable condition of the mesh having no backlash. Therefore, it is important to pay particular attention to axial plays. From Table II-4 **(2)**, we understand that axial thrust force, F_a, changes from positive to negative in the range of teeth ratio from 1.5 to 2.0 when a gear carries force on the convex side. The precise turning point of axial thrust force, F_a ,is at the teeth ratio z_1 / z_2 = 1.57357.

Table II-3 Values of Axial Thrust Force, Fa, and Radial Force, Fr

(1) Pinion

Meshing Tooth Face	Ratio of Number of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave Side of Tooth	$\frac{80.9}{-18.1}$	$\frac{82.9}{-1.9}$	$\frac{82.5}{8.4}$	$\frac{81.5}{15.2}$	$\frac{80.5}{20.0}$	$\frac{78.7}{26.1}$	$\frac{77.4}{29.8}$
Convex Side of Tooth	$\frac{-18.1}{80.9}$	$\frac{-33.6}{75.8}$	$\frac{-42.8}{71.1}$	$\frac{-48.5}{67.3}$	$\frac{-52.4}{64.3}$	$\frac{-57.2}{60.1}$	$\frac{-59.9}{57.3}$

(2) Gear

Meshing Tooth Face	Ratio of Number of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave Side of Tooth	$\frac{80.9}{-18.1}$	$\frac{75.8}{-33.6}$	$\frac{71.1}{-42.8}$	$\frac{67.3}{-48.5}$	$\frac{64.3}{-52.4}$	$\frac{60.1}{-57.2}$	$\frac{57.3}{-59.9}$
Convex Side of Tooth	$\frac{-18.1}{80.9}$	$\frac{-1.9}{82.9}$	$\frac{8.4}{82.5}$	$\frac{15.2}{81.5}$	$\frac{20.0}{80.5}$	$\frac{26.1}{78.7}$	$\frac{29.8}{77.4}$

Figure II–10 describes the forces for a pair of spiral bevel gears with shaft angle Σ = 90°, pressure angle α_n = 20°, spiral angle β_m = 35° and the teeth ratio, *u*, ranging from 1 to 1.57357.

Figure II–11 expresses the forces of another pair of spiral bevel gears taken with the teeth ratio equal to or larger than 1.57357.

$\Sigma = 90°$, $\alpha n = 20°$, $\beta m = 35°$, $u < 1.57357$.

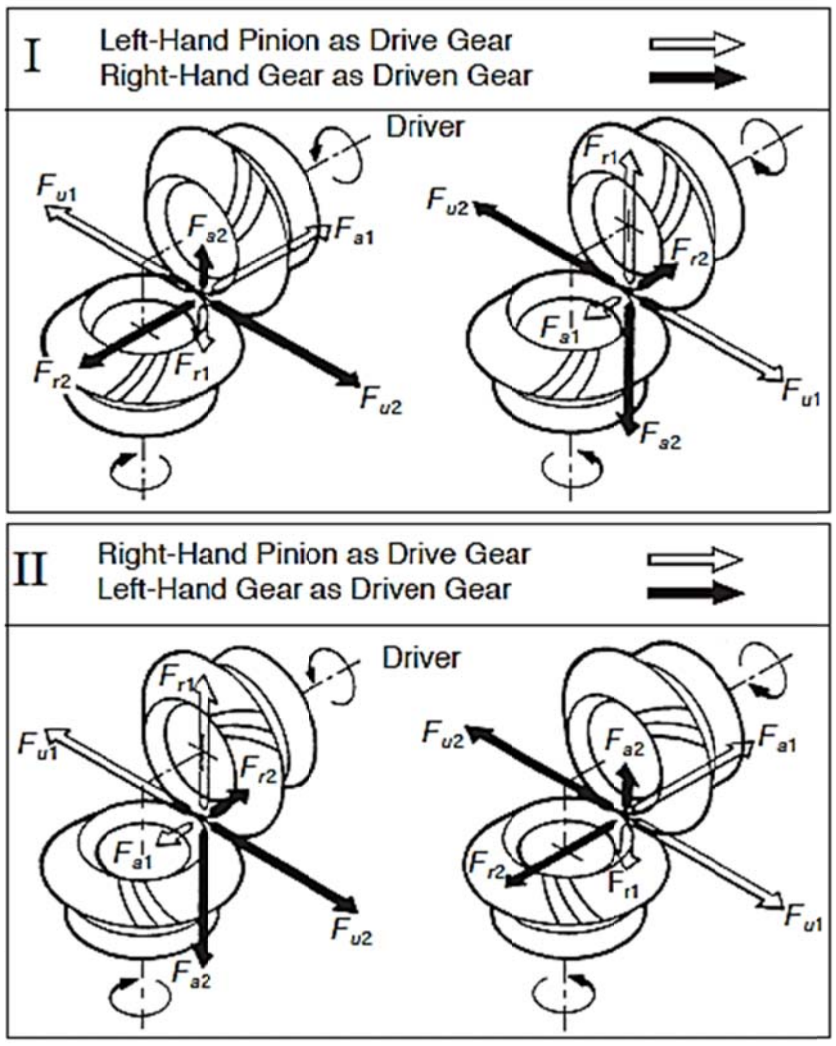

Fig. II–10 The Direction of Forces Carried by Spiral Bevel Gears (1)

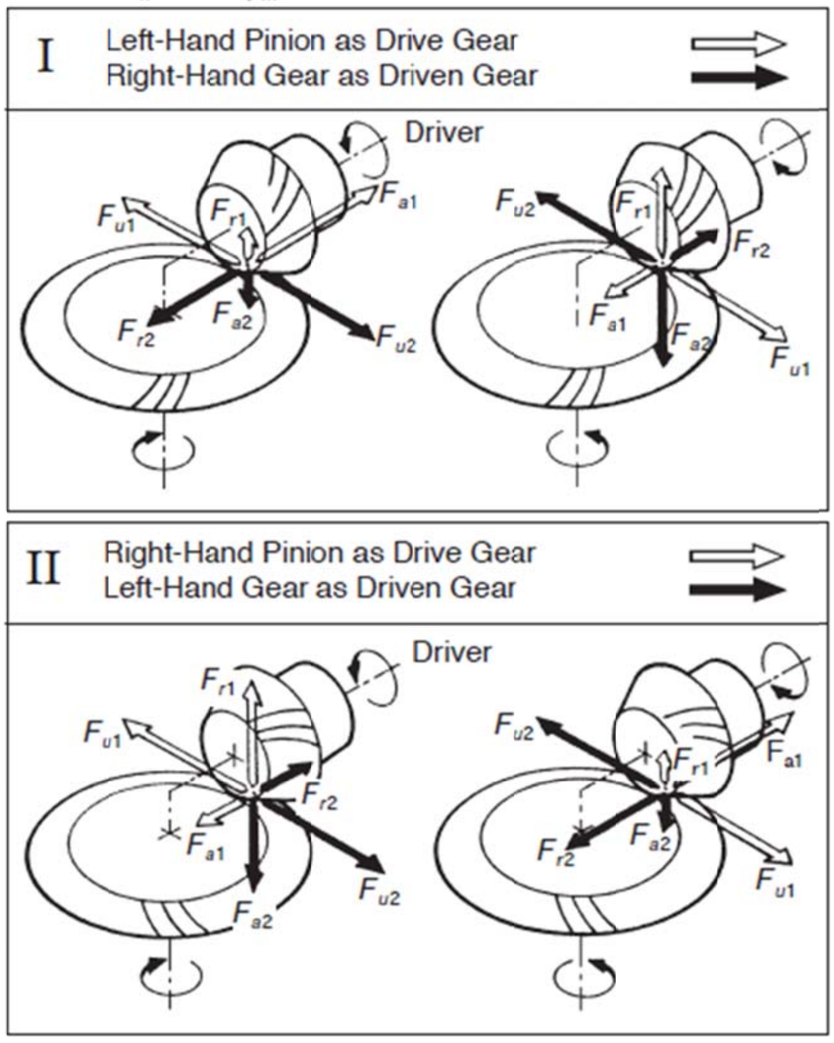

Fig. II–11 The Direction of Forces Carried by Spiral Bevel Gears (2)

II.6.Forces In A Worm Gear Mesh

II.6.1. Worm as the Driver

For the case of a worm as the driver, Figure II–12, the transmission force, F_n, which is normal to the tooth surface at the pitch circle can be resolved into components F_1 and Fr_1.

$$F_1 = F_n \cos \alpha_n \qquad\qquad F_{r1} = F_n \sin \alpha_n$$

At the pitch surface of the worm, there is, in addition to the tangential component, F_1, a friction sliding force on the tooth surface, μ*Fn*. These two forces can be resolved into the circular and axial directions as:

$$F_{u1} = F_1 \sin \gamma + F_n \mu \cos \gamma$$
$$F_{u1} = F_1 \sin \gamma - F_n \mu \cos \gamma$$

and by substitution, the result is:

$$F_{u1} = F_n (cos\alpha_n \sin \gamma + \mu \; cos\gamma)$$
$$F_{a1} = F_n (cos\alpha_n \cos \gamma + \mu \; sin\gamma)$$
$$F_{r1} = F_n sin\alpha_n$$

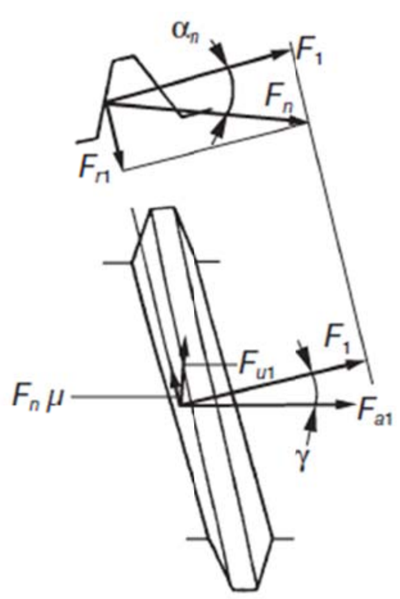

Fig. II–12 Forces Acting on the Tooth Surface of a Worm

Figure II–13 presents the direction of forces in a worm gear mesh with a shaft angle Σ = 90°. These forces relate as follows:

$F_{a1} = F_{u2}$, $F_{u1} = F_{a2}$, $F_{r1} = F_{r2}$

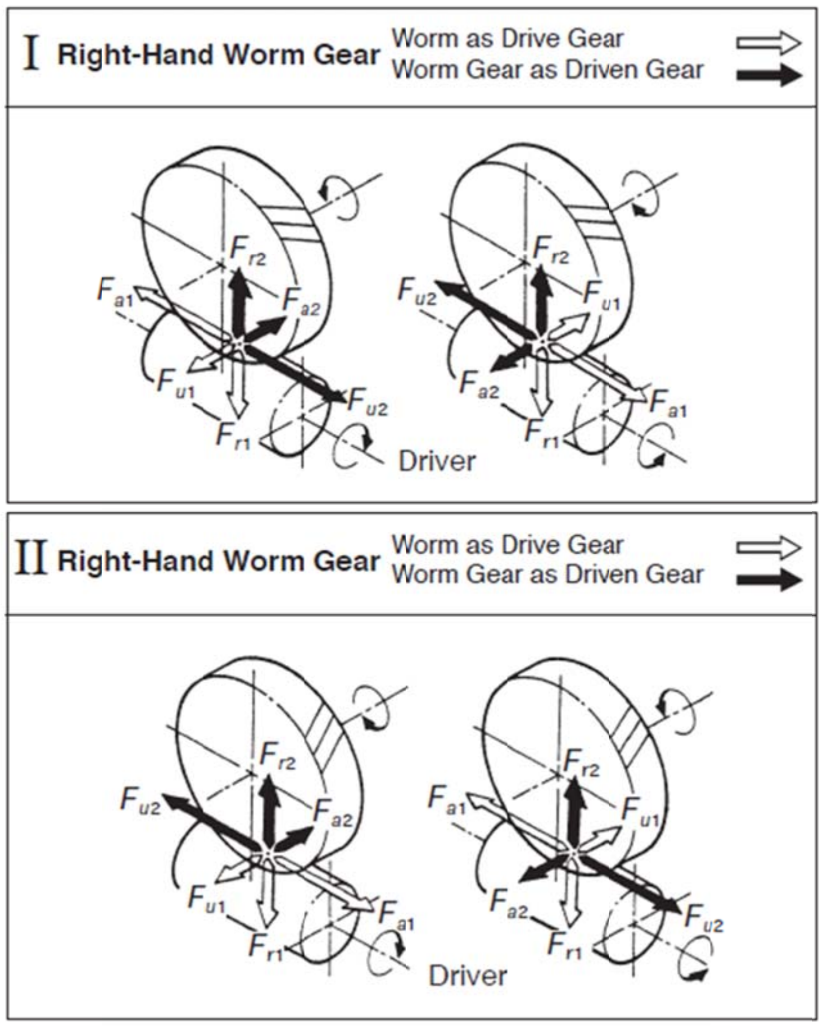

Fig. II–13 Direction of Forces in a Worm Gear Mesh

The coefficient of friction has a great effect on the transmission of a worm gear. **Bellow Equation** presents the efficiency when the worm is the driver.

$$\eta_R = \frac{T_1}{T_{1i}} = \frac{F_{u2}}{F_{u2}} tan\,\gamma = \frac{cos\alpha_n\,cos\gamma - \mu\,sin\gamma}{cos\alpha_n sin\gamma + \mu\,cos\gamma}\,tan\gamma$$

II.6.2. Worm Gear as the Driver

For the case of a worm gear as the driver, the forces are as in Figure II–14 and Bellow per **Equations**

$$F_{u2} = F_n(cos\alpha_n\,cos\gamma + \mu\,sin\gamma)$$
$$F_{a2} = F_n(cos\alpha_n\,sin\gamma - \mu\,cos\gamma)$$
$$F_{r2} = F_n\,sin\alpha_n$$

When the worm and worm gear are at 90° shaft angle, **bellow Equations** apply. Then, when the worm gear is the driver, the transmission efficiency η_I is expressed as bellow per.

$$\eta_R = \frac{T_{1i}}{T_2} = \frac{F_{u1}}{F_{u2}tan\,\gamma} = \frac{cos\alpha_n\,sin\gamma - \mu\,cos\gamma}{cos\alpha_n cos\gamma + \mu\,sin\gamma}\,(\frac{1}{tan\gamma})$$

The equations concerning worm and worm gear forces contain the coefficient μ. This indicates the coefficient of friction is very important in the transmission of power.

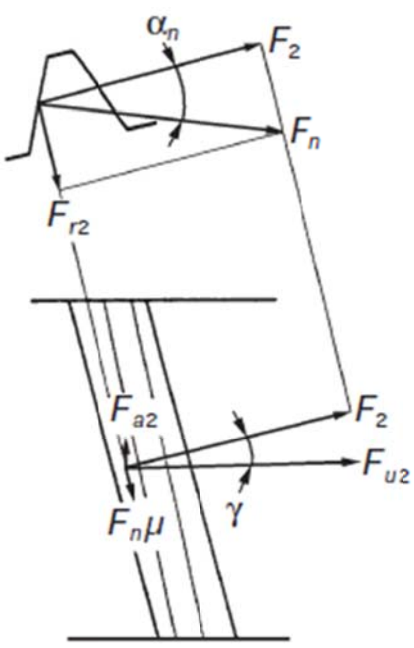

Fig. II–14 Forces in a Worm Gear Mesh

II.7.Forces In A Screw Gear Mesh

The forces in a screw gear mesh are similar to those in a worm gear mesh. For screw gears that have a shaft angle Σ = 90°, merely replace the worm's lead angle γ, , with the screw gear's helix angle β1.

In the general case when the shaft angle is not 90°, as in Figure II–15, the driver screw gear has the same forces as for a worm mesh. These are expressed in bellow Equations.

$$F_{u1} = F_n(cos\alpha_n \, cos\beta 1 + \mu \, sin\beta 1)$$
$$F_{a1} = F_n(cos\alpha_n \, sin\beta 1 - \mu \, cos\beta 1)$$
$$F_{r1} = F_n \, sin\alpha_n$$

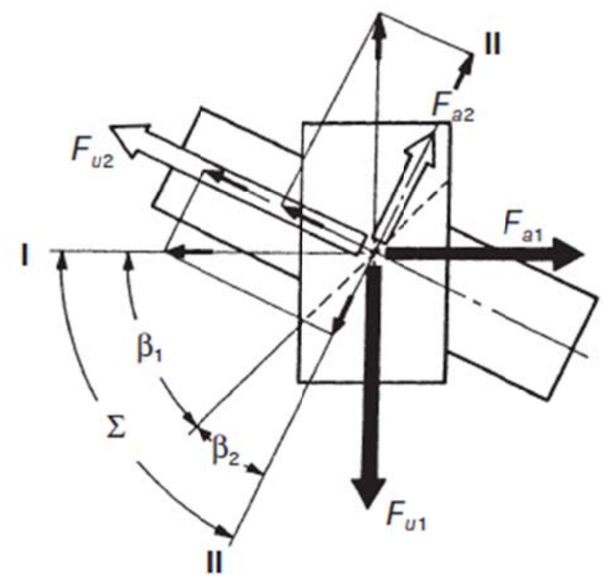

Fig. II–15 The Forces in a Screw Gear Mesh

Forces acting on the driven gear can be calculated by

$$F_{u2} = F_{a1} \sin\Sigma + F_{u1} \cos\Sigma$$
$$F_{u2} = F_{a1} \sin\Sigma - F_{u1} \cos\Sigma$$
$$F_{r2} = F_{r1}$$

If the Σ term **is** 90°, it becomes identical. Figure II–16presents the direction of forces in a screw gear mesh when the shaft angle Σ = 90° and β_1 = β_2 = 45°.

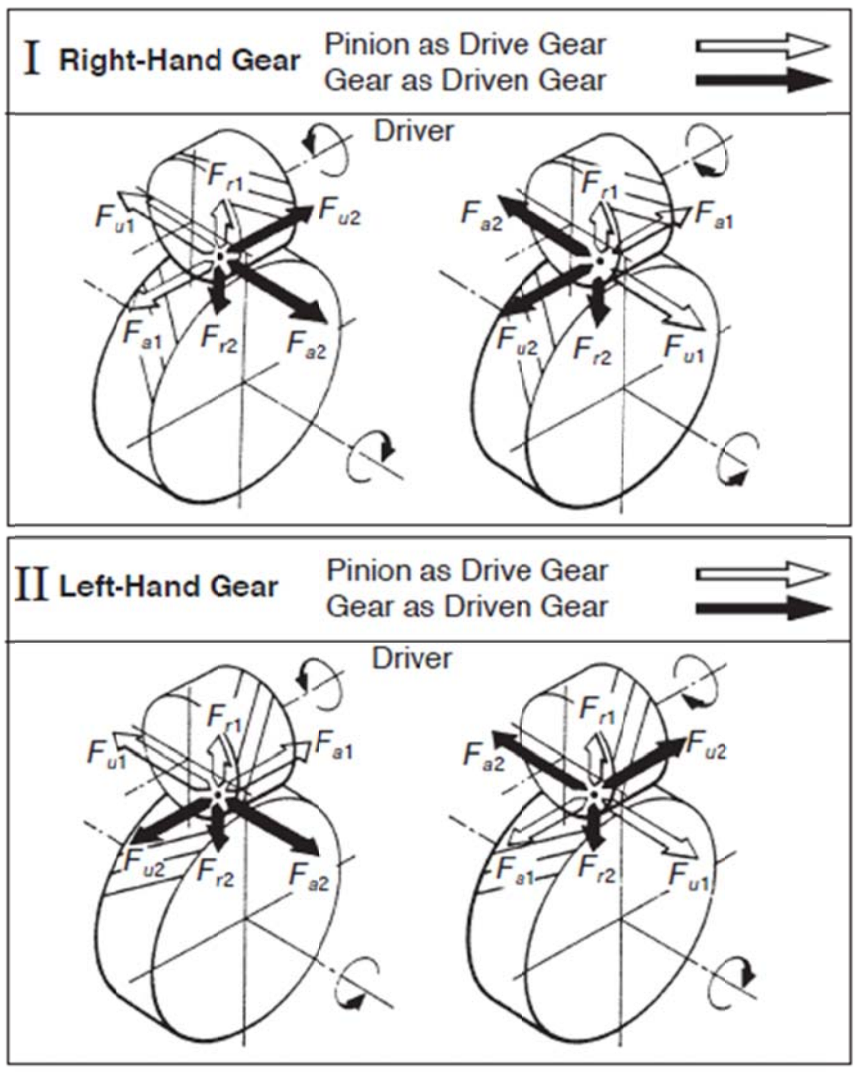

Fig. II–16 Directions of Forces in a Screw Gear Mesh